Chapter 10

Stability and Frequency Compensation

Negative feedback finds wide application in the processing of analog signals. The properties of feedback described in Chapter 8 allow precise operations by suppressing variations of the open-loop characteristics. Feedback systems, however, suffer from potential instability, that is, they may oscillate.

In this chapter, we deal with the stability and frequency compensation of linear feedback systems to the extent necessary to understand design issues of analog feedback circuits. Beginning with a review of stability criteria and the concept of phase margin, we study frequency compensation, introducing various techniques suited to different op amp topologies. We also analyze the impact of frequency compensation on the slew rate of two-stage op amps.

10.1 General Considerations

Let us consider the negative feedback system shown in Fig. 10.1, where β is assumed constant. Writing the closed-loop transfer function as

$$\frac{Y}{X}(s) = \frac{H(s)}{1+\beta H(s)}, \tag{10.1}$$

we note that if $\beta H(s = j\omega_1) = -1$, the "gain" goes to infinity, and the circuit can amplify its own noise until it eventually begins to oscillate. In other words, if $\beta H(j\omega_1) = -1$, then

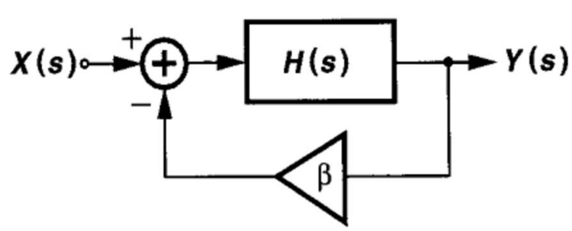

Figure 10.1 Basic negative-feedback system.

the circuit may oscillate at frequency ω_1. This condition can be expressed as

$$|\beta H(j\omega_1)| = 1 \tag{10.2}$$

$$\angle\beta H(j\omega_1) = -180^\circ, \tag{10.3}$$

which are called "Barkhausen's Criteria." Note that the total phase shift around the loop at ω_1 is 360° because *negative* feedback itself introduces 180° of phase shift. The 360° phase shift is necessary for oscillation since the feedback signal must add *in phase* to the original noise to allow oscillation buildup. By the same token, a loop gain of unity (or greater) is also required to enable growth of the oscillation amplitude.

In summary, a negative feedback system may oscillate at ω_1 if (1) the phase shift around the loop at this frequency is so much that the feedback becomes *positive* and (2) the loop gain is still enough to allow signal buildup. Illustrated in Fig. 10.2, the situation can be viewed as excessive loop gain at the frequency for which the phase shift reaches -180° or, equivalently, excessive phase at the frequency for which the loop gain drops to unity. Thus, to avoid instability, we must minimize the total phase shift so that for $|\beta H| = 1$, $\angle\beta H$ is still more positive than -180°. In this chapter, we assume β is less than or equal to unity and does not depend on the frequency.

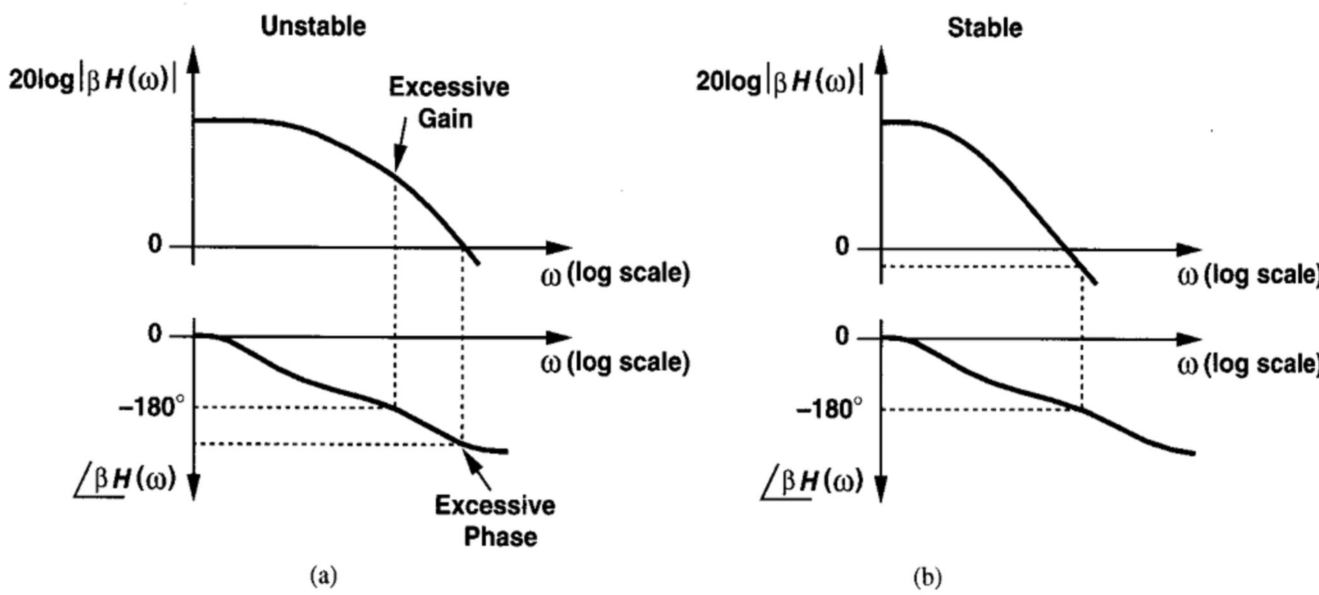

Figure 10.2 Bode plots of loop gain for unstable and stable systems.

The frequencies at which the magnitude and phase of the loop gain are equal to unity and -180°, respectively, play a critical role in the stability and are called the "gain crossover point" and the "phase crossover point," respectively. In a stable system, the gain crossover must occur well before the phase crossover. For the sake of brevity, we denote the gain crossover by GX and the phase crossover by PX. Note that if β is reduced (i.e., less feedback is applied), then the magnitude plots of Fig. 10.2 are shifted down, thereby moving the gain crossover closer to the origin and making the feedback system more stable. Thus, the

worst-case stability corresponds to $\beta = 1$, i.e, unity-gain feedback. For this reason, we often analyze the magnitude and phase plots for $\beta H = H$.

Before studying more specific cases, let us review a few basic rules of constructing Bode plots. A Bode plot illustrates the asymptotic behavior of the magnitude and phase of a complex function according to the magnitude of the poles and zeros. The following two rules are used. (1) The slope of the magnitude plot changes by +20 dB/dec at every zero frequency and by −20 dB/dec at every pole frequency. (2) For a pole (zero) frequency of ω_m, the phase begins to fall (rise) at approximately $0.1\omega_m$, experiences a change of −45° (+45°) at ω_m, and approaches a change of −90° (+90°) at approximately $10\omega_m$. The key point here is that the phase may be much more significantly affected by high-frequency poles and zeros than the magnitude is.

It is also instructive to plot the location of the poles of a closed-loop system on a complex plane. Expressing each pole frequency as $s_p = j\omega_p + \sigma_p$ and noting that the impulse response of the system includes a term $\exp(j\omega_p + \sigma_p)t$, we observe that if s_p falls in the right half plane, i.e., if $\sigma_p > 0$, then the system is likely to oscillate because its time-domain response exhibits a growing exponential [Fig. 10.3(a)]. Even if $\sigma_p = 0$, the system may sustain oscillations [Fig. 10.3(b)]. Conversely, if the poles lie in the left half plane, all time-domain exponential terms decay to zero [Fig. 10.3(c)].[1] In practice, we plot

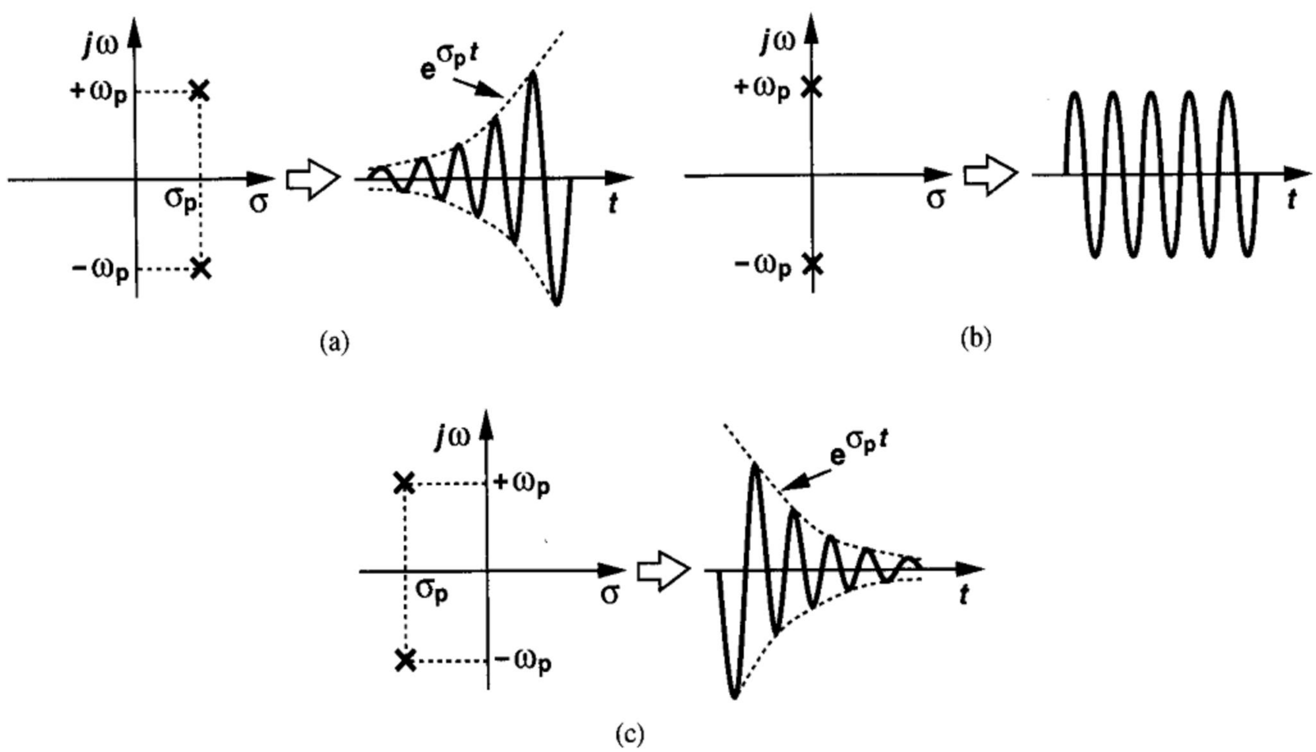

Figure 10.3 Time-domain response of a system versus the position of poles, (a) unstable with growing amplitude, (b) unstable with constant-amplitude oscillation, (c) stable.

[1] We ignore the effect of zeros for now.

the location of the poles as the loop gain varies, thereby revealing how close to oscillation the system may come. Such a plot is called a "root locus."

We now study a feedback system incorporating a one-pole feedforward amplifier. Assuming $H(s) = A_0/(1 + s/\omega_0)$, we have from (10.1),

$$\frac{Y}{X}(s) = \frac{\dfrac{A_0}{1+\beta A_0}}{1 + \dfrac{s}{\omega_0(1+\beta A_0)}}. \tag{10.4}$$

In order to analyze the stability behavior, we plot $|\beta H(s = j\omega)|$ and $\angle\beta H(s = j\omega)$ (Fig. 10.4), observing that a single pole cannot contribute a phase shift greater than 90° and the system is unconditionally stable for all non-negative values of β. Note that $\angle\beta H$ is independent of β.

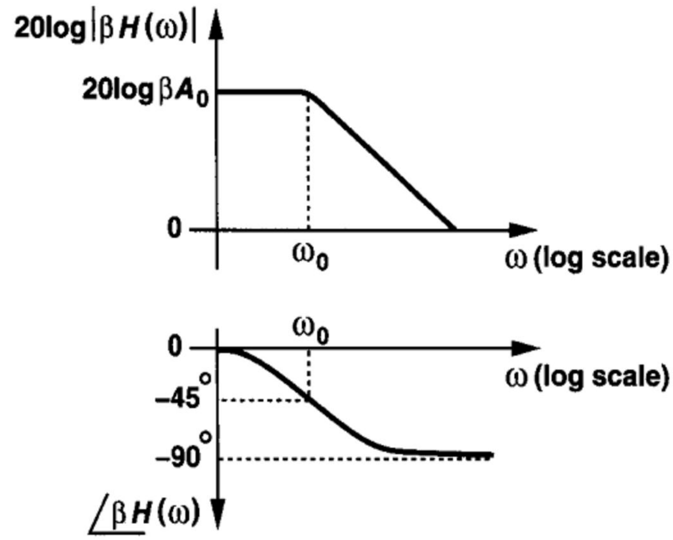

Figure 10.4 Bode plots of loop gain for a one-pole system.

Example 10.1

Construct the root locus for a one-pole system.

Solution

Equation (10.4) implies that the closed-loop system has a pole $s_p = -\omega_0(1+\beta A_0)$, i.e., a real-valued pole in the left half plane that moves away from the origin as the loop gain increases (Fig. 10.5).

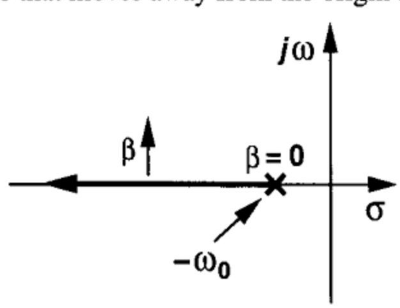

Figure 10.5

10.2 Multipole Systems

Our study of op amps in Chapter 9 indicates that such circuits generally contain multiple poles. In two-stage op amps, for example, each gain stage introduces a "dominant" pole. It is therefore important to study a feedback system whose core amplifier exhibits more than one pole.

Let us consider a two-pole system first. For stability considerations, we plot $|\beta H|$ and $\angle\beta H$ as a function of the frequency. Shown in Fig. 10.6, the magnitude begins to drop at 20 dB/dec at $\omega = \omega_{p1}$ and at 40 dB/dec at $\omega = \omega_{p2}$. Also, the phase begins to change at $\omega = 0.1\omega_{p1}$, reaches $-45°$ at $\omega = \omega_{p1}$ and $-90°$ at $\omega = 10\omega_{p1}$, begins to change again at $\omega = 0.1\omega_{p2}$ (if $0.1\omega_{p2} > 10\omega_{p1}$), reaches $-135°$ at $\omega = \omega_{p2}$, and asymptotically approaches $-180°$. The system is therefore stable because $|\beta H|$ drops to below unity at a frequency for which $\angle\beta H < -180°$.

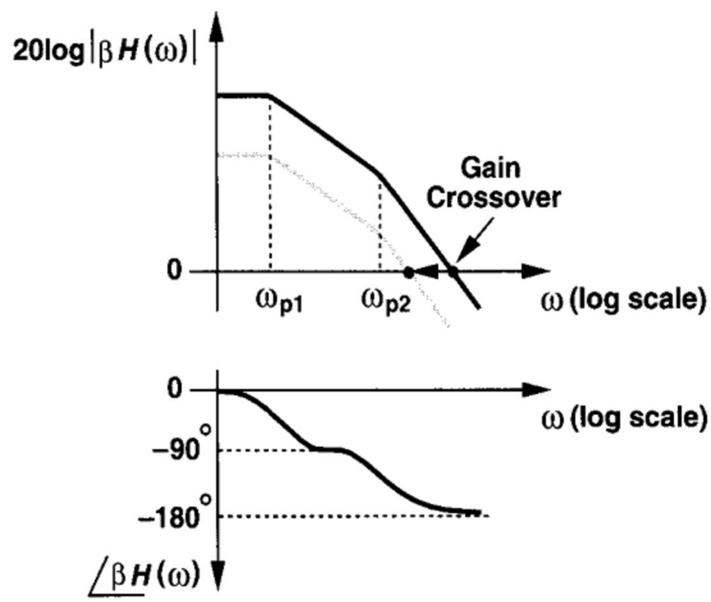

Figure 10.6 Bode plots of loop gain for a two-pole system.

What happens if the feedback is made "weaker?" To reduce the amount of feedback, we decrease β, obtaining the gray magnitude plot in Fig. 10.6. For a logarithmic vertical axis, a change in β translates the magnitude plot vertically. Note that the phase plot does not change. The key point is that as the feedback becomes weaker, the gain crossover point moves toward the origin while the phase crossover point remains constant, resulting in a more stable system. The stability is obtained at the cost of weaker feedback.

Example 10.2

Construct the root locus for a two-pole system.

Solution

Writing the open-loop transfer function as:

$$H(s) = \frac{A_0}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}, \tag{10.5}$$

we have

$$\frac{Y}{X}(s) = \frac{A_0}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)+\beta A_0} \tag{10.6}$$

$$= \frac{A_0\omega_{p1}\omega_{p2}}{s^2+(\omega_{p1}+\omega_{p2})s+(1+\beta A_0)\omega_{p1}\omega_{p2}}. \tag{10.7}$$

Thus, the closed-loop poles are given by

$$s_{1,2} = \frac{-(\omega_{p1}+\omega_{p2}) \pm \sqrt{(\omega_{p1}+\omega_{p2})^2 - 4(1+\beta A_0)\omega_{p1}\omega_{p2}}}{2}. \tag{10.8}$$

As expected, for $\beta = 0$, $s_{1,2} = -\omega_{p1}, -\omega_{p2}$. As β increases, the term under the square root drops, taking on a value of zero for

$$\beta_1 = \frac{1}{A_0}\frac{(\omega_{p1}-\omega_{p2})^2}{4\omega_{p1}\omega_{p2}}. \tag{10.9}$$

As shown in Fig. 10.7, the poles begin at $-\omega_{p1}$ and $-\omega_{p2}$, move toward each other, coincide for $\beta = \beta_1$, and become complex for $\beta > \beta_1$.

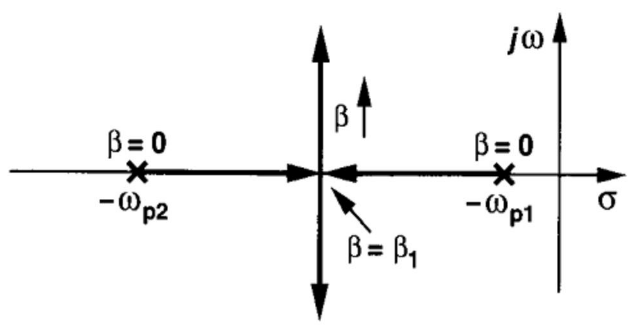

Figure 10.7

The foregoing calculations point to the complexity of the algebra required to construct a root locus for higher-order systems. For this reason, many root locus techniques have been devised so as to minimize such computations [1].

We now study a three-pole system. Shown in Fig. 10.8 are the Bode plots of the magnitude and phase of the loop gain. The third pole gives rise to additional phase shift, possibly moving the phase crossover to frequencies lower than the gain crossover and leading to oscillation.

Since the third pole also decreases the *magnitude* of the loop gain at a greater rate, the reader may wonder why the gain crossover does not move as much as the phase crossover does. As mentioned before, the phase begins to change at approximately one-tenth of the pole frequency whereas the magnitude begins to drop only near the pole frequency. For this reason, additional poles (and zeros) impact the phase to a much greater extent than they do the magnitude.

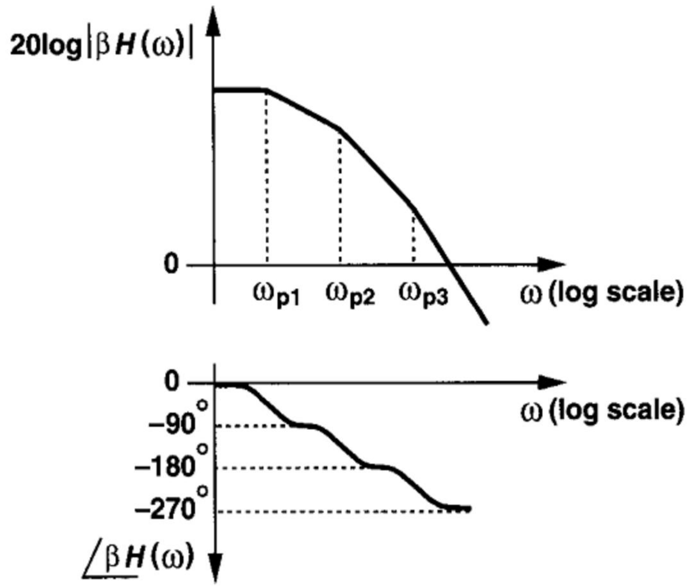

Figure 10.8 Bode plots of loop gain for a three-pole system.

As with a two-pole system, if the feedback factor in Fig. 10.8 decreases, the circuit becomes more stable because the gain crossover moves toward the origin while the phase crossover remains constant.

10.3 Phase Margin

Our foregoing study indicates that to ensure stability, $|\beta H|$ must drop to unity before $\angle\beta H$ crosses $-180°$. We may naturally ask: how far should PX be from GX? Let us first consider a "marginal" case where, as depicted in Fig. 10.9(a), GX is only slightly below PX; sharp peak for example, at GX the phase equals $-175°$. How does the closed-loop system respond in this case? Noting that at GX, $\beta H(j\omega_1) = 1 \times \exp(-j175°)$, we have

$$\frac{Y}{X}(j\omega_1) = \frac{H(j\omega_1)}{1+\beta H(j\omega_1)} \tag{10.10}$$

$$= \frac{\frac{1}{\beta}\exp(-j175°)}{1+\exp(-j175°)} \tag{10.11}$$

$$= \frac{1}{\beta}\cdot\frac{-0.9962 - j0.0872}{0.0038 - j0.0872}, \tag{10.12}$$

and hence

$$\left|\frac{Y}{X}(j\omega_1)\right| = \frac{1}{\beta}\cdot\frac{1}{0.0872} \tag{10.13}$$

$$\approx \frac{11.5}{\beta}. \tag{10.14}$$

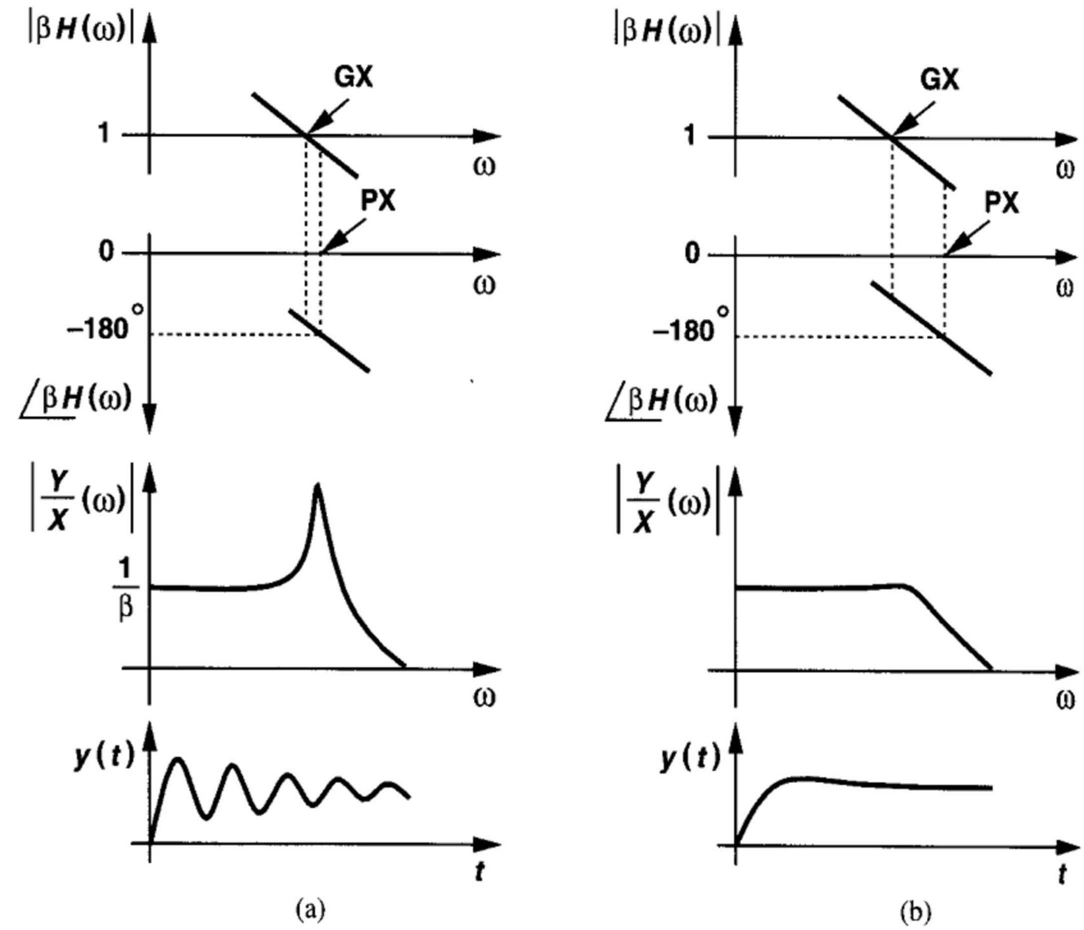

Figure 10.9 Closed-loop frequency and time response for (a) small and (b) large margin between gain and phase crossover points.

Since at low frequencies, $|Y/X| \approx 1/\beta$, the closed-loop frequency response exhibits a sharp peak in the vicinity of $\omega = \omega_1$. In other words, the closed-loop system is near oscillation and its step response exhibits a very underdamped behavior. This point also reveals that a second-order system may suffer from ringing although it is stable.

Now suppose, as shown in Fig. 10.9(b), GX precedes PX by a greater margin. Then, we expect a relatively "well-behaved" closed-loop response in both the frequency domain and the time domain. It is therefore plausible to conclude that the greater the spacing between GX and PX (while GX remains below PX), the more stable the feedback system. Alternatively, the phase of βH at the gain crossover frequency can serve as a measure of stability: the smaller $|\angle \beta H|$ at this point, the more stable the system.

This observation leads us to the concept of "phase margin" (PM), defined as $PM = 180^\circ + \angle \beta H(\omega = \omega_1)$, where ω_1 is the gain crossover frequency.

Example 10.3

A two-pole feedback system is designed such that $|\beta H(\omega_{p2})| = 1$ and $|\omega_{p1}| \ll |\omega_{p2}|$ (Fig. 10.10). What is the phase margin?

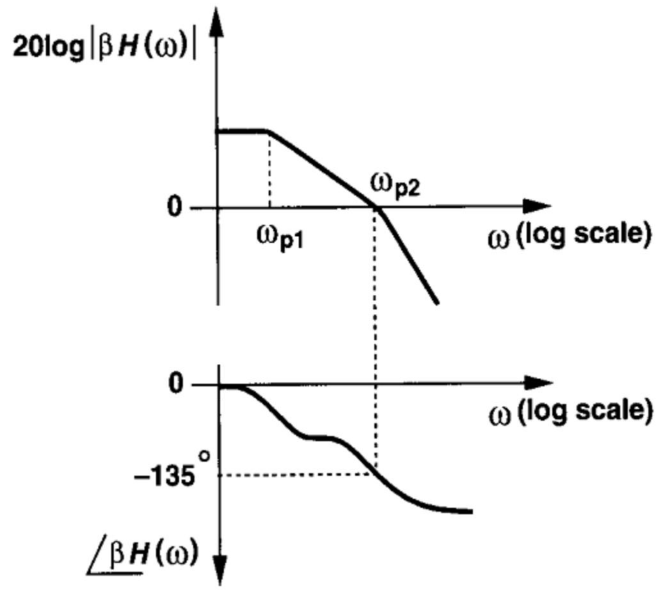

Figure 10.10

Solution

Since $\angle \beta H$ reaches $-135°$ at $\omega = \omega_{p2}$, the phase margin is equal to 45°.

How much phase margin is adequate? It is instructive to examine the closed-loop frequency response for different phase margins [1]. For $PM = 45°$, at the gain crossover frequency $\angle \beta H(\omega_1) = -135°$ and $|\beta H(\omega_1)| = 1$ (Fig. 10.11), yielding

$$\frac{Y}{X} = \frac{H(j\omega_1)}{1 + 1 \times \exp(-j135°)} \tag{10.15}$$

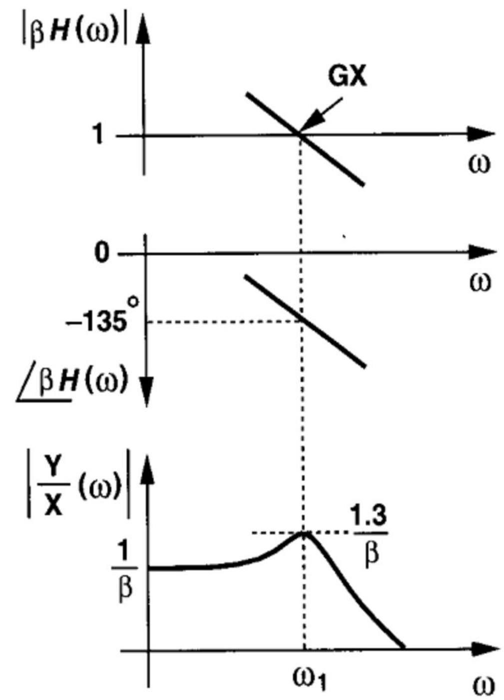

Figure 10.11 Closed-loop frequency response for 45° phase margin.

$$= \frac{H(j\omega_1)}{0.29 - 0.71j}. \tag{10.16}$$

It follows that

$$\left|\frac{Y}{X}\right| = \frac{1}{\beta} \cdot \frac{1}{|0.29 - 0.71j|} \tag{10.17}$$

$$\approx \frac{1.3}{\beta}. \tag{10.18}$$

Consequently, the frequency response of the feedback system suffers from a 30% peak at $\omega = \omega_1$.

It can be shown that for $PM = 60°$, $Y(j\omega_1)/X(j\omega_1) = 1/\beta$, suggesting a negligible frequency peaking. This typically means that the step response of the feedback system exhibits little ringing, providing a fast settling. For greater phase margins, the system is more stable but the time response slows down (Fig. 10.12). Thus, $PM = 60°$ is typically considered the optimum value.

The concept of phase margin is well-suited to the design of circuits that process *small* signals. In practice, the large-signal step response of feedback amplifiers does not follow the illustration of Fig. 10.12. This is not only due to slewing but also because of the nonlinear behavior resulting from large excursions in the bias voltages and currents of the amplifier. Such excursions in fact cause the pole and zero frequencies to *vary* during the transient, leading to a complicated time response. Thus, for large-signal applications, time-domain simulations of the closed-loop system prove more relevant and useful than small-signal ac computations of the open-loop amplifier.

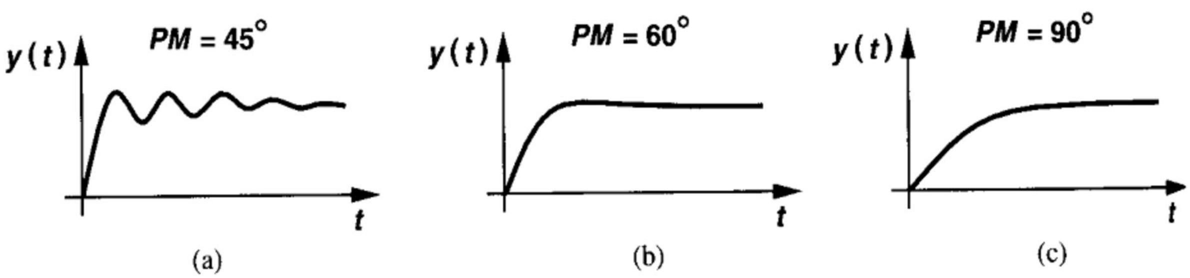

Figure 10.12 Closed-loop time response for 45°, 60°, and 90° phase margins.

As an example of a feedback circuit exhibiting a reasonable phase margin but poor settling behavior, consider the unity-gain amplifier of Fig. 10.13, where the aspect ratio of all transistors is equal to 50 μm / 0.6 μm. With the choice of the device dimensions, bias currents, and capacitor values shown here, SPICE yields a phase margin of approximately 65° and a unity-gain frequency of 150 MHz. The large-signal step response, however, suffers from significant ringing.

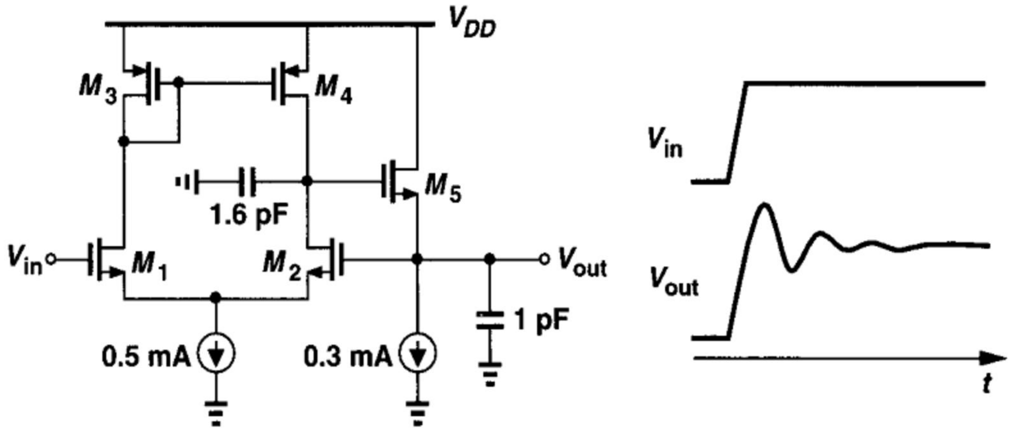

Figure 10.13 Unity-gain buffer.

10.4 Frequency Compensation

Typical op amp circuits contain many poles. In a folded-cascode topology, for example, both the folding node and the output node contribute poles. For this reason, op amps must usually be "compensated," that is, their open-loop transfer function must be modified such that the closed-loop circuit is stable and the time response is well-behaved.

The need for compensation arises because $|\beta H|$ does not drop to unity well before $\angle\beta H$ reaches $-180°$. We then postulate that stability can be achieved by (1) minimizing the overall phase shift, thus pushing the phase crossover *out* [Fig. 10.14(a)]; or (2) dropping the gain, thereby pushing the gain crossover *in* [Fig. 10.14(b)]. The first approach requires that we attempt to minimize the number of poles in the signal path by proper design. Since each additional stage contributes at least one pole, this means the number of stages must be

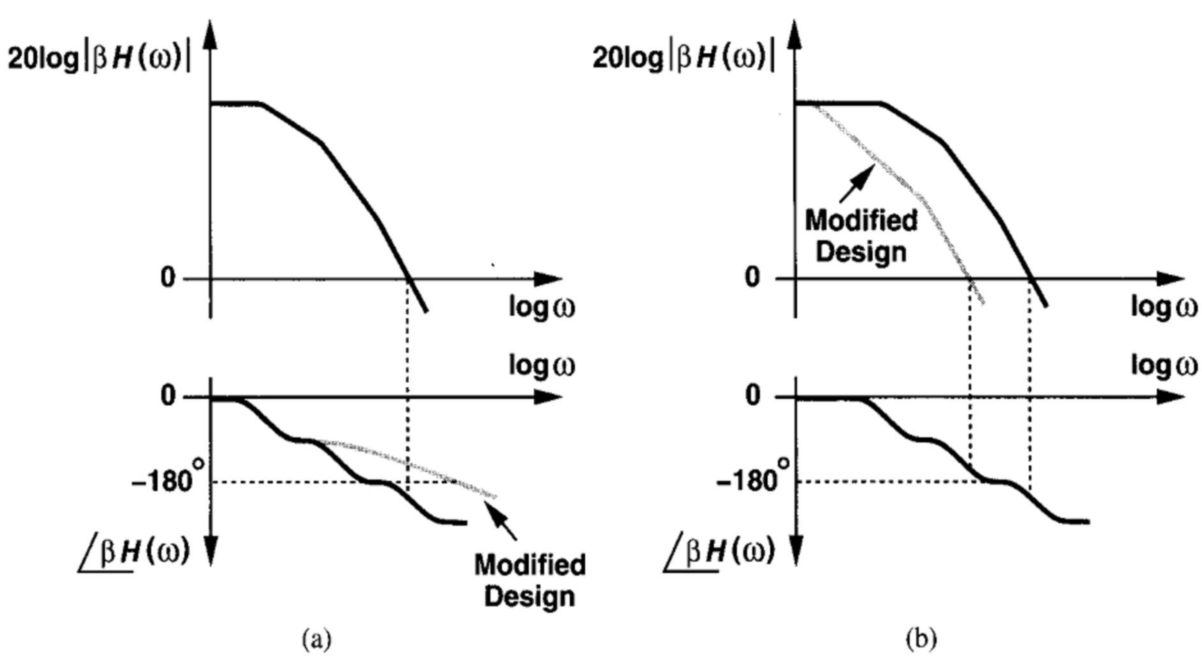

Figure 10.14 Frequency compensation by (a) moving PX out, (b) pushing GX in.