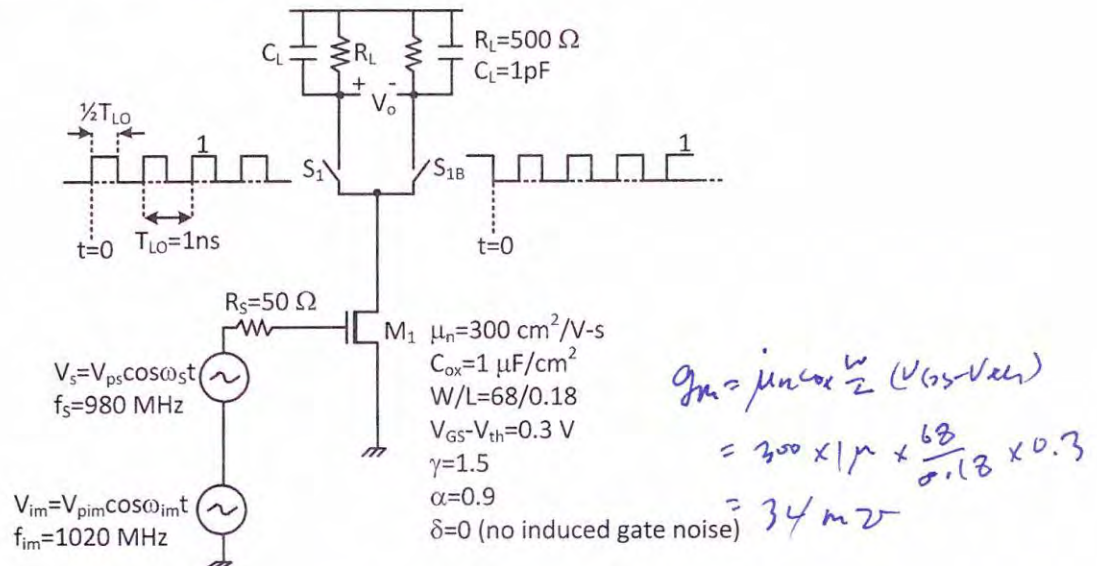


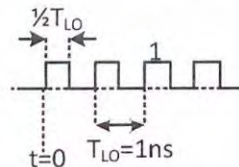
1. In the mixer shown below, DC characteristic of the NMOS, M1, is set by square-law characteristic, i.e.

$$I_{DS} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2.$$

Assume that $R_s \ll 1/\omega C_g$ and NMOS has only drain thermal noise current, i.e., no gate induced noise and parasitic gate resistance. The drain thermal noise coefficient is γ . S_1 and S_{1B} are ideal differential switches driven by ideal rectangular pulse trains whose duty cycle is 50%. R_L is noisy load resistor.



*You may need this series expression for question #1.



$$\text{rect}(t) = \frac{1}{2} + \sum_{n=1}^{n=\infty} \left(\frac{2}{(2n-1)\pi} \sin(2n-1)\omega_{LO}t \right)$$

- 1) Assume no image signal ($V_{pim}=0$) and $V_{ps}=1mV_{peak}$. Calculate **rms** output signal power (S_o) and output noise power ($N_o/\Delta f$) density and noise factor F (15 pt).

(Note: $K=1.38 \times 10^{-23}$ J/K, $T=300$ K)

$$V_o = \frac{2}{\pi} g_m R_L V_p \rightarrow V_o^2 = \left(\frac{2}{\pi} g_m R_L V_p \right)^2$$

$$\rightarrow \overline{V_o^2} = \left(\frac{2}{\pi} g_m R_L V_p \right)^2 \frac{1}{2}$$

$$\rightarrow S_o = \overline{V_o^2} \cdot \frac{1}{R_L} = \left(\frac{2}{\pi} g_m R_L V_p \right)^2 \frac{1}{2 R_L}$$

$$= \left(\frac{2}{\pi} \cdot 34m \cdot 500 \cdot 1m \right)^2 \cdot \frac{1}{2 \cdot 500} = \underline{117.13 nW}$$

$$= -39.31 dBm$$

$$\overline{V_{no}}^2 = 4KT \left(R_s \cdot g_m^2 + \frac{r}{\alpha} \cdot g_m + \frac{2}{R_L} \right) R_L^2$$

$$= 4 \cdot 1.38 \times 10^{-23} \cdot 300 \left(50 \cdot (34m)^2 + \frac{1.5}{0.9} \cdot 34m + \frac{2}{500} \right) 500^2$$

$$= \underline{490.45 \times 10^{-18} V_{rms}^2 / Hz}$$

- 2) Repeat 1) when W/L increased twice while maintaining same bias current (10 pt).

$$F = \frac{1}{\left(\frac{2}{\pi} g_m R_L \right)^2} \cdot \frac{\overline{V_{no}}^2}{\overline{V_{ni}}^2} = \frac{1}{\left(\frac{2}{\pi} g_m R_L \right)^2} \cdot \frac{4KT \left(R_s \cdot g_m^2 + \frac{r}{\alpha} g_m + \frac{2}{R_L} \right) R_L^2}{4KT R_s}$$

$$= \left(\frac{\pi}{2} \right)^2 \left(1 + \frac{r}{\alpha} \cdot \frac{1}{g_m R_s} + \frac{2}{g_m^2 R_L R_s} \right) = \underline{5.057} \Rightarrow \underline{7.04 dB}$$

$$W/L \uparrow \times 2 \rightarrow g_m \uparrow \times \sqrt{2}$$

$$\Rightarrow S_o \uparrow \times 2$$

$$\Rightarrow F_{new} = \left(\frac{\pi}{2} \right)^2 \left(1 + \frac{r}{\alpha} \cdot \frac{1}{g_m R_s} \cdot \frac{1}{\sqrt{2}} + \frac{2}{g_m^2 R_L R_s} \times \frac{1}{\sqrt{2}} \right) = \underline{4.26}$$

$$\Rightarrow \underline{6.3 dB}$$

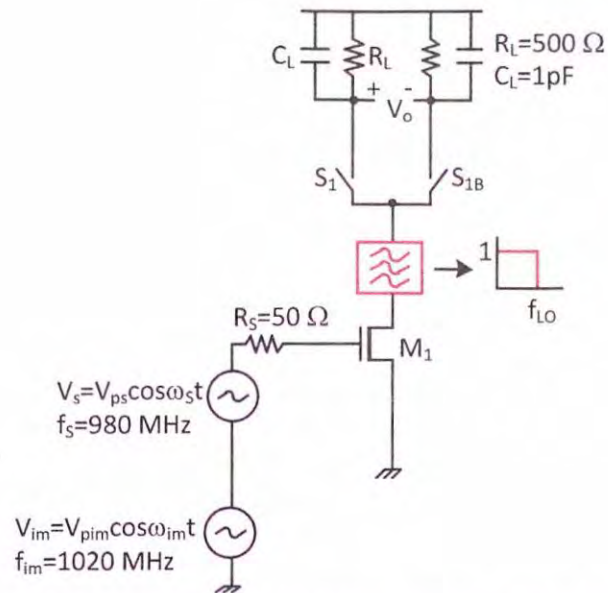
- 3) What's output signal power if the image tone has same magnitude as signal, i.e., $V_{pim}=V_{ps}=1mV_{peak}$ (5 pt) ?

$$V_o = \frac{4}{\pi} \left(\sin \omega_{Lo} t \cdot V_{ps} \cos \omega_{st} + \sin \omega_{Lo} t \cdot V_{pim} \cos \omega_{im} t \right)$$

$$= \frac{2}{\pi} V_{ps} \left(\underbrace{\sin(\omega_{Lo} - \omega_{st}) t}_{= +\omega_{IF}} + \underbrace{\sin(\omega_{Lo} - \omega_{im}) t}_{= -\omega_{IF}} \right)$$

$$= \frac{2}{\pi} V_{ps} \left(\sin \omega_{IF} t - \sin \omega_{IF} t \right)$$

$$= 0.$$



- 4) Now, in order to remove the image tone, let's apply ideal brick-wall type low pass filter (LPF) as shown above. Calculate output noise power and noise factor. Assume that the LPF is noiseless (5 pt).

(Note: the image rejection filter will also reject noise and help to improve SNR. In general, the M1 can be regarded as LNA and the switches (S1 and S1B) as mixer, respectively. You could understand the role of a filter between LNA and mixer through this example.)

$$\overline{V_{no}}^2 = \left(\frac{2}{\pi}\right)^2 \cdot 4KT \left(R_S \cdot g_m^2 + \frac{1}{2} g_m \right) R_L^2 + \frac{4KT R_L \times 2}{\text{No-filtering of } R_L \text{ noise}}$$

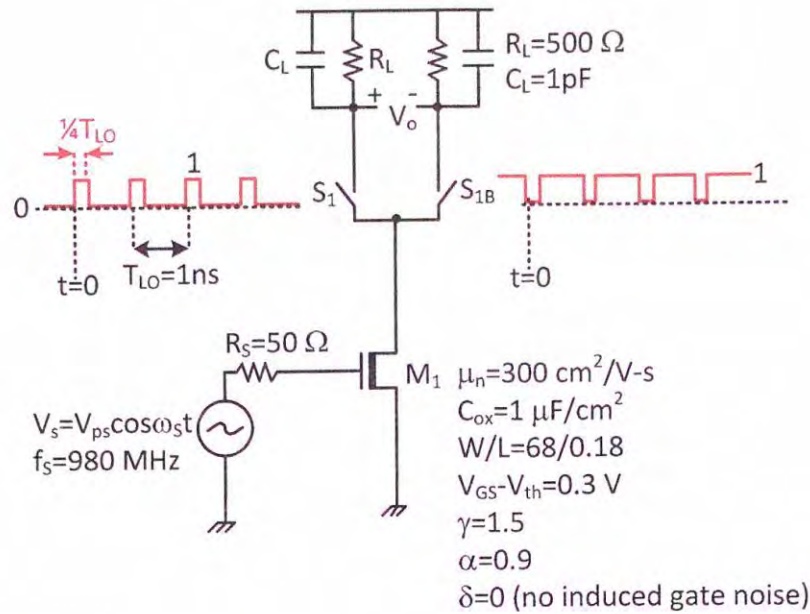
\uparrow
 due to filtering

$$= 208.62 \times 10^{-18} \text{ V}_{rms}^2 / \text{Hz}$$

$$F = \frac{1}{\left(\frac{2}{\pi} g_m R_L\right)^2} \cdot \frac{\overline{V_{no}}^2}{\overline{V_{ni}}^2}$$

$$= \frac{1}{\left(\frac{2}{\pi} \cdot 3.4 \text{ m} \times 500\right)^2} \cdot \frac{208.62 \times 10^{-18}}{828 \times 10^{-21}}$$

$$= 2.15 \Rightarrow 3.327 \text{ dB} \Rightarrow \text{about 3 dB improvement.}$$



- 5) Now let's change the LO clock duty cycle from 50% to 25% as shown above, and all the other conditions are same as 1) and no image signal. Calculate **rms** output signal power (S_o) and output noise power (N_o) and noise factor F (15 pt).
(Note: non-50% duty cycle also generates RF leakage to IF output node due to DC component of LO waveforms.)

Fundamental tone

$$50\% \rightarrow \frac{2}{\pi}$$

$$25\% \rightarrow \frac{2}{\pi} \sin\left(\pi \cdot \frac{1}{4}\right) = \frac{2}{\pi} \cdot \frac{1}{\sqrt{2}}$$

$$\therefore S_{o, \text{new}} = S_{o, \text{old}} \times \left(\sin \frac{\pi}{4}\right)^2 = 117.13 \text{ nW} \times \frac{1}{2} = 58.565 \text{ nW}$$

Noise power will be the same as in 50% duty-cycle

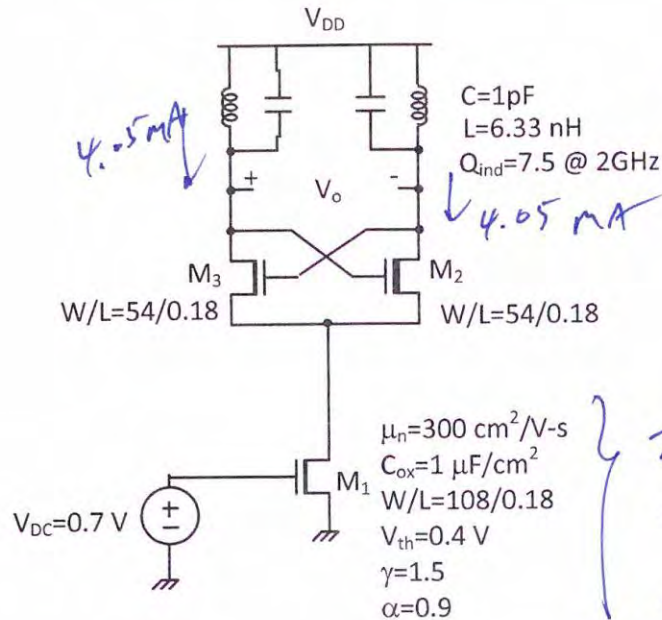
$$\therefore F_{\text{new}} = F_{\text{old}} \times \frac{1}{\left(\sin \frac{\pi}{4}\right)^2} = \frac{5.057}{1/2} = 10.114$$

$$\Rightarrow 10.049 \text{ dB}$$

2. In the VCO shown below, DC characteristic of the NMOS, M1-3, is set by square-law characteristic,

$$I_{DS} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2.$$

Assume that C_{gs} of M2-3 is negligible compared with C in the LC-tank. For simplicity, let's further assume that dominant noise sources are M1 and parasitic resistance from LC-tank, and noises from M2-3 are negligible. M1 has only drain thermal noise current, i.e., no gate induced noise and parasitic gate resistance.



$$\begin{aligned} Q_{ind} &= 7.5 \\ R_{ind} &= Q_{ind} \cdot \omega_{osc} \cdot L \\ &= 7.5 \times 2\pi \cdot 2 \text{ GHz} \cdot 6.33 \text{ nH} \\ &= 596.59 \Omega \end{aligned}$$

$$\begin{aligned} I_{DC} &= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th})^2 \\ &= \frac{1}{2} \cdot 300 \times 1 \mu \times \frac{108}{0.18} \times 0.3^2 \\ &= 8.1 \text{ mA} \end{aligned}$$

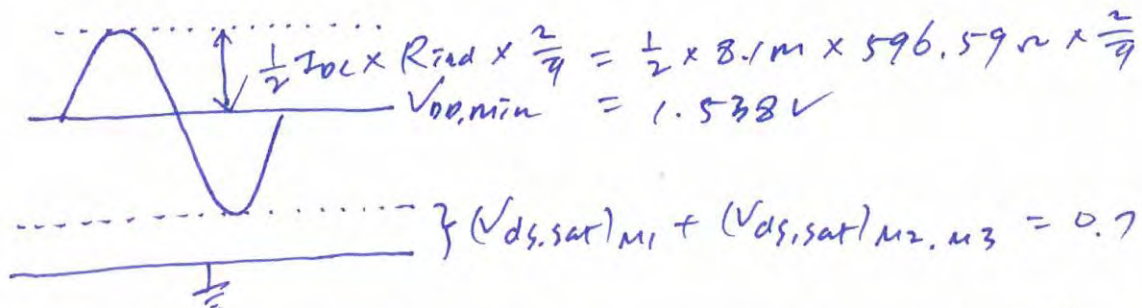
- 1) Calculate the possible maximum output signal swing in peak-to-peak value and set minimum supply voltage V_{DD} that allows the maximum output swing (5 pt).

(Note: Each NMOS needs at least minimum $V_{ds,sat}$ during full output swing excursion to sustain oscillation.)

$$(V_{ds,sat})_{M1} = V_{GS} - V_{th} = 0.7 - 0.4 = 0.3 \text{ V}$$

$$\begin{aligned} (V_{ds,sat})_{M2, M3} @ I_{DS} = 8.1 \text{ mA} &= (V_{ds,sat})_{M1} \times \sqrt{2} \\ &= 0.424 \text{ V} \end{aligned}$$

because size of $M2, M3$ is half of $M1$.



$$\therefore V_{DD, min} = 1.538 + 0.724 = 2.262 \text{ V}$$

- 2) Under the bias condition determined in 1), calculate phase noise contributions from parasitic resistance from inductor at 10 MHz offset frequency (10 pt).

$$\begin{aligned}
 (\overline{\Phi_n^2})_{\text{due to } R_{\text{ind}}} &= \left(\frac{1}{2Q \frac{\omega}{\omega_0}} \right)^2 \times \frac{4kT R_{\text{ind}} \times 2}{\frac{1}{2} V_p^2 \times 4} \quad \text{differential \& no-correlation} \\
 &= \left(\frac{1}{2 \times 7.5 \frac{2\pi}{10\text{M}}} \right)^2 \times \frac{4 \times 1.38 \times 10^{-23} \times 300 \times 596.6 \times 2}{\frac{1}{2} \cdot (1.538)^2 \times 4} \quad \text{differential \& correlation} \\
 &= 742.52 \times 10^{-18} \text{ rad}^2/\text{Hz} \\
 &= -151.29 \text{ dBc/Hz @ 10 MHz offset}
 \end{aligned}$$

- 3) Under the bias condition determined in 1), calculate phase noise contributions from M1 at 10 MHz offset frequency (10 pt)

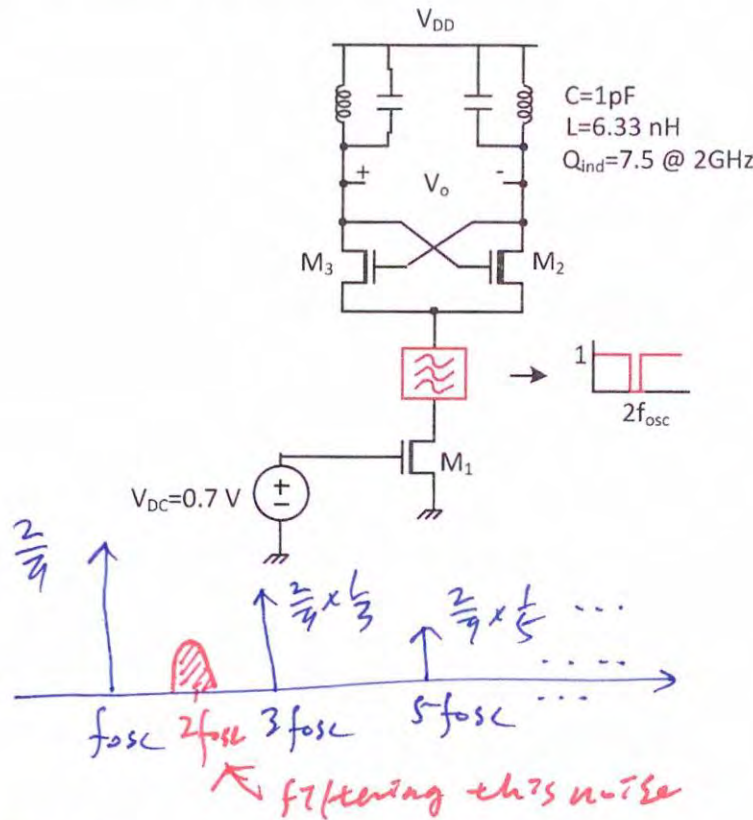
(Note: You don't need to calculate phase noise contribution due to flicker noise of M1, since at 10MHz offset dominant noise source will be thermal noise of M1.)

$$\begin{aligned}
 (\overline{\Phi_n^2})_{\text{due to } M1} &= \left(\frac{1}{2Q \frac{\omega}{\omega_0}} \right)^2 \times \frac{4kT \frac{r}{2} g_{m1} \cdot R_{\text{ind}}}{\frac{1}{2} V_p^2 \times 4} \\
 &= \left(\frac{1}{2 \times 7.5 \frac{2\pi}{10\text{M}}} \right)^2 \times \frac{4 \times 1.38 \times 10^{-23} \times \frac{1.5}{0.9} \times 54\text{m} \times 596.59}{\frac{1}{2} (1.538)^2 \times 4} \\
 &= 19.934 \times 10^{-15} \text{ rad}^2/\text{Hz} \\
 &= -137. \text{ dBc/Hz @ 10 MHz offset}
 \end{aligned}$$

- 4) Under the bias condition determined in 1), calculate total phase noise at 10 MHz offset frequency (5 pt).

$$\begin{aligned}
 (\overline{\Phi_n^2})_{\text{total}} &= 742.52 \times 10^{-18} + 19.934 \times 10^{-15} \\
 &\approx 19.934 \times 10^{-15} \text{ rad}^2/\text{Hz} \\
 &= -137 \text{ dBc/Hz @ 10 MHz offset} \\
 &= -117 \text{ dBc/Hz @ 1 MHz offset} \\
 &\Rightarrow \text{This is a typical number for NMOS vco at 20kHz range.}
 \end{aligned}$$

- 5) Now, let's use ideal filter which will reject noise at $2f_{osc}$ from M1 completely (f_{osc} is output oscillation frequency). Assume the filter is noiseless. Calculate phase noise contribution from M1 at 10MHz offset. How much improvement will it achieve compared with 3) through filtering the second harmonic noise? (20 pt).



$$\overline{v_{no}}^2 = 4kT \frac{r}{2} g_{m1} \left(1 - \left(\frac{2}{\pi} \right)^2 \left(1^2 + \frac{1}{3^2} \right) \right)$$

double side band noise per tone

details $\Rightarrow \overline{v_{no}}^2 = 2 \times 2kT \frac{r}{2} g_m \left(\frac{2}{\pi} \right)^2 \left\{ \sum_{n=1}^{\infty} \left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots \right) \times 2 - \left(1 + \frac{1}{3^2} \right) \right\}$

2-sided spectrum *1-side noise current power* *power of fundamental & harmonic tones* *rejection by filtering*

$$= 4kT \frac{r}{2} g_m - 4kT \frac{r}{2} g_m \left(\frac{2}{\pi} \right)^2 \left(1 + \frac{1}{3^2} \right)$$

$$\therefore (\Phi_n)_{due to M1} = \left(\frac{1}{2Q \frac{\Delta \omega}{\omega_0}} \right)^2 \frac{4kT \frac{r}{2} g_m R_L^2 \left(1 - \left(\frac{2}{\pi} \right)^2 \left(1 + \frac{1}{9} \right) \right)}{\frac{1}{2} V_p^2 \times 4}$$

$$= 10.958 \times 10^{-10} \text{ rad}^2/\text{Hz} = -139.6 \text{ dBc/Hz}$$

$\Rightarrow 2.6 \text{ dB improvement.}$