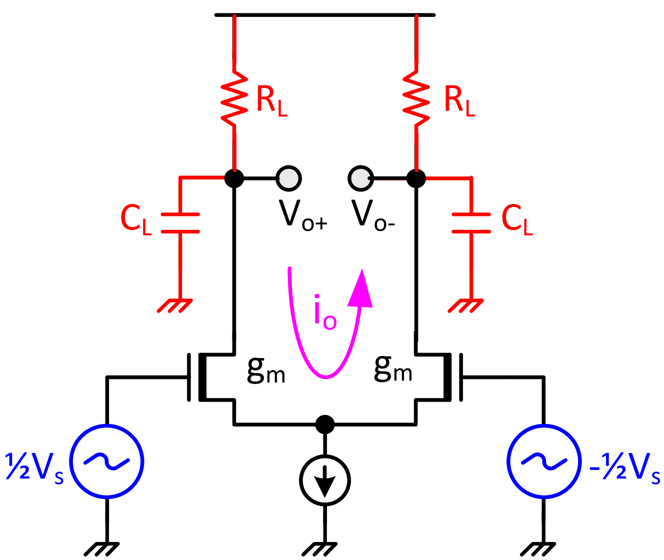
Differential Amplifier: Frequency Response

- In typical analog systems, input is driven by another feedback amplifier which has very low source impedance. Therefore, input node becomes non-significant parasitic pole in most cases.
- Dominant pole (3-dB BW) is determined by a load pole (load resistance and capacitance).



$$\frac{v_{o+}}{v_s} = \underbrace{\left(-\frac{1}{2}g_m R_L\right)}_{DC-gain}\underbrace{\left(\frac{1}{1+\frac{s}{\omega_p}}\right)}_{Pole-frequency}, \omega_p = \frac{1}{R_L C_L}$$

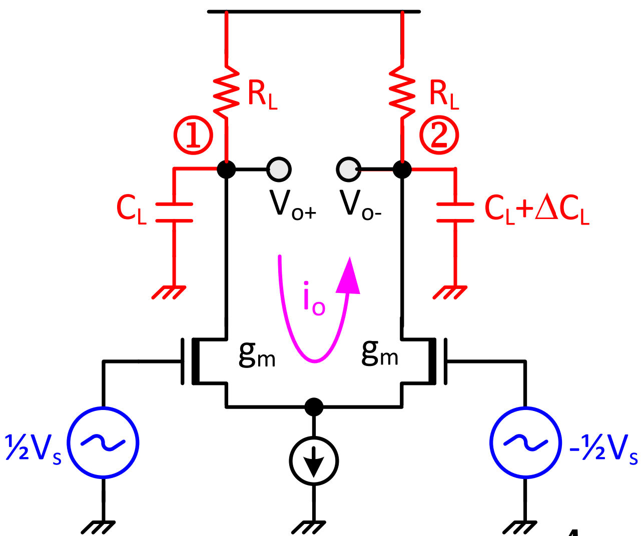
$$\frac{v_{o-}}{v_s} = \underbrace{\left(+\frac{1}{2}g_m R_L\right)}_{DC-gain}\underbrace{\left(\frac{1}{1+\frac{s}{\omega_p}}\right)}_{Pole-frequency}, \omega_p = \frac{1}{R_L C_L}$$

$$A_v = \frac{v_{o+} - v_{o-}}{v_s} = \frac{v_{o+}}{v_s} - \frac{v_{o-}}{v_s} = \underbrace{(-g_m R_L)}_{DC-gain}\underbrace{\left(\frac{1}{1+\frac{s}{\omega_p}}\right)}_{Pole-frequency}, \omega_p = \frac{1}{R_L C_L}$$

- With perfectly matched loads, differential amplifier will have one pole.

Differential Amplifier: Pole-Zero Doublet (1)

- What happens if the load are not matched exactly ?
- Let's assume that the load capacitances have a slight mismatch of ΔC_L.



$$\frac{v_{o+}}{v_s} = \underbrace{\left(-\frac{1}{2}g_m R_L\right)}_{DC-gain} \underbrace{\left(\frac{1}{1+\frac{s}{\omega_{p1}}}\right)}_{Pole-frequency\ at\ node\ ①}, \omega_{p1} = \frac{1}{R_L C_L}$$

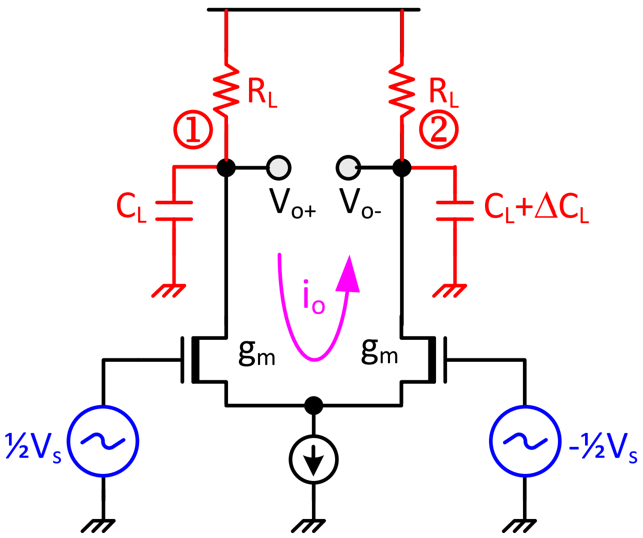
$$\frac{v_{o-}}{v_s} = \underbrace{\left(+\frac{1}{2}g_m R_L\right)}_{DC-gain} \underbrace{\left(\frac{1}{1+\frac{s}{\omega_{p2}}}\right)}_{Pole-frequency\ at\ node\ ②}, \omega_{p2} = \frac{1}{R_L C_L\left(1+\frac{\Delta C_L}{C_L}\right)}$$

$$A_v = \frac{v_{o+} - v_{o-}}{v_s} = \frac{v_{o+}}{v_s} - \frac{v_{o-}}{v_s} = \left(-\frac{1}{2}g_m R_L\right)\left(\frac{1}{1+\frac{s}{\omega_{p1}}} + \frac{1}{1+\frac{s}{\omega_{p2}}}\right)$$

$$= (-g_m R_L)\left(\frac{1+s\frac{1}{2}\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}\right) = \underbrace{(-g_m R_L)\left(\frac{1}{1+\frac{s}{\omega_{p1}}}\right)}_{\substack{origianl\ response \\ without\ mismatch}} \underbrace{\left(\frac{1+s\frac{1}{2}\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)}{\left(1+\frac{s}{\omega_{p2}}\right)}\right)}_{\substack{additional\ pole-zero\ pair \\ due\ to\ mismatch}}$$

Differential Amplifier: Pole-Zero Doublet (2)

- From previous result, mismatch generates another pole-zero pair called as "doublet".
- But most typical cases of differential amplifier designs , mismatch is very small enough to be negligible, $\omega_{p1} \approx \omega_{p2}$.



$$A_v = \underbrace{(-g_m R_L)\left(\frac{1}{1+\frac{s}{\omega_{p1}}}\right)}_{\substack{\text{origianl response} \\ \text{without mismatch}}} \underbrace{\left(\frac{1+s\frac{1}{2}\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)}{\left(1+\frac{s}{\omega_{p2}}\right)}\right)}_{\substack{\text{additional pole-zero pair} \\ \text{due to mismatch} \\ \text{called "doublet"}}}$$

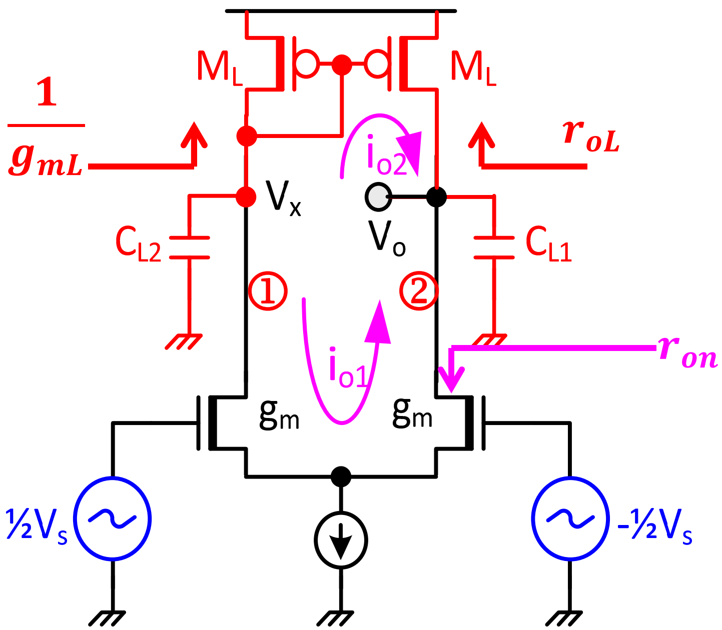
$$A_v \approx \underbrace{(-g_m R_L)\left(\frac{1}{1+\frac{s}{\omega_{p1}}}\right)}_{\substack{\text{origianl response} \\ \text{without mismatch}}} \underbrace{\left(\frac{1+s\frac{1}{2}\left(\frac{1}{\omega_{p2}}+\frac{1}{\omega_{p2}}\right)}{\left(1+\frac{s}{\omega_{p2}}\right)}\right)}_{\substack{\text{typical differential amplifiers,} \\ \omega_{p1} \approx \omega_{p2}}}$$

$$= (-g_m R_L)\left(\frac{1}{1+\frac{s}{\omega_{p1}}}\right)$$

- Key observation here is that "Any mismatch in differential amplifier loads will generate another polo-zero pair (doublet)".

Differential Amplifier with Active Load (1)

- Differential amplifier with active loading is **heavily mismatched case ⇒ "pole-zero doublet".**



$$v_x = i_{o1} \times \frac{1}{g_{mL}} \times \underbrace{\left(\frac{1}{1+\frac{s}{\omega_{px}}}\right)}_{Pole-frequency\ at\ node\ ①}, \omega_{px} = \frac{g_{mL}}{C_{L2}}$$

$$i_{o2} = g_{mL} \times v_x$$
$$= g_{mL} \times \left[i_{o1} \times \frac{1}{g_{mL}} \times \left(\frac{1}{1+\frac{s}{\omega_{px}}}\right)\right]$$
$$= i_{o1}\left(\frac{1}{1+\frac{s}{\omega_{px}}}\right)$$

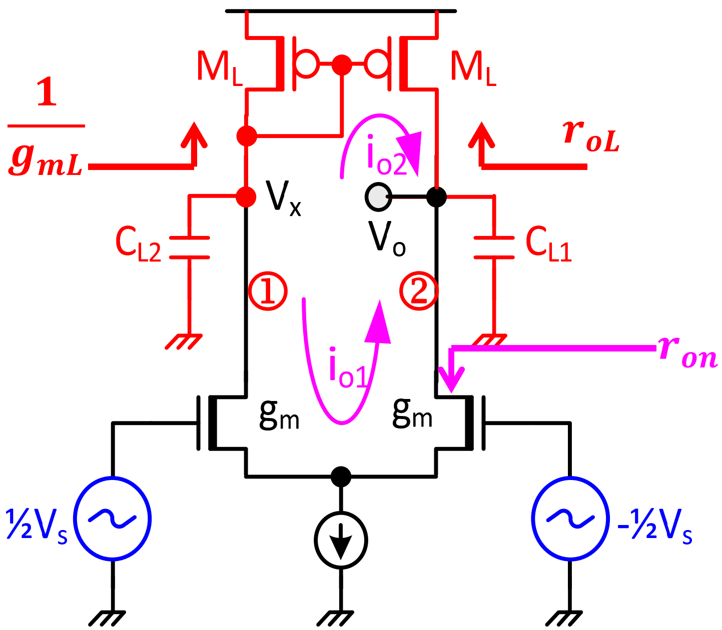
$$v_o = (i_{o1} + i_{o2}) \times (r_{on} \parallel r_{oL}) \times \underbrace{\left(\frac{1}{1+\frac{s}{\omega_p}}\right)}_{Pole-frequency\ at\ node\ ②}, \omega_p = \frac{1}{(r_{on} \parallel r_{oL})C_{L1}}$$

$$= i_{o1}(r_{on} \parallel r_{oL}) \times \left(1 + \frac{1}{1+\frac{s}{\omega_{px}}}\right)\left(\frac{1}{1+\frac{s}{\omega_p}}\right)$$

$$= 2i_{o1}(r_{on} \parallel r_{oL}) \times \frac{\left(1+\frac{s}{2\omega_{px}}\right)}{\left(1+\frac{s}{\omega_p}\right)\left(1+\frac{s}{\omega_{px}}\right)}$$

Differential Amplifier with Active Load (2)

- Differential amplifier with active loading is **heavily mismatched case ⇒ "pole-zero doublet".**



$$v_o = 2 i_{o1}\left(r_{on} \parallel r_{oL}\right) \times \frac{\overbrace{\left(1+\frac{s}{2\omega_{px}}\right)}^{\text{zero-freqeuncy}=2\times\text{pole at node ①}}}{\underbrace{\left(1+\frac{s}{\omega_p}\right)}_{\substack{\text{Pole-frequency}\\ \text{at output node}}}\underbrace{\left(1+\frac{s}{\omega_{px}}\right)}_{\substack{\text{Second}\\ \text{pole-frequency}\\ \text{from node ①}}}}$$

"pole-zero doublet"

- In differential amplifier with active-load,

1) Output node contributes dominant pole
2) Non-dominant pole (the second pole) comes from the other path
3) There is a zero frequency which is 2x larger than the non-dominant pole frequency.

⇒ 2-pole & 1-zero system

- Note that 2x frequency difference is often regarded as "close frequency", and the system can be approximated as one-pole system frequently.