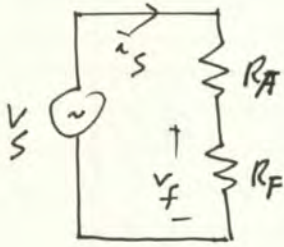


* 2-resistor feedback (series) ~~series~~

equivalent feedback system

i) Without R_F , i_S will be developed as

$$i_S = \frac{V_S}{R_A}$$

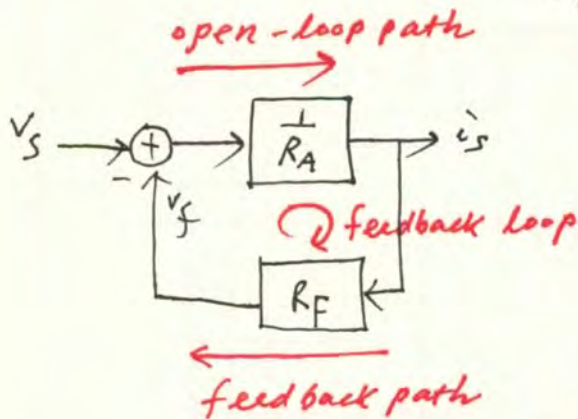
ii) When R_F included, R_F will detect i_S and develop V_F .And the V_F will be subtracted from V_S .

$$\text{The resultant } i_S = \frac{V_S - V_F}{R_A} \quad \text{--- (A)}$$

$$\Rightarrow V_F = i_S R_F \quad \text{--- apply to (A)}$$

$$\Rightarrow i_S R_A = V_S - V_F = V_S - i_S R_F$$

$$\therefore i_S = \frac{V_S}{R_A + R_F} = \frac{V_S}{R_A} \frac{1}{\left(1 + \frac{R_F}{R_A}\right)}$$



open loop gain, $A_{op} = \frac{1}{R_A}$
 feedback factor, $f = R_F$
 loop gain, $T = A_{op} f = \frac{R_F}{R_A}$
 closed-loop gain (A_S)

~~closed-loop gain~~
~~(2)~~

$$\begin{aligned}
 &= \frac{i_S}{V_S} = \frac{A_{op}}{1+T} = \frac{A_{op}}{1+A_{op}f} \\
 &= \frac{\frac{1}{R_A}}{1 + \frac{R_F}{R_A}} = \frac{1}{R_A + R_F}
 \end{aligned}$$

⇒ Without R_F , $Z_{in} = R_A$

With R_F , $Z_{in} = R_A + R_F = R_A \left(1 + \frac{R_F}{R_A}\right)$
 $= R_A (1 + T)$

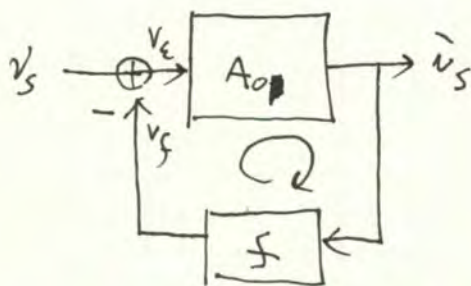
Z_{in} increases by a factor of $(1 + T)$.
 (why?)

⇒ Without R_F , $Z_{out} = R_A$

With R_F , $Z_{out} = R_A + R_F = R_A \left(1 + \frac{R_F}{R_A}\right)$
 $= R_A (1 + T)$

Z_{out} increases by a factor of $(1 + T)$.
 (why?)

< Generalization >



$$v_s - v_s = v_e = \frac{1}{1 + T} v_s$$

NOTE: input is Voltage
 output is Current

① Loop gain $= T = A_{op} \cdot f$

② closed loop gain $= A_f = \frac{A_{op}}{1 + T} = \frac{A_{op}}{1 + A_{op} f}$

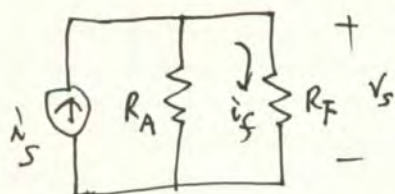
1st-order approximation
 $\approx \frac{1}{f} \left(1 - \frac{1}{T}\right) \text{ (if } T \gg 1) = \frac{1}{f} \left(\frac{1}{1 + \frac{1}{A_{op} f}}\right) = \frac{1}{f} \left(\frac{1}{1 + \frac{1}{T}}\right)$

2nd-order approximation
 $\approx \frac{1}{f} \text{ (if } T \gg 1)$

③ $Z_{in f} = Z_{in} (1 + T)$ ← makes more ideal voltage driving.

④ $Z_{out} = Z_{out} (1 + T)$

⊗ 2-resistor feedback (parallel)



equivalent
feedback
system.

i) Without R_F , V_s will be developed as

$$V_s = i_s R_A.$$

ii) when R_F paralleled,

R_F will detect V_s and generate i_f .

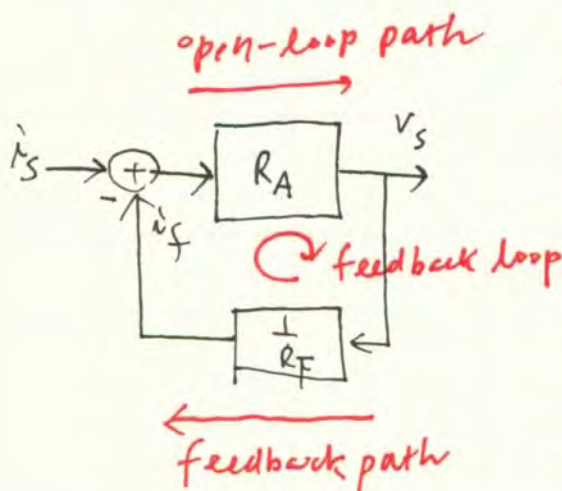
And the i_f will be subtracted from i_s .

The resultant $V_s = (i_s - i_f) R_A$ — (A)

$$\Rightarrow i_f = \frac{V_s}{R_F} \quad \text{apply to (A)}$$

$$\Rightarrow V_s = \left(i_s - \frac{V_s}{R_F} \right) R_A$$

$$\therefore V_s = \frac{i_s R_A}{1 + \frac{R_A}{R_F}}$$



open loop gain, $A_o = R_A$
feedback factor, $f = \frac{1}{R_F}$

$$\text{loop gain, } T = A_o f = R_A / R_F$$

Closed loop gain (A_f)

$$= \frac{V_s}{i_s} = \frac{A_o}{1 + T} = \frac{R_A}{1 + \frac{R_A}{R_F}}$$

$$= \frac{R_A \cdot R_F}{R_A + R_F} = R_A \parallel R_F$$

\Rightarrow Without R_F , $Z_{in} = R_A$

with R_F , $Z_{in} = R_A \parallel R_F = \frac{R_A \cdot R_F}{R_A + R_F} = \frac{R_A}{1 + \frac{R_A}{R_F}}$

Z_{in} decreases
by a factor of
(1+T).

(Why?)

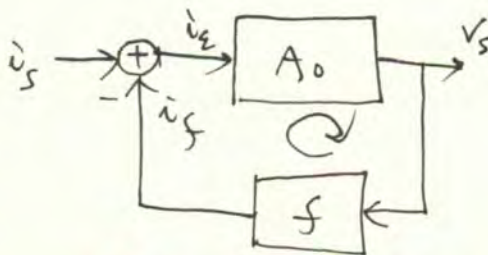
$$= \frac{R_A}{1+T}$$

\Rightarrow without R_F , $Z_{out} = R_A$

with R_F , $Z_{out} = R_A \parallel R_F = \frac{R_A}{1 + \frac{R_A}{R_F}} = \frac{R_A}{1+T}$

Z_{in} decreases
by a factor of (1+T).
(Why?)

< generalization >



$$\begin{aligned} i_s - i_f &= i_e \\ &= \frac{1}{1+T} \times i_s \end{aligned}$$

NOTE: Input is current
output is voltage

① loop gain, $T = A_o f$

② closed loop gain, $A_f = \frac{A_o}{1+T} = \frac{A_o}{1+A_o f}$

$$= \frac{1}{f} \left(\frac{1}{1 + \frac{1}{A_o f}} \right) = \frac{1}{f} \left(\frac{1}{1 + \frac{1}{T}} \right)$$

$$\approx \frac{1}{f} \left(1 - \frac{1}{T} \right) \quad \text{③ } T \gg 1$$

$$\approx \frac{1}{f}$$

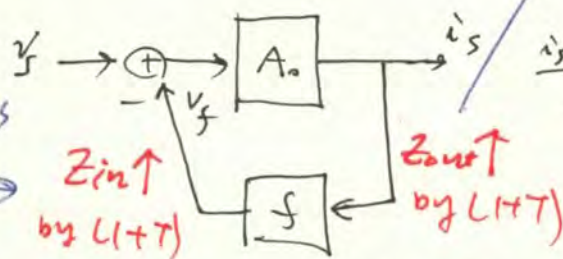
③ $Z_{in f} = Z_{in} \frac{1}{1+T}$ ← makes more ideal current driving.

④ $Z_{out f} = Z_{out} \frac{1}{1+T}$

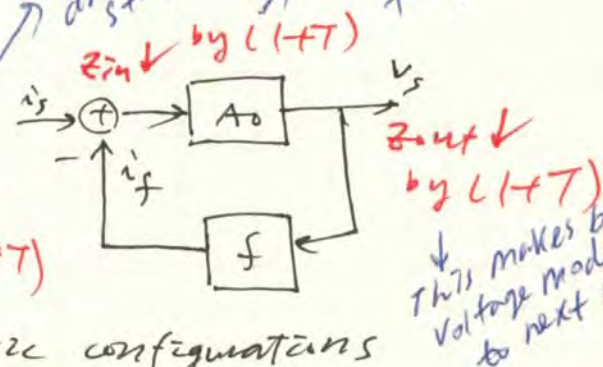
Feedback Basics

(*) feedback configurations

this makes better voltage driving for previous stage.

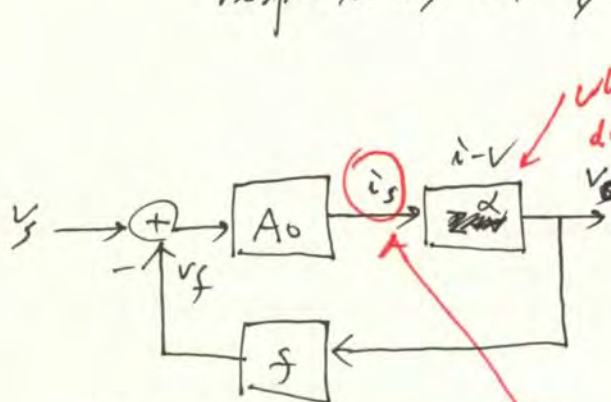


this makes better current mode driving to next stage.



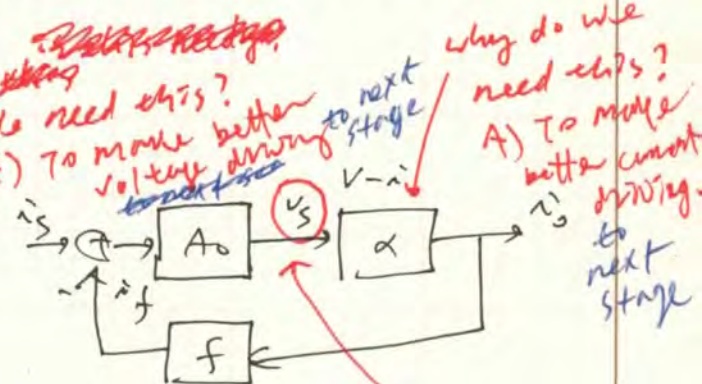
this makes better voltage mode driving to next stage.

- \Rightarrow these are two basic configurations
- \Rightarrow driving source should be either voltage or current.
- \Rightarrow if driving source is voltage, immediate output response is current, which is first case.
- \Rightarrow if driving source is current, immediate output response is voltage, which is the second case.



- \Rightarrow $i-v$ converter can be used to convert i_s to V_o .
- \Rightarrow But, still the point of feedback operation is to decrease i_s .
- \Rightarrow decreasing i_s results in decreasing V_o .

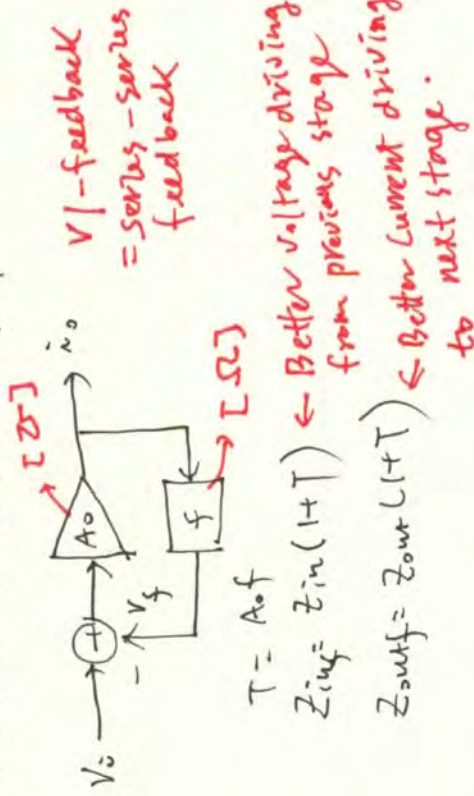
$$\Rightarrow \begin{aligned} Z_{in} &\uparrow \times (1+T) \\ Z_{out} &\downarrow \times (1+T) \end{aligned}$$



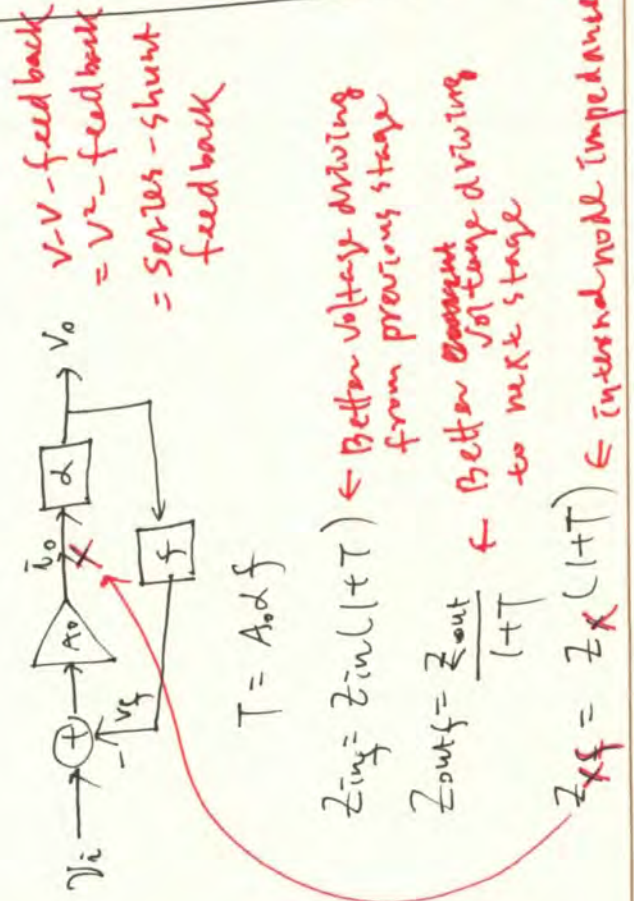
- \Rightarrow $v-i$ converter can be used to convert V_s to i_o .
- \Rightarrow But, still the point of feedback operation is to decrease V_s .
- \Rightarrow decreasing V_s results in decreasing i_o .

$$\Rightarrow \begin{aligned} Z_{in} &\downarrow \times (1+T) \\ Z_{out} &\uparrow \times (1+T) \end{aligned}$$

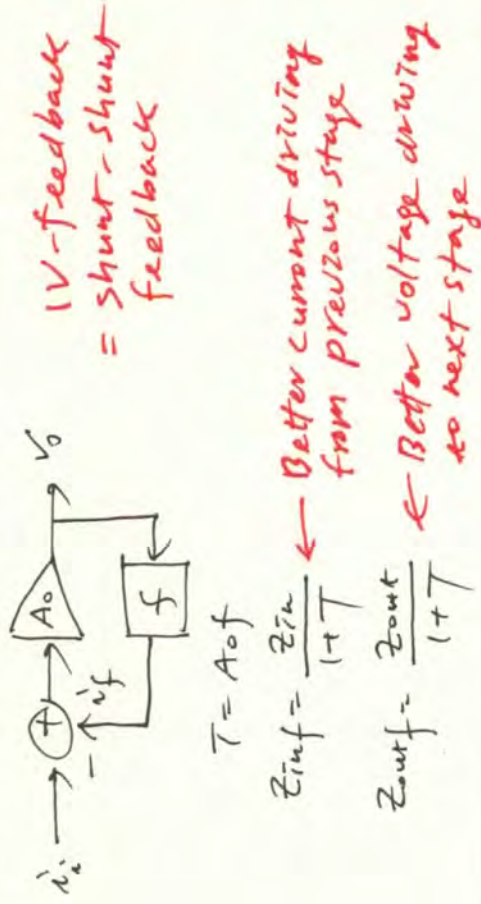
1 Transconductance amplifier



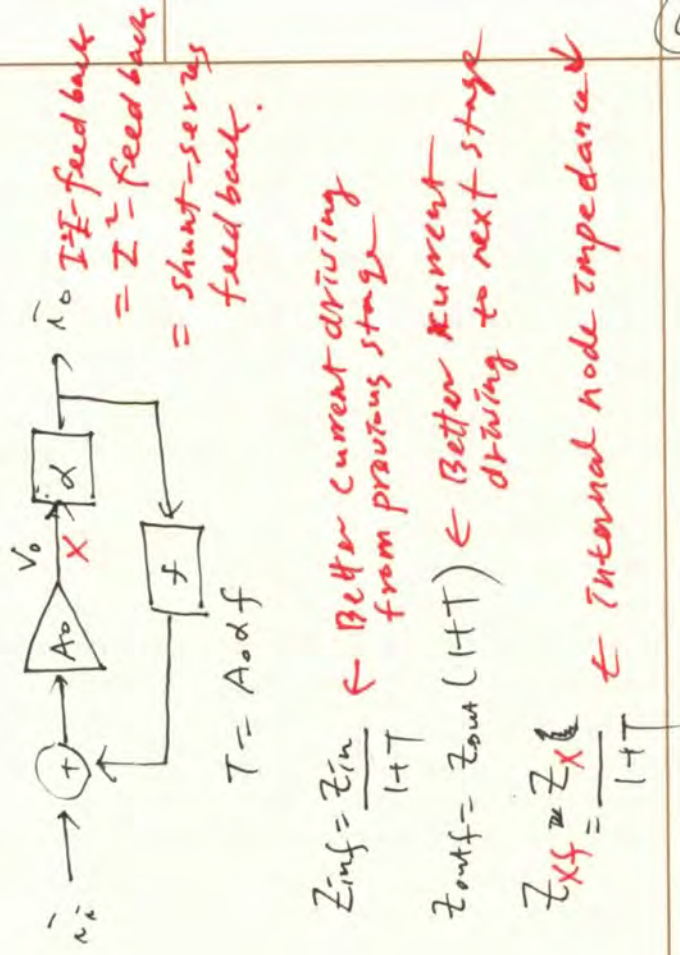
2 Voltage gain amplifier



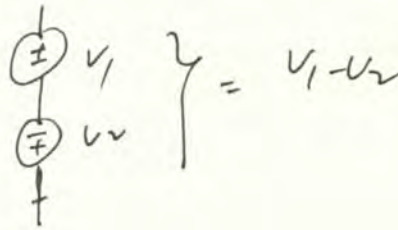
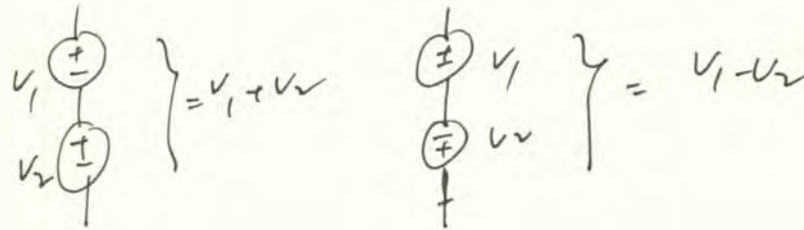
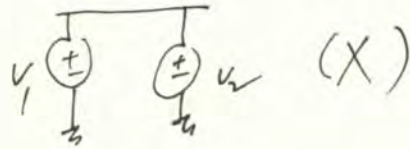
3 Transimpedance amplifier



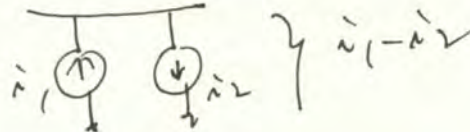
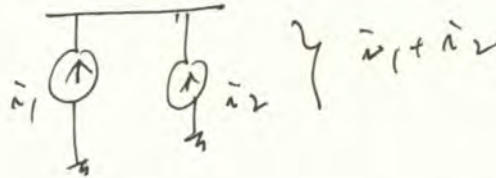
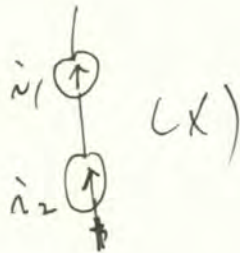
4 Current gain amplifier



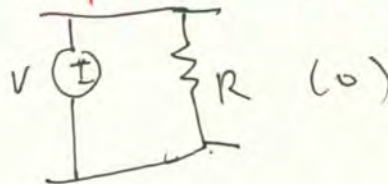
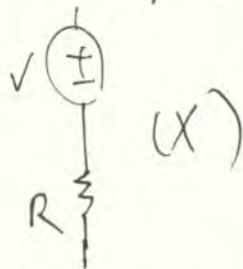
⊕ voltage addition/subtraction \Rightarrow series network.



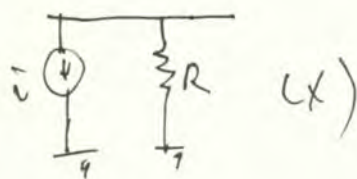
⊗ Current addition/subtraction = parallel (shunt) network



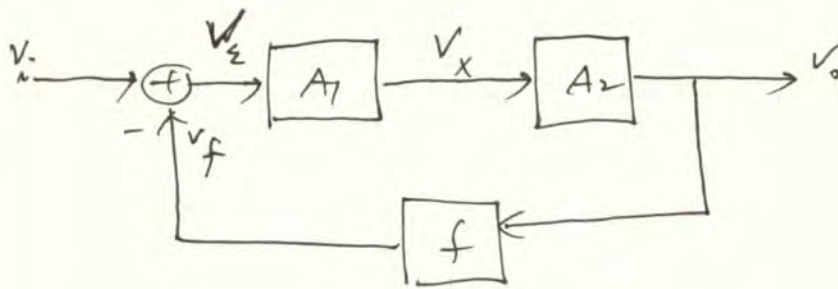
⊗ voltage sensing \Rightarrow parallel network (shunt network)



⊗ Current sensing \Rightarrow series network



⊛ Remember this!

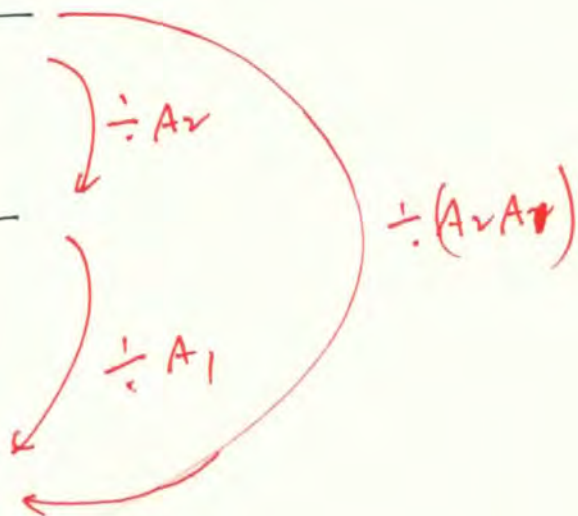


- loop gain, $T = A_1 A_2 f$

$$- A_f = \frac{V_o}{V_i} = \frac{A_1 A_2}{1 + T}$$

$$- \frac{V_x}{V_i} = \frac{A_1}{1 + T}$$

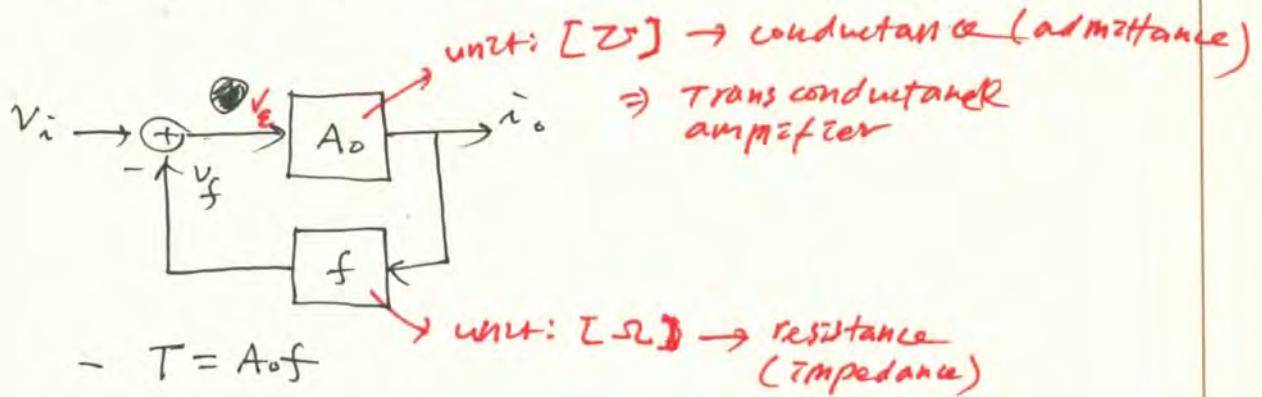
$$- \frac{V_z}{V_i} = \frac{1}{1 + T}$$



⊛ We will study more complex form of feedback in active filter design section.

⊛ study "Mason's Gain Formula".

⊛ V-I feedback (series-series feedback)



$$T = A_o f$$

$$A_f = \frac{i_o}{V_i} = \frac{A_o}{1+T} = \frac{A_o}{1+A_o f} \approx \frac{1}{f} \left(1 - \frac{1}{A_o f}\right) \approx \frac{1}{f}$$

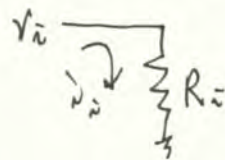
$$\rightarrow i_o = \frac{A_o}{1+T} V_i$$

$$\rightarrow V_e = \frac{1}{1+T} V_i$$

(if perfect feedback)
 $V_e = 0, V_i = V_f$

① Z_{in} & $Z_{in f}$

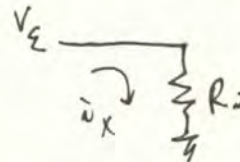
i) open-loop



$$i_i = \frac{V_i}{R_i}$$

$$Z_{in} = \frac{V_i}{i_i} = R_i$$

ii) closed-loop



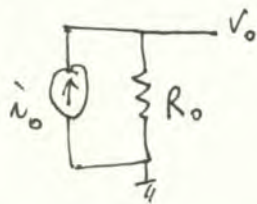
$$i_x = \frac{V_e}{R_i} = \frac{V_i}{(1+T) R_i}$$

$$Z_{in f} = \frac{V_i}{i_x} = (1+T) R_i$$

To compare input impedance,
 the same reference of V_i
 is chosen.

② Z_{out} & Z_{outf}

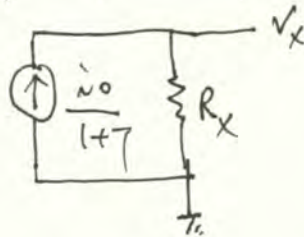
i) open-loop



$$V_o = \hat{i}_o R_o$$

$$Z_{out} = \frac{V_o}{\hat{i}_o} = R_o$$

ii) closed-loop



$$V_x = \frac{\hat{i}_o}{1+T} R_x = \frac{V_o}{(1+T) R_o} \cdot R_x$$

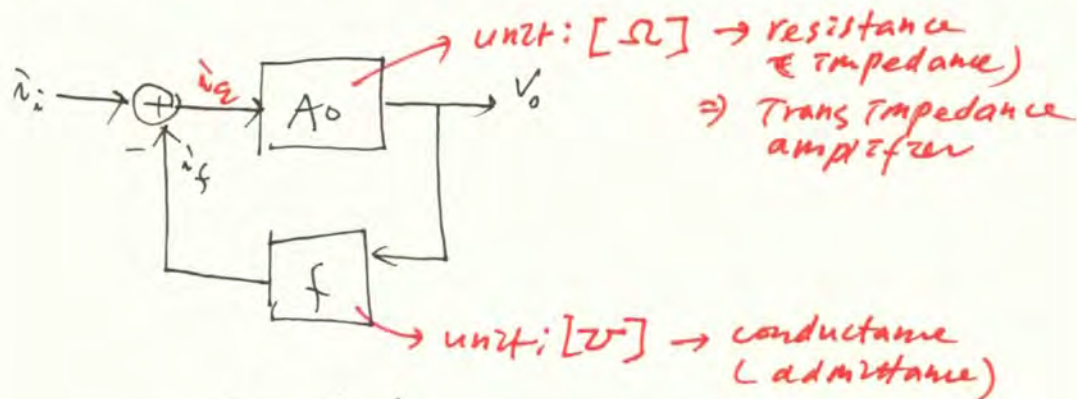
$$\Rightarrow R_x = (1+T) R_o \frac{V_x}{V_o}$$

To compare these two impedances, we need to set $V_o = V_x$ (same reference).

$$\therefore R_x = (1+T) R_o$$

$$\Rightarrow Z_{outf} = R_x = (1+T) R_o$$

⊗ I-V feedback (shunt-shunt feedback)



$$T = A_o f$$

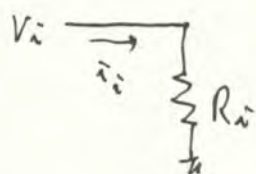
$$A_f = \frac{v_o}{i_i} = \frac{A_o}{1+T} = \frac{A_o}{1+A_o f} \approx \frac{1}{f} \left(1 - \frac{1}{A_o f}\right) \approx \frac{1}{f}$$

$$\rightarrow v_o = \frac{A_o}{1+T} i_i$$

$$\rightarrow i_x = \frac{1}{1+T} i_i \quad \left(\textcircled{if} \text{ perfect feedback } i_x = 0, i_i = i_f \right)$$

① Z_{in} & Z_{inf}

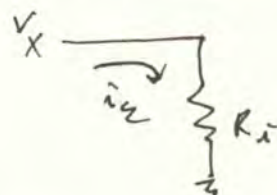
i) open-loop



$$V_i = i_i R_i$$

$$Z_{in} = \frac{V_i}{i_i} = R_i$$

ii) closed-loop



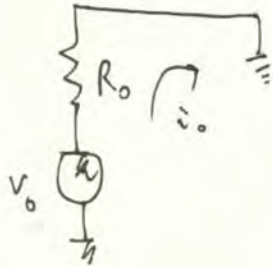
$$V_x = i_x R_i = \frac{i_i}{1+T} R_i$$

$$Z_{inf} = \frac{V_x}{i_x} = \frac{R_i}{1+T}$$

To compare input impedance, the same reference of i_i is chosen.

② Z_{out} & Z_{outf}

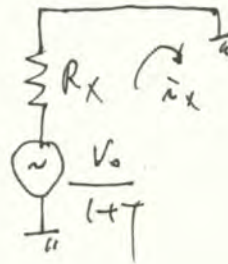
i) open-loop



$$\tilde{i}_o = \frac{v_o}{R_o}$$

$$Z_{out} = \frac{v_o}{\tilde{i}_o} = R_o$$

ii) closed-loop



$$\tilde{i}_x = \frac{v_o}{(1+T)} \frac{1}{R_x} = \frac{\tilde{i}_o R_o}{(1+T)} \frac{1}{R_x}$$

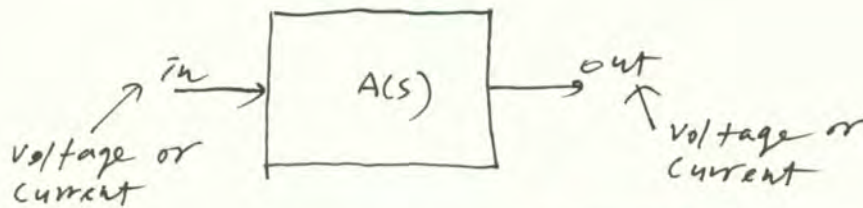
$$\Rightarrow R_x = \frac{R_o}{1+T} \frac{\tilde{i}_o}{\tilde{i}_x}$$

To compare these two impedances, we need to set $\tilde{i}_x = \tilde{i}_o$ (same reference)

$$\therefore R_x = \frac{R_o}{1+T}$$

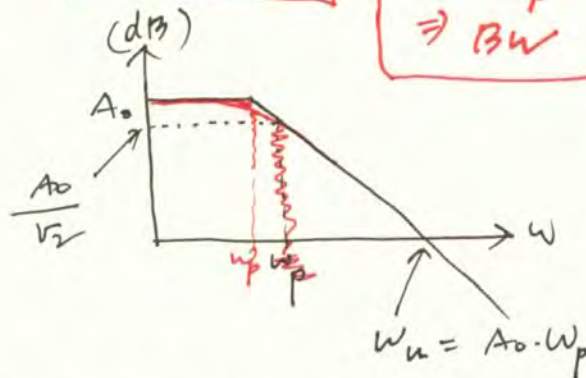
$$\Rightarrow Z_{outf} = R_x = \frac{R_o}{1+T}$$

(*) one-pole system



$$\Rightarrow A(s) = \frac{A_0}{1 + s\tau_p} = \frac{A_0}{1 + s/\omega_p} = \frac{A_0 \omega_p}{s + \omega_p} = \frac{\omega_u}{s + \omega_p}$$

τ_p Time constant
 $\omega_p = \frac{1}{\tau_p} \Rightarrow BW$
 $\omega_u = A_0 \omega_p$
 $Gain \times BW = GBW = \omega_u$



\Rightarrow These equations give a good intuition for understanding a system in view of Gain, BW and frequency response.

\Rightarrow Another useful expression

$$A(s) = \frac{\omega_u}{s + \omega_p} = \frac{\omega_u}{s + \frac{\omega_u}{Q}}$$

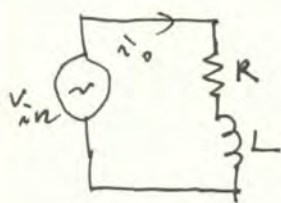
This equation gives a good intuition for understanding a system in view of energy.

\hookrightarrow Pole Quality factor

$$Q = \frac{\omega}{\omega_p} = \omega \tau_p$$

\Downarrow
 Q is a metric to express a stored energy in a system, which is relative amount to dissipated energy by the system.

examples

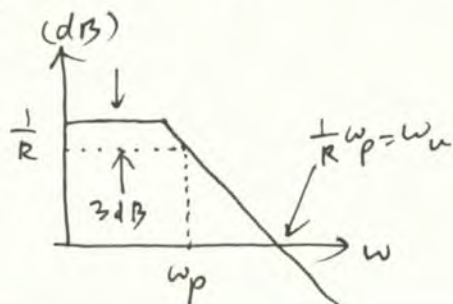


$$A(s) = \frac{v_o}{v_{in}} = \frac{1}{R + sL} = \frac{1}{R} \cdot \frac{1}{(1 + sL/R)}$$

$$= \frac{1}{R} \cdot \frac{1}{1 + sZ_p}, \quad Z_p = \frac{L}{R}$$

$$= \frac{1}{R} \cdot \frac{1}{1 + s/\omega_p}, \quad \omega_p = \frac{1}{Z_p}$$

$$= \frac{\frac{1}{R} \cdot \omega_p}{s + \omega_p} = \frac{\omega_u}{s + \omega_p}$$



what's the meaning of Z_p ?

remember

$$Q = \frac{\text{Peak Energy stored in L or C}}{\text{Energy dissipated by R per unit radian}}$$

$$= \frac{\frac{1}{2} L \hat{i}_0^2}{\frac{1}{2} \hat{i}_0^2 R \times T \times \frac{1}{2\pi}}$$

$$= \frac{\frac{1}{2} L \hat{i}_0^2}{\frac{1}{2} \hat{i}_0^2 R \times \frac{1}{\omega}}$$

$$= \omega \frac{L}{R} = \omega Z_p = \frac{\omega}{\omega_p}$$

$Q \uparrow \rightarrow Z_p \uparrow$
 \rightarrow larger inertia energy
 \rightarrow slow response time
 \rightarrow slow speed.

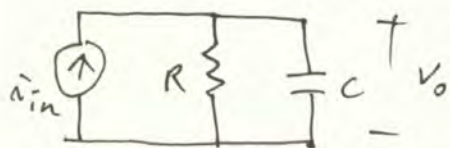
$Q=1$
 \Rightarrow energy balance between stored & dissipated energy

\Rightarrow higher $Q \rightarrow$ larger stored energy than dissipated energy by R

$Q = \frac{1}{2} \frac{\epsilon}{\text{dissipated } \epsilon}$
 \Rightarrow stored ϵ is 50% of dissipated ϵ .

\Rightarrow (if) $R=0 \rightarrow Q=\infty \rightarrow$ pure L
 \rightarrow pure energy storage element.

$$\therefore A(s) = \frac{1}{R} \frac{1}{1 + sZ_p} = \frac{1}{R} \frac{1}{1 + s/\omega_p} = \frac{\omega_u}{s + \frac{\omega}{Q}}$$

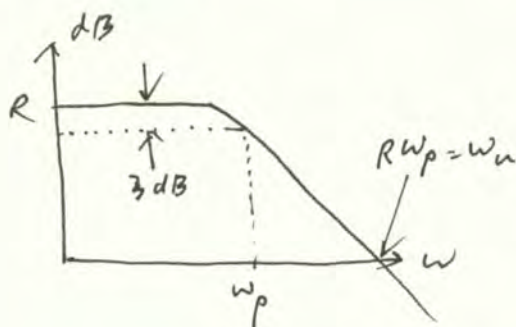
examples

$$A(s) = \frac{v_o}{\hat{v}_{in}} = \frac{1}{\frac{1}{R} + sC} = \frac{R}{1 + sCR}$$

$$= R \frac{1}{1 + sZ_p}, \quad \boxed{Z_p = RC}$$

$$= R \frac{1}{1 + s/\omega_p}, \quad \boxed{\omega_p = \frac{1}{Z_p}}$$

$$= \frac{R\omega_p}{s + \omega_p} = \frac{\omega_u}{s + \omega_p}$$



$$Q = \frac{\text{Peak Energy stored in L or C}}{\text{Energy dissipated by R per unit radian}}$$

$$= \frac{\frac{1}{2} C V_o^2}{\frac{1}{2} \frac{V_o^2}{R} \times T \times \frac{1}{2\pi}}$$

$$= \frac{\frac{1}{2} C V_o^2}{\frac{1}{2} \frac{V_o^2}{R} \cdot \frac{1}{\omega}}$$

$$= \omega C R = \omega Z_p = \frac{\omega_u}{\omega_p}$$

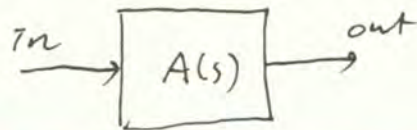
$Q \uparrow \rightarrow Z_p \uparrow$
 \rightarrow large inertia
 energy
 \rightarrow slow response time
 \rightarrow slow speed.

\Rightarrow higher $Q \rightarrow$ higher stored energy than dissipated energy by R

\Rightarrow (CF) $R = \infty \rightarrow Q = \infty \rightarrow$ pure C
 \rightarrow pure energy storage element

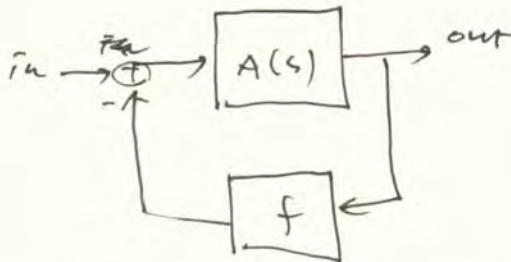
$$\therefore A(s) = \frac{R}{1 + sZ_p} = \frac{R}{1 + s/\omega_p} = \frac{\omega_u}{s + \frac{\omega_u}{Q}}$$

(*) one-pole system with feedback



understand meaning of each expression.

$$\Rightarrow A(s) = \frac{A_0}{1+sZ_p} = \frac{A_0}{1+s/w_p} = \frac{w_n}{s+w_p} = \frac{w_n}{s+\frac{w}{Q}}$$



$$\Rightarrow A_f(s) = \frac{A(s)}{1+T} = \frac{A(s)}{1+A(s)f} = \frac{\frac{A_0}{1+sZ_p}}{1+\frac{A_0}{1+sZ_p}f}$$

$$= \frac{A_0}{(1+A_0f) + sZ_p}$$

$$= \frac{A_0}{1+A_0f} \frac{1}{1+s \frac{Z_p}{1+A_0f}}$$

$$= \frac{A_0}{1+T_0} \frac{1}{1+s \left(\frac{Z_p}{1+T_0} \right)} \quad \leftarrow T_0 = A_0f \text{ (DC loop gain)}$$

Time constant decreases by a factor of $(1+T_0)$.

\Rightarrow can you explain this in more intuitive way?

$$\therefore A_f(s) = \frac{A_0}{1+T_0} \frac{1}{1+s \frac{Z_p}{1+T_0}} = \frac{A_0}{1+T_0} \frac{1}{1+s \frac{Z_p}{w_p(1+T_0)}} = \frac{w_n = A_0 w_p}{s + \frac{w}{1+T_0}}$$

$Z_p \downarrow \times (1+T_0)$

$w_p \uparrow \times (1+T_0)$

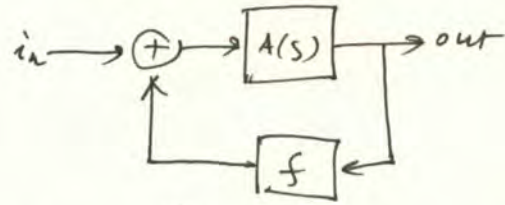
$Q \downarrow \times (1+T_0)$

Negative feedbacks

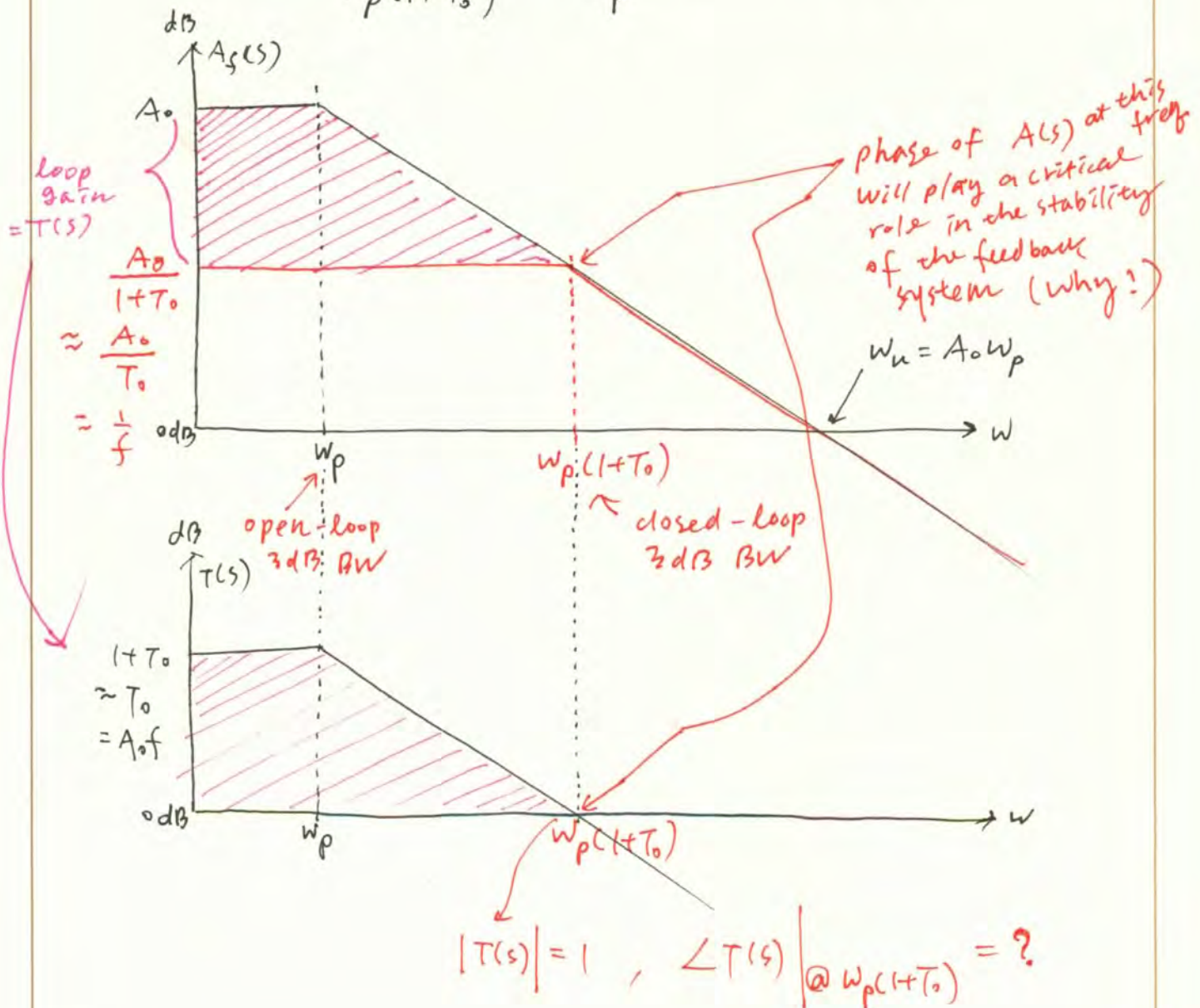
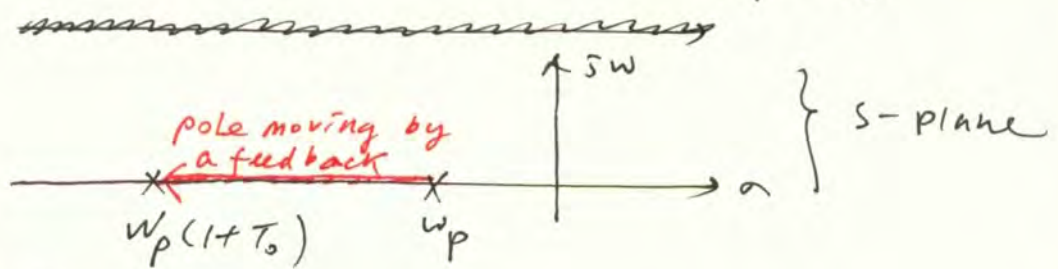
- ① decreased Z_p by a factor of $(1+T_0)$
- ② increase w_p by a factor of $(1+T_0)$
- ③ decrease Q by a factor of $(1+T_0)$

17.

< Closed-loop >



$$\Rightarrow A_f(s) = \frac{A_o}{1 + T_o} \cdot \frac{1}{1 + s/\omega_p(1 + T_o)}$$



$$\textcircled{1} \quad A(s) = \frac{A_0}{1+s/\omega_p} \rightarrow \begin{cases} |A(s)| = \frac{A_0}{\sqrt{1+\frac{\omega^2}{\omega_p^2}}} \\ \angle A(s) = -\tan^{-1}\left(\frac{\omega}{\omega_p}\right) \end{cases}$$

$\rightarrow @ \omega = \omega_p \rightarrow \angle A(s) = -45^\circ$
 $@ \omega = \infty \rightarrow \angle A(s) = -90^\circ$

$$\textcircled{2} \quad T(s) = A(s)f = \frac{A_0 f}{1+s/\omega_p} \rightarrow \begin{cases} |T(s)| = |A(s)|f \\ \rightarrow \text{Just a scaled version of } A(s) \\ \angle T(s) = \angle A(s) = -\tan^{-1}\left(\frac{\omega}{\omega_p}\right) \\ \rightarrow \text{the same as that of } A(s). \end{cases}$$

$$\textcircled{3} \quad A_f(s) = \frac{A(s)}{1+T(s)} \rightarrow \textcircled{if} \quad |T(s)| = 1 \rightarrow 0 \text{ dB}$$

and $\angle T(s) = -180^\circ$
 $\rightarrow \text{This means } T(s) = -1$

$A_f(s) = \infty$
 $\rightarrow \text{unstable.}$

Phase Margin (ϕ_M)

\Rightarrow A measure of a stability in feedback systems.

\Rightarrow How much far away of $\angle T(s)$ from -180° @ $\omega = \omega_p(1+T_0)$

$$\Rightarrow \phi_M = 180^\circ - |\angle T(s)| @ \omega = \omega_p(1+T_0)$$

most electronic circuits & systems

$$\phi_M > 60^\circ$$

(practically $\phi_M > 90^\circ$)

$$= 180^\circ - \tan^{-1}\left(\frac{\omega_p(1+T_0)}{\omega_p}\right)$$

$$= 180^\circ - \tan^{-1}(1+T_0)$$

$$= 180^\circ - \tan^{-1}(T_0)$$

$$= 180^\circ - \tan^{-1}(A_0 f)$$

for one-pole system,
 $\phi_M > 90^\circ$

$$A_f(s) = \frac{A(s)}{1+T(s)}$$

if $|T(s)|$ is smaller than 1 when $\angle T(s) = -180^\circ$
~~at $\omega = \omega_{crossover}$~~ $A_f(s)$ will also be stable.



Gain Margin (G_M)

\Rightarrow Alternative way of measuring stability of a feedback system (But not popular way)

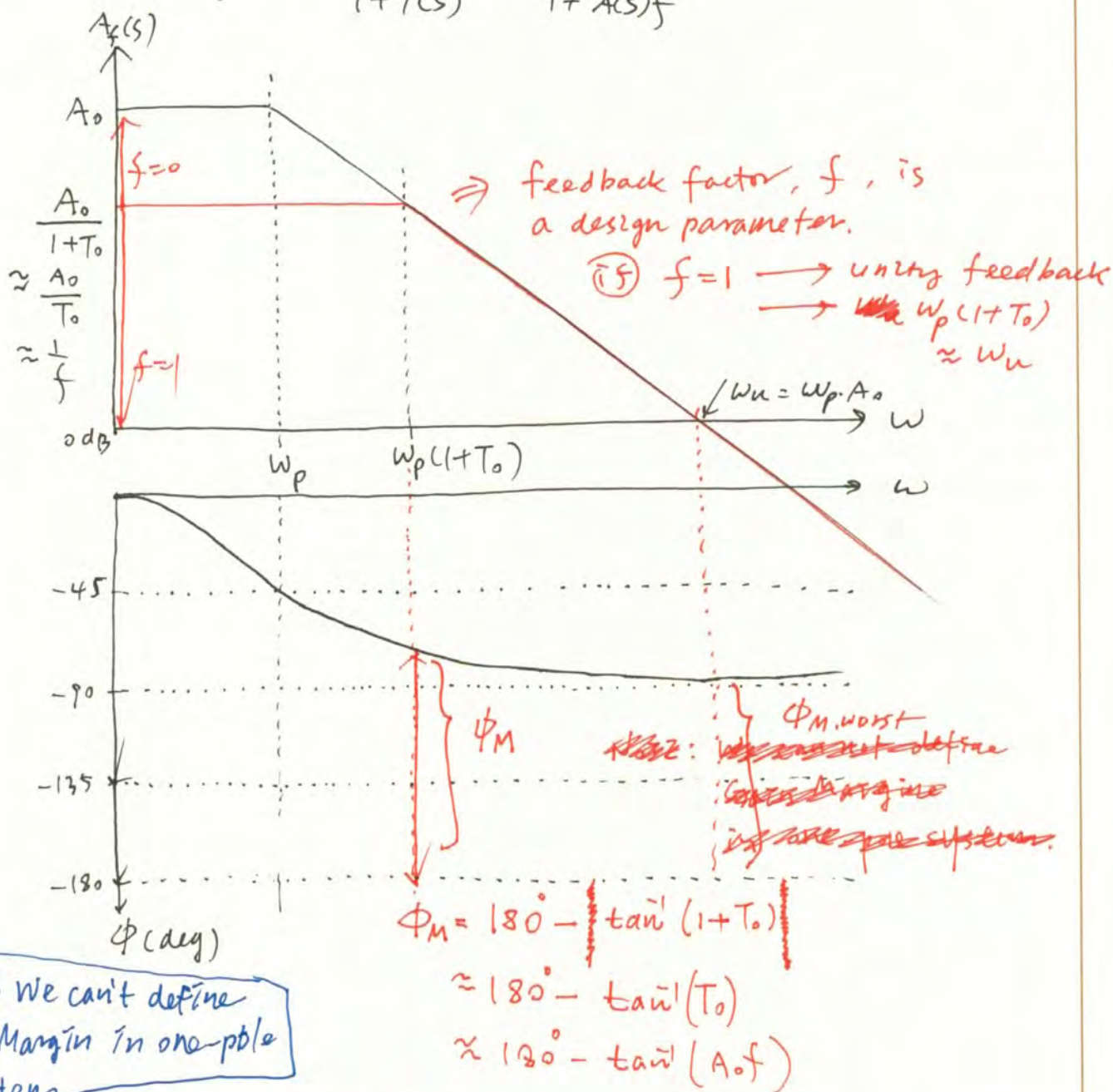
\Rightarrow ~~How much smaller~~
 by how much $|T(s)|$ is smaller than 1
~~at $\omega = \omega_{crossover}$~~ when $\angle T(s) = -180^\circ$

$$\Rightarrow G_M = \frac{1}{|T(s)| \text{ when } \angle T(s) = -180^\circ}$$

\Rightarrow usually dB-scale.

exampleone-pole system with feedback

$$A_f(s) = \frac{A(s)}{1+T(s)} = \frac{A(s)}{1+A(s)f}$$



NOTE: We can't define Gain Margin in one-pole system

\Rightarrow Worst case ϕ_M is when $f=1$

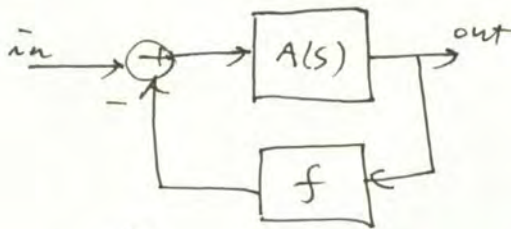
$$\Rightarrow \phi_{M, \text{worst}} = 180^\circ - \tan^{-1}\left(\frac{\omega_u}{\omega_p}\right) = 180^\circ - \tan^{-1}(A_0)$$

$$\Rightarrow \phi_{M, \text{worst}} > 90^\circ$$

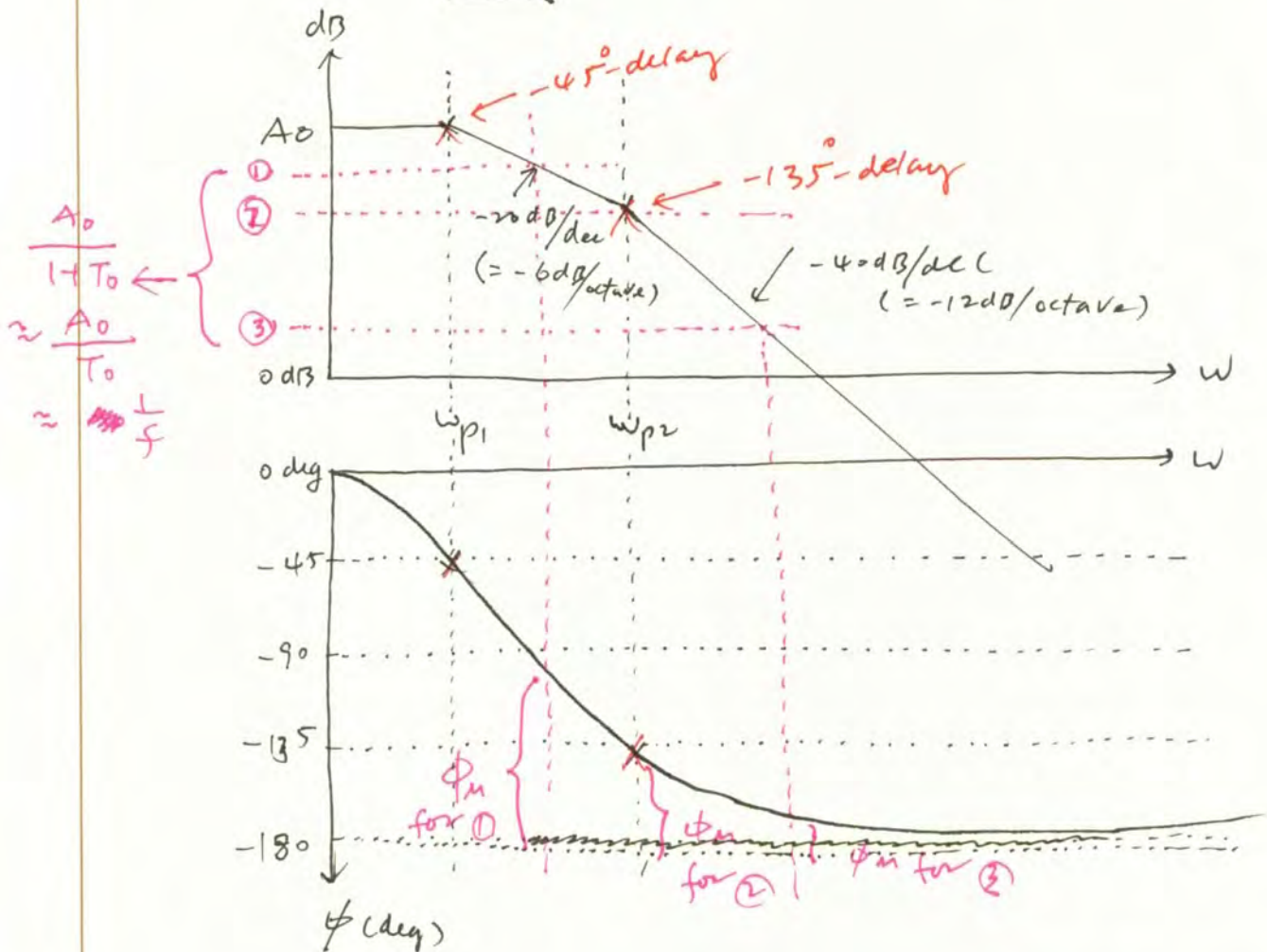
\Rightarrow Absolutely stable for any feedback factor of f

example

uncompensated two-pole system.

~~Two-pole system~~ Two-pole system with feedback

$$A(s) = \frac{A_0}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})}$$

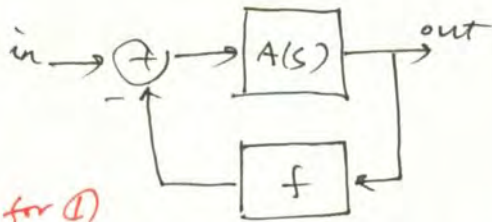
 \Rightarrow examples for three different cases of f

$\Rightarrow \phi_m > 0$ (In a strict sense 2-pole system is stable for any value of f . But usually we have extra parasitic poles which causes $\phi_m < 0$)

 \Rightarrow We cannot define Gain Margin in 2-pole system.

Example

three-pole system with feedback

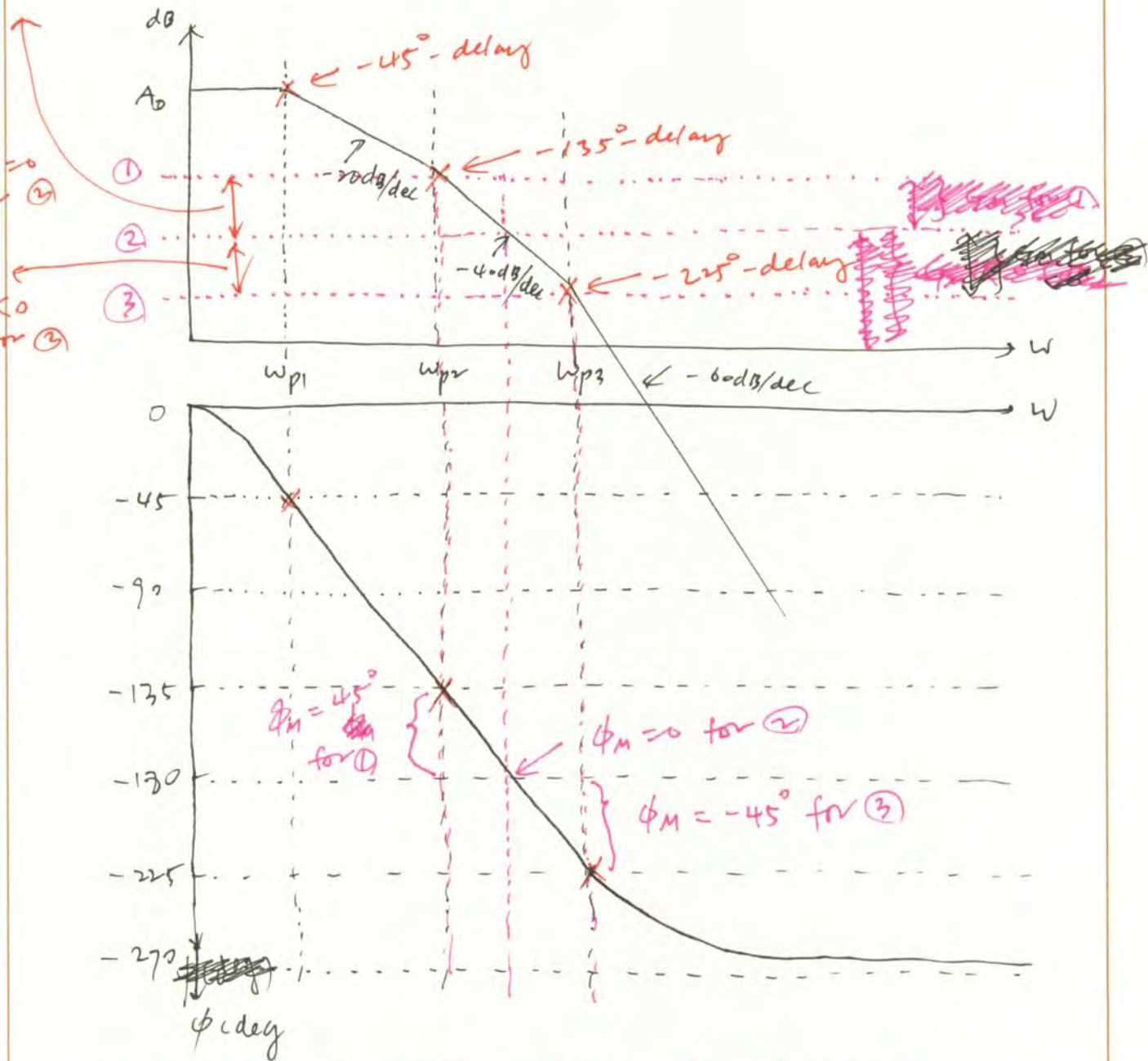


$$A(s) = \frac{A_0}{(1+s/\omega_{p1})(1+s/\omega_{p2})(1+s/\omega_{p3})}$$

① $G_M \rightarrow 0$ for ①

② $G_M \rightarrow 0$ for ②

③ $G_M \rightarrow 0$ for ③



\Rightarrow 2-pole, 3-pole systems are not common.

\Rightarrow They need frequency compensation.

① pole-splitting

effectively 1-pole system

effectively 1-pole system

③ zero-insertion (2-pole 1-zero, 3-pole 2-zero)