Noise-Gain Tradeoff in RF SiGe HBTs

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Abstract—**This work examines the tradeoff between noise figure and gain in SiGe HBT RF technology. Based on the linear noisy two-port theory, analytical expressions of the minimum noise figure, the optimum source admittance, and the associated gain are derived. For many current RF applications of SiGe HBTs operating at frequencies comparable to or smaller than $f_T/\sqrt{\beta}$, β must be increased through SiGe profile optimization to further reduce noise. This noise improvement, however, also results in a degradation of associated gain, as experimentally observed. The implications of the noise-gain tradeoff on RF integrated circuit design are discussed.**

Keywords—*SiGe HBT, RF noise, associated gain, bipolar technology, RF circuits.*

I. Introduction

Mobile wireless communication links demand very sensitive receiver circuits. Low-noise amplifiers are typically used to amplify the incoming RF signal to overcome the noise of subsequent stages, while introducing a minimum amount of noise [1]. Much research has been devoted to the understanding of transistor noise, and has led to effective guidelines for designing low-noise transistors [2]-[7]. In the case of bipolar transistors, a higher DC gain (β), a higher cut-off frequency (f_T), and a lower base resistance (r_b) are desired to reduce noise [2]-[7]. These requirements, however, often result in limiting constraints when translated into device design using Si bipolar technology. For instance, decreasing r_b necessarily causes a drop in β for a Si BJT. Thanks to the additional design freedom offered by bandgap engineering, SiGe HBTs can simultaneously achieve high β, high f_T, and low r_b, thus lessening these constraints.

An important but rarely addressed question is how the device optimization for low noise affects the power gain capability of the transistor. Ultimately, the main function of a LNA is to amplify the incoming RF signal while adding as little as possible noise. A noiseless transistor is useless, however, without sufficient gain, and vice versa. This work is aimed at answering the above fundamental question, and was largely motivated by the intriguing experimental observation that a Si BJT has higher (better) associated gain despite its higher (worse) noise figure than three types of SiGe HBTs designed explicitly for low noise (described below). Associated gain is defined as the highest power gain achievable when the transistor source is noise-matched for minimum noise figure, and represents the power gain capability of a transistor when used for low-noise amplifiers. The devices used in this work were fabricated using IBM's SiGe HBT RF technology, the details of which can be found in [8].

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II. Noise Parameters and Associated Gain

The RF noise performance of a transistor is characterized by the minimum noise figure NF_{min}, the optimum source admittance $Y_{s,opt}$, and the noise resistance R_n. The significance of each parameter can be understood by examining the noise figure equation for an arbitrary source admittance Y_s [9]:

$$NF = NF_{min} + \frac{R_n}{G_s}\left|Y_s - Y_{s,opt}\right|^2 \tag{1}$$

where G_s is the real part of Y_s. Noise figure reaches its minimum when $Y_s = Y_{s,opt}$, while R_n determines the sensitivity of noise figure to deviations from $Y_{s,opt}$.

The smallest possible NF_{min} is obviously desired. A small R_n is also desired when the source is intentionally terminated at a value different from $Y_{s,opt}$ in order to have a higher gain than at $Y_{s,opt}$. In general, the optimum Y_s for minimum noise figure (noise matching) differs from the optimum Y_s for maximum power transfer (gain matching). The "closeness" of noise matching and gain matching conditions determines the associated gain G_a, defined as the maximum available gain for a noise matching source termination ($Y_s = Y_{s,opt}$).

NF_{min}, $Y_{s,opt} = G_{s,opt} + jB_{s,opt}$, and R_n are functions of the input noise current i_n, the input noise voltage v_n, and their cross-correlation $v_n i_n^*$, all of which can be expressed using the physical noise sources and the Y-parameters, as detailed below. Denoting the spectral densities of i_n, v_n, and $i_n v_n^*$ as S_{in}, S_{vn}, and $S_{i_n v_n^*}$, one obtains the following equations for R_n, $G_{s,opt}$, $B_{s,opt}$ and NF_{min} from the linear noisy two-port theory [9]:

$$R_n = \frac{S_{vn}}{4kT} \tag{2}$$

$$G_{s,opt} = \sqrt{\frac{S_{in}}{S_{vn}} - \left[\frac{\Im(S_{invn^*})}{S_{vn}}\right]^2} \tag{3}$$

$$B_{s,opt} = -\frac{\Im(S_{invn^*})}{S_{vn}} \tag{4}$$

$$NF_{min} = 1 + 2R_n\left(G_{s,opt} + \frac{\Re(S_{invn^*})}{S_{vn}}\right) \tag{5}$$

where $\Re$ and $\Im$ stand for the real and imaginary parts, respectively.

The associated gain is obtained from the tranducer gain G_t under source noise matching $Y_S = Y_{s,opt}$ and load conjugate matching $Y_L = Y_{out}^*$. G_t is given by [10]:

$$G_t = \frac{4G_L G_S |Y_{21}|^2}{|Y_{11} + Y_S|^2 |Y_{out} + Y_L|^2} \tag{6}$$

Substituting $Y_L = Y_{out}^*$, and $Y_S = Y_{s,opt}$ into Eq. (6) leads to:

$$G_a = \left| \frac{Y_{21}}{Y_{11} + Y_{s,opt}^S} \right|^2 \frac{G_{s,opt}}{G_{out}} \tag{7}$$

where

$$G_{out} = \Re(Y_{out}) \tag{8}$$

and

$$Y_{out} = Y_{22} - \frac{Y_{12}Y_{21}}{Y_{11} + Y_{s,opt}} \tag{9}$$

To examine the impact of device parameters (e.g., β) on NF_{min} and G_a, we need to express the spectral densities of the input noise current and voltage, as well as the Y-parameters, in terms of these device parameters.

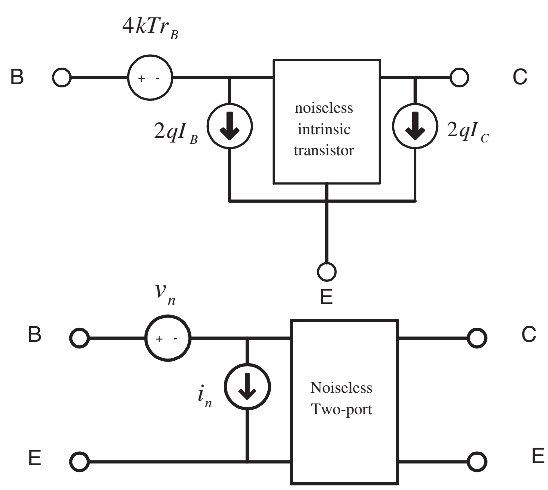

Fig. 1. Schematic of the noise sources in a transistor and its chain noisy two-port representation.

A. Input Noise Current and Voltage

The primary RF noise sources in a bipolar transistor are the base current shot noise $2qI_B$, the collector current shot noise $2qI_C$, and the base resistance-induced thermal noise $4kTr_B$, as shown in Fig. 1. The spectral densities of the input noise current (S_{in}), input noise voltage (S_{vn}), and their cross-correlation (S_{invn^*}) are given by [11]:

$$S_{in} = 2q\frac{I_C}{\beta} + \frac{2qI_C}{\left|\frac{Y_{21}}{Y_{11}}\right|^2} \tag{10}$$

$$S_{vn} = 4kTr_B + \frac{2qI_C}{|Y_{21}|^2} \tag{11}$$

$$S_{invn*} = \frac{2qI_C Y_{11}}{|Y_{21}|^2} \tag{12}$$

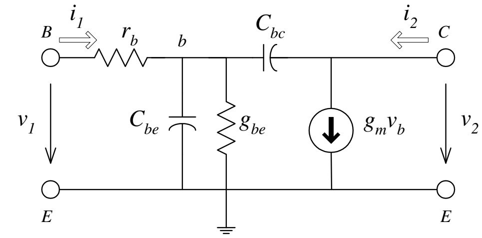

Fig. 2. Equivalent circuit for Y-parameter derivation.

B. Y-parameters

The Y-parameters can be expressed in terms of device parameters by examining the small-signal equivalent circuit shown in Fig. 2. At frequencies smaller than f_T, the base resistance is not important for the input impedance, and thus can be neglected for simplicity, even though it is significant as a noise voltage generator. The $I - V$ relation becomes:

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} g_{be} + j\omega(C_{be} + C_{bc}) & -j\omega C_{bc} \\ g_m - j\omega C_{bc} & j\omega C_{bc} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \tag{13}$$

The Y-parameters are thus obtained as:

$$Y_{11} = \frac{g_m}{\beta} + j\omega C_i \tag{14}$$

$$Y_{12} = -j\omega C_{bc} \tag{15}$$

$$y_{21} \approx g_m \tag{16}$$

$$y_{22} = j\omega C_{bc} \tag{17}$$

where $g_m = qkT/I_C$, $C_i = C_{be} + C_{bc}$. C_{be} consists of the EB depletion capacitance C_{te} and the EB diffusion capacitance $g_m\tau$ ($C_{be} = C_{te} + g_m\tau$), with τ being the transit time. C_i is related to f_T through $f_T = g_m/2\pi C_i$.

C. Noise Parameters and Associated Gain

S_{in}, S_{vn}, and S_{invn^*} can be expressed in terms of I_C (or g_m), β, C_i, and r_b by substituting Eqs. (14)-(17) into Eqs. (10)-(12):

$$S_{in} = 2qI_C \left[\frac{1}{\beta} + \left(\frac{\omega C_i}{g_m}\right)^2\right] \tag{18}$$

$$S_{vn} = 4kT\left(r_b + \frac{1}{2g_m}\right) \tag{19}$$

$$S_{invn^*} = 2kT\left(\frac{1}{\beta} + \frac{j\omega C_i}{g_m}\right) \tag{20}$$

where $I_C = g_m kT/q$ was used. With Eqs. (18)–(20), we can determine R_n, $G_{s,opt}$, $B_{s,opt}$, and NF_{min} from Eqs. (2)–(5):

$$R_n = r_b + \frac{1}{2g_m} \tag{21}$$

$$G_{s,opt} = \sqrt{\frac{g_m}{2R_n}\frac{1}{\beta} + \frac{(\omega C_i)^2}{2g_m R_n}\left(1 - \frac{1}{2g_m R_n}\right)} \tag{22}$$

$$B_{s,opt} = -\frac{\omega C_i}{2g_m R_n} \tag{23}$$

$$\begin{aligned} NF_{min} &= 1 + \frac{1}{\beta} + \sqrt{A} \\ A &= \frac{2g_m r_b + 1}{\beta} + \frac{2(r_b + \frac{1}{2g_m})(\omega C_i)^2}{g_m} \frac{2g_m r_b}{2g_m r_b + 1} \end{aligned} \tag{24}$$

Eq. (24) clearly indicates that a higher β, a lower C_i (hence a higher f_T), and a lower r_b are desired to reduce NF_{min}. In typical circuit applications, $g_m r_b \gg 1/2$, and Eq. (24) can be further simplified to:

$$NF_{min} = 1 + \frac{1}{\beta} + \sqrt{2g_m r_b}\sqrt{\frac{1}{\beta} + \left(\frac{f}{f_T}\right)^2} \tag{25}$$

The two terms inside the second square root become equal at $f = f_T/\sqrt{\beta}$, which defines a transition of NF_{min} from a white noise behavior (independent of frequency) to a 10dB/decade increase as the frequency increases. Current SiGe HBT RF technologies typically provide a peak f_T from 50–70GHz and a β of around 100. The operating frequencies of many wireless application systems, such as 950MHz for GSM, and 2.4GHz for Bluetooth, are well below $f_T/\sqrt{\beta}$, making NF_{min} low because $(f/f_T)^2 \ll 1/\beta$. As a result, the $1/\beta$ term inside the square root dominates, and a further increase of f_T does not help much in reducing NF_{min}. At a given technology generation, base doping and r_b is fixed, and the only practical way to further reduce noise is to increase β by SiGe profile optimization, because of the dominance of the β related terms. This is consistent with the results obtained from numerical simulation in [11].

D. Associated Gain

Finally, we can express the associated gain in terms of β, C_{bc}, C_i, g_m, and r_b by substitution of Eqs. (22) and (23) into Eqs. (7)–(9):

$$G_a = \frac{1}{\omega^2 C_{bc} C_i r_b}\sqrt{\frac{g_m r_b + 1/2}{2}\frac{g_m^2}{\beta} + \frac{(\omega C_i)^2}{2} g_m r_b} \tag{26}$$

where Eq. (21) was used. The derivation of Eq. (26) is mathematically complicated but conceptually straightforward. The G_a values directly calculated from Eq. (26) agree with the G_a values numerically calculated using Eqs. (7)–(9) and Eqs. (22)–(23), thus verifying the accuracy of the derivation. A number of very important observations can be made from Eq. (26):

1. G_a increases with I_C through the g_m term, indicating that we need a certain amount of I_C to have sufficient associated gain.

2. G_a *decreases* with increasing β through the first term inside the square root. This suggests that any reduction of noise figure due to β increase must result in a reduction of associated gain. The impact of β on G_a is important at lower frequencies, and becomes less important at higher frequencies.

3. G_a decreases with increasing frequency. At relatively lower frequencies, the first term inside the square root dominates, making the frequency dependence of G_a close to $1/\omega^2$ (or -20dB/decade).

4. At higher frequencies, the second term inside the square root dominates, making the frequency dependence close to $1/\omega$ (or -10dB/decade). An inspection of Eq. (26) shows that the two terms inside the square root become equal at $f = f_T/\sqrt{\beta}$.

5. For many current wireless applications of RF SiGe HBT technologies, such as GSM at 950MHz and Bluetooth at 2.4GHz, the operating frequency is either close to or far less than $f_T/\sqrt{\beta}$, as discussed above. The impact of β on G_a is therefore expected to be appreciable. Even at lower I_C values where f_T is far less than the peak (e.g. f_T=10GHz), these operating frequencies are comparable to $f_T/\sqrt{\beta}$. Therefore, β still has an appreciable role in determining G_a.

6. A smaller transit time is desired to increase G_a because of smaller C_i, and is consistent with conventional device design approaches for noise reduction.

7. A smaller CB capacitance C_{bc} is desired to increase G_a.

III. Experimental Results

Eq. (26) can be used to explain the experimental observation of higher associated gain in Si BJTs than in SiGe HBTs. Four devices considered include: a SiGe control HBT, two SiGe HBTs explicitly optimized for low noise (LN1 and LN2), and a Si BJT [11]. All of the devices were fabricated in the same wafer lot under identical processing conditions, and have the same base doping level.

The SiGe profiles in LN1 and LN2 HBTs were optimized explicitly to reduce NF_{min} at 2GHz, which is a relatively "low" frequency compared to $f_T/\sqrt{\beta}$. An inspection of Eq. (25) shows that the terms associated with β dominates NF_{min} at such low frequencies. As a result, the only practical way to reduce NF_{min} is to increase β by SiGe profile optimization (r_b is fixed at a given technology generation). To keep the total amount of Ge within the stability constraint, the retrograding of Ge into the collector has to be reduced, making the high injection f_T roll-off worse. However, by careful simulation, the high injection f_T roll-off in LN1 and LN2 was minimized such that NF_{min} was improved without sacrificing the peak f_T and f_{max} [11].

Compared to the control SiGe HBT, LN1 and LN2 have a higher average Ge content and a higher Ge gradient in the neutral base, but less Ge retrograding into the collector. Consequently, LN1 and LN2 have much higher β than the control HBT, a slightly higher f_T, but a stronger (worse) f_T roll-off at high injection. The higher β and f_T translate into a reduction of NF_{min}. The peak Ge mole fraction is 10%, 14%, and 18% in the SiGe control, LN1 and LN2 HBTs, respectively. Fig. 3 shows the doping and Ge profiles for the 18% peak Ge low-noise profile LN2 measured by SIMS.

The noise parameters (NF_{min}, $Y_{s,opt}$ and R_n) and associated gain (G_a) were measured at 2GHz for a $0.5 \times 20 \times 2\mu m^2$ unit cell using a NP-5B noise measurement system from ATN Microwave. Fig. 4 shows the measured NF_{min} versus I_C at 2GHz. The measured NF_{min} of LN1 and LN2 HBTs are 0.2dB lower than that of the control SiGe HBT. The Si BJT has the highest NF_{min} because of its lower β and higher τ, as expected. Fig. 5 shows the measured associated gain as a function of I_C. Interestingly, the Si BJT has the highest associated gain at lower current densities where LNAs are typically biased in this technology (e.g., $I_C = 2mA$), despite its higher (worse) minimum noise figure. This can be understood using the associated gain expression given in Eq. (26).

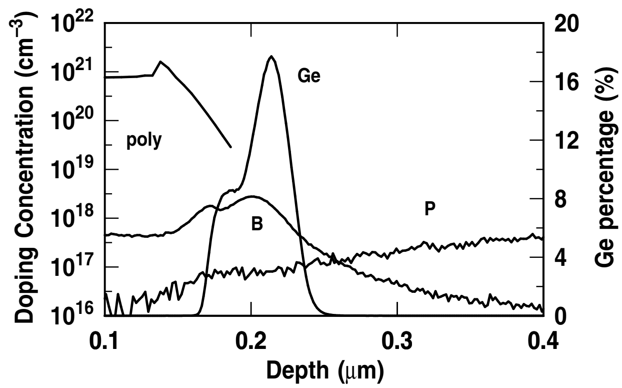

Fig. 3. Measured SIMS doping and Ge profiles for the fabricated SiGe low noise profile LN2.

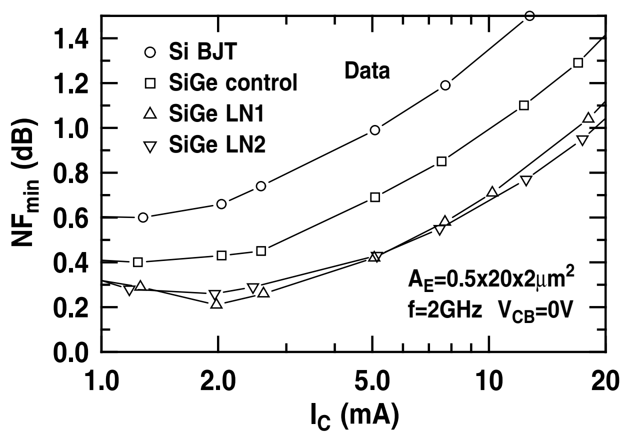

Fig. 4. Measured (NF_{min}) versus I_C at 2GHz for the Si BJT, SiGe control, and two low-noise SiGe HBTs.

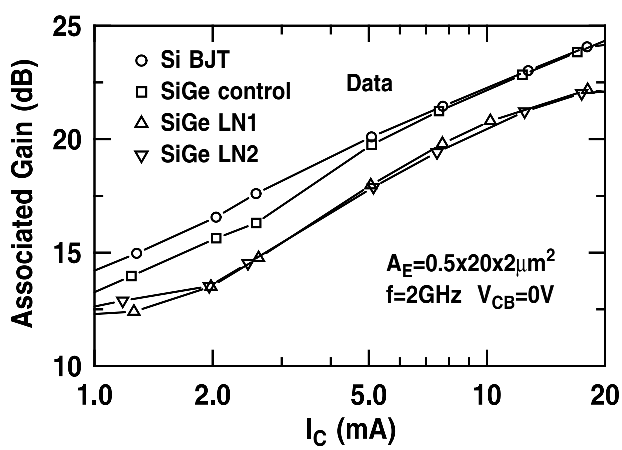

Fig. 5. Measured associate gain versus I_C at 2GHz for the Si BJT, SiGe control, and two low-noise SiGe HBTs.

Fig. 6 shows NF_{min} versus I_C calculated using Eq. (24). Device parameters β, τ, C_{te}, C_{bc} and r_b were extracted from the measured DC and S-parameters from 500MHz–40GHz, and were shown in Table I. The parameter extraction method of [12] was used. The measured β is I_C dependent, and the dependence varies with SiGe profile. The β at I_C=2mA are shown in Table I. The calculated NF_{min} values are higher than the measured NF_{min} values. The relative comparison between transistor profiles, however, is well-predicted by Eq. (24). Fig. 7 shows G_a versus I_C calculated using Eq. (26). The calculated G_a values are higher than the measured G_a values. The calculated I_C dependence, and relative comparison between devices, however, are close to the experimental results. One source of discrepancy may come from the $g_m = qI_C/kT$ assumption, while g_m is less than qI_C/kT in reality for the bias considered.

TABLE I
DEVICE PARAMETERS USED FOR NF_{min} AND G_a CALCULATION.
$A_E = 0.5 \times 20 \times 2\mu m^2$.

Parameter	Si BJT	SiGe control	SiGe LN1	SiGe LN2
β (I_C=2mA)	55	80	245	280
$C_{bc}(pF)$	0.1	0.1	0.1	0.1
$C_{te}(pF)$	0.27	0.29	0.26	0.28
r_b (Ω)	7.0	7.0	7.0	7.0
τ (ps)	3.9	2.86	2.7	2.7

The inherent reduction of G_a due to the β increase used to improve NF_{min} is not necessarily a disadvantage, for the following

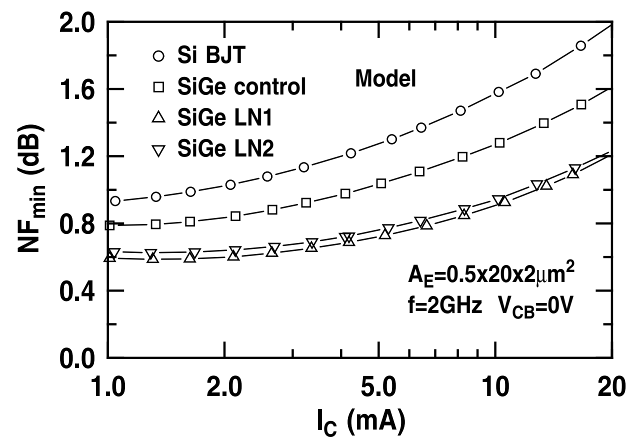

Fig. 6. Calculated NF_{min} versus I_C at 2GHz for the Si BJT, SiGe control, and two low-noise SiGe HBTs.

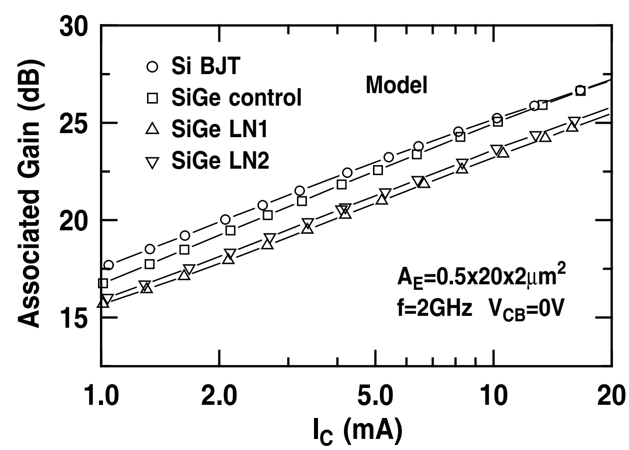

Fig. 7. Calculated associate gain versus I_C at 2GHz for the Si BJT, SiGe control, and two low-noise SiGe HBTs.

reasons:

1. Despite the inevitable noise-gain tradeoff, there is still sufficient gain (13dB for 1mA I_C) to overcome the noise of the following receiver stages, which is typically limited by the noise of the downconverting mixer.

2. The mixers fabricated using the same low-noise SiGe HBTs naturally have lower noise, as demonstrated by the measured circuit results [13], thus reducing the gain requirement on the low-noise amplifiers.

3. A higher β also effectively reduces the $1/f$ noise, thus reducing the residual phase noise of the LNA, which is of great importance for direct conversion receivers [14], as well as the oscillator phase noise [15] [16]. These additional advantages of using low-noise, high-β SiGe HBTs need to be taken into account in designing RF integrated circuits.

IV. Conclusion

We have examined the tradeoff between noise figure and associated gain in a SiGe HBT RF technology. The minimum noise figure NF_{min} and associated gain G_a were both expressed as a function of fundamental transistor parameters, including β, C_{be}, C_{bc}, g_m, r_b, and τ. A higher β decreases (improves) NF_{min}, but necessarily decreases (degrades) G_a as well. A lower C_{bc}, a smaller τ, and a lower r_b are desired to improve either NF_{min} or G_a. The frequency dependence of G_a was shown to be close to -20dB/decade at lower frequencies, and close to -10dB/decade at higher frequencies. For many current RF applications of SiGe HBT technology, the operating frequency is comparable to or smaller than $f_T/\sqrt{\beta}$, making the increase of β through SiGe profile optimization the only practical way to further reduce noise for a given technology generation. This noise improvement, however, also results in an inherent degradation of associated gain. The derived model was used to explain the experimental observation that a Si BJT can have a higher associated gain than a SiGe HBT, despite its worse noise figure.

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References

[1] T.H. Lee, *The Design of CMOS Radio-Frequency Integrated Circuits*, Cambridge University Press, 1998.

[2] E.G. Nielson, "Behavior of noise figure in junction transistors," *Proc. IRE*, vol. 45, no. 7, pp. 957-963, July 1957.

[3] A. van der Ziel, "Noise in solid-state devices and lasers," *Proc. IEEE*, vol. 58, no. 8, pp. 1178-1206, August 1970.

[4] R.J. Hawkins, *Solid-State Electronics*, vol. 20, no. 3, p. 191, 1977.

[5] H. Fukui, *Low-Noise Microwave Transistors and Amplifiers*, New York, IEEE Press, 1981.

[6] F. Herzel and D. Heinemann, "A novel approach to HF-characterization of heterojunction bipolar transistors," in *Simulation of Semiconductor Devices and Processes*, H. Ryssel and P. Pichler, Eds., vol. 6, pp. 88-101, 1995.

[7] S.P. Voinigescu, M.C. Maliepaard, J.L. Showell, G.E. Babcock, D. Marchesan, M. Schroter, P. Schvan, and D.L. Harame, "A scalable high-frequency noise model for bipolar transistors with application to optimal transistor sizing for low-noise amplifier design," *IEEE Journal of Solid-State Circuits*, vol 32, no. 9, pp. 1430-1438, September 1997.

[8] S. Subbanna *et al.*, "Integration and Design Issues in Combining Very-High-Speed Silicon-Germanium Bipolar Transistors and ULSI CMOS for System-on-a-Chip Applications," *Tech. Dig. IEDM*, pp. 845-848, 1999.

[9] H.A. Haus *et al.*, "Representation of noise in linear twoports," *Proc. IRE*, vol. 48, no. 8, pp. 69-74, Jan 1960.

[10] R.S. Carson, *High Frequency Amplifiers*, New York:Wiley, 1975.

[11] G.F. Niu, S. Zhang, J.D. Cressler, A.J. Joseph, J.S. Fairbanks, L.E. Larson, C.S. Webster, W.E. Ansley, and D.L. Harame, "Noise modeling and SiGe profile design tradeoffs for RF applications," *IEEE Trans. Electron Devices*, pp. 2037-2044, 2000.

[12] P. Baureis, and D. Seitzer, "Parameter extraction for HBT's temperature dependent large signal equivalent circuit model," *Tech. Dig. GaAs IC Symposium*, pp. 263-266, 1993.

[13] L. Sheng, J. Jensen, and L. Larson, "A Si/SiGe HBT sub-harmonic mixer/downconverter," *Proc. IEEE BCTM*, pp. 71-74, 1999.

[14] A. Abidi, "Low-power radio frequency IC's for portable communications," in *RF/Microwave Circuit Design for Wireless Communications*, L. Larson Ed. Norwood, MA: Artech House, 1996, pp. 43-98.

[15] D.B. Leeson, "A simple model of feedback oscillator noise spectrum," *Proc. IEEE*, vol. 54, no. 2, pp. 329-330, Feb 1966.

[16] G. Niu, Z. Jin, J.D. Cressler, R. Rapeta, A. Joseph, and D. Harame, "Transistor noise in SiGe HBT RF technology," to appear in *IEEE Journal of Solid-State Circuits*, 2001.