1/30/2014

BAYES NETS: I-Maps, P-maps.

1 I-map : Independency map

$I(G) = \{$ set of indep. assumptions encoded in $G\}$

$I(P) = \{$ set of independencies in $P\}$

If $I(G) \subseteq I(P) \Rightarrow G$ is an I-map of P

e.g

Coin 1	Coin 2	Prob P_1	Prob P_2
H	H	0.25	0.5
H	T	0.25	0
T	H	0.25	0
T	T	0.25	0.5

$I(P_1) = \{C_1 \perp C_2\}$ $\quad I(P_2) = \phi$

G_1: C_1 $\quad C_2$ $\qquad G_2$: $C_1 \longrightarrow C_2$

$I(G_1) = C_1 \perp C_2$ $\quad I(G_2) = \phi$

For P_1: both G_1 & G_2 are I-maps

For P_2: only G_2 is an I-map

(2) Minimal I-map: Defn

G is an I-map of P

& removing an edge from G makes it no longer an I-map of P

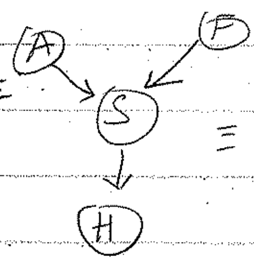
e.g say $I(P) = \{X \perp Y \mid Z\}$

$G: \; X \rightarrow Z \rightarrow Y$

drop this edge & you introduce an indep $X \perp Y$ & $Z \perp Y$ (Not true in P)

(2.5) ~~Mu~~ Minimal I-map not always 'simple' or unique

Say "true" model or $I(P) = I(G^*) =$

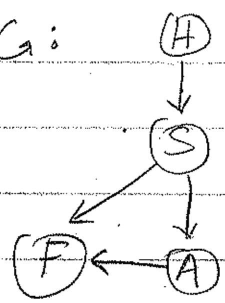


$\equiv A \perp F$

$A, F \perp H \mid S$

so on

Now consider G:



Now $I(G) \subseteq I(P)$

e.g $A, F \perp H \mid S \in I(G)$

but $A \perp F \notin I(G)$

But can't drop an edge from G (no longer an I-map)

So both G & G^* are minimal I-maps of P

but G is not very "natural" or "simple"

③ Perfect-Maps P-Map

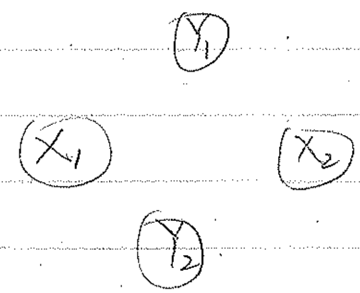
G is a P-map of distribution P

iff

$I(G) = I(P)$

However P-Maps don't always exist!

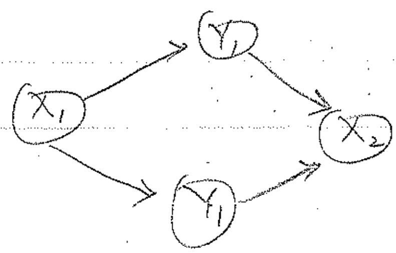
e.g



Say $I(P) = \{ X_1 \perp X_2 \mid Y_1, Y_2 ;$
$Y_1 \perp Y_2 \mid X_1, X_2 \}$

Can't construct a DAG

e.g



$X_1 \perp X_2 \mid Y_1, Y_2$

but $Y_1 \perp Y_2 \mid X_1, X_2$ Not true!

$\therefore$ v-structure!

This counter-example needs Markov Nets or MRFs (later in class!)