CHAPTER

4 Noise Analysis and Modelling

To develop good analog circuit design techniques, a basic understanding of noise sources and analysis is required. Another motivation to study noise analysis is to learn basic concepts of random signals for a proper understanding of oversampling converters. The purpose of this chapter is to present some fundamentals of noise analysis followed by an introduction to electronic noise sources and circuit analysis.

It should be mentioned here that this chapter deals with *inherent noise* as opposed to *interference noise*. Interference noise is a result of unwanted interaction between the circuit and the outside world, or between different parts of the circuit itself. This type of noise may or may not appear as random signals. Examples are power supply noise on ground wires (such as a 60-Hz hum) or electromagnetic interference between wires. Interference noise can be significantly reduced by careful circuit wiring or layout. In contrast, inherent noise refers to random noise signals that can be reduced but never eliminated since this noise is due to fundamental properties of the circuits. Some examples of inherent noise are thermal, shot, and flicker noise. Inherent noise is only moderately affected by circuit wiring or layout, such as using multiple base contacts to change the resistance value of the base of a transistor. However, inherent noise can be significantly reduced through proper circuit design, such as changing the circuit structure or increasing the power consumption.

The outline of this chapter is as follows: First, a time-domain analysis of noise signals (or other random signals) is presented. Here, basic concepts such as rms value, signal-to-noise ratio, and noise summation are presented. Next, a frequency-domain analysis of noise is presented. As with deterministic signals, analysis in the frequency domain results in more powerful analysis tools than does analysis that remains strictly in the time domain. Noise models for circuit elements are then presented, and finally, two circuit noise analyses are performed to give the reader some experience in such analysis.

4.1 TIME-DOMAIN ANALYSIS

Since inherent noise signals are random in nature, we define here some basic tools to effectively deal with random signals. Specifically, in this section, we define the following terms in the time domain: rms value, SNR, dBm, and noise summation.

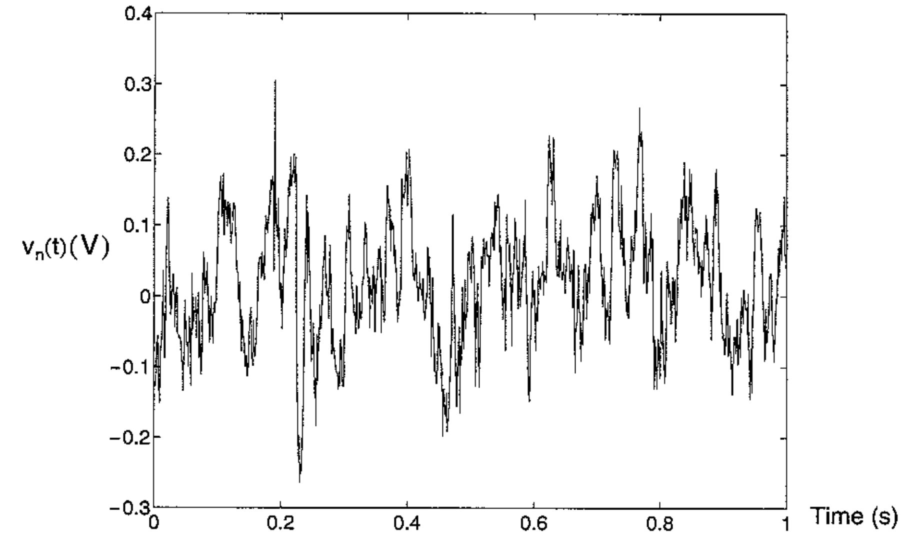


Fig. 4.1 Example of a voltage noise signal (time domain).

An example of a random noise signal in the time domain is shown in Fig. 4.1. Although this signal here is a voltage signal, it could just as easily be current noise or some other quantity. It should be noted that this noise signal appears to have an average value of zero. In fact, *throughout this chapter we will assume all noise signals have a mean value of zero*, which simplifies many of the definitions and is also valid in most physical systems.

Rms Value

Consider a noise voltage, $v_n(t)$, such as that shown in Fig. 4.1, or a noise current, $i_n(t)$. The *rms*, or *root mean square*, voltage value is defined[1] as

$$V_{n(rms)} \equiv \left[\frac{1}{T}\int_0^T v_n^2(t)\,dt\right]^{1/2} \tag{4.1}$$

where T is a suitable averaging time interval. Typically, a longer T gives a more accurate rms measurement. Similarly, the rms current value is defined as

$$I_{n(rms)} \equiv \left[\frac{1}{T}\int_0^T i_n^2(t)\,dt\right]^{1/2} \tag{4.2}$$

The benefit in knowing the rms value of a signal is that it indicates the *normalized noise power* of the signal. Specifically, if the random signal $v_n(t)$ is applied to a 1-Ω resistor, the average power dissipated, P_{diss}, in watts, equals the normalized noise

1. For those more rigorously inclined, we assume throughout this chapter that random signals are ergodic, implying their ensemble averages can be approximated by their time averages.

power and is given by

$$P_{diss} = \frac{V^2_{n(rms)}}{1\ \Omega} = V^2_{n(rms)} \tag{4.3}$$

This relationship implies that the power dissipated by a resistor is the same whether a random signal or a dc level of k volts (rms) is applied across it. For example, a noise signal with an rms value of 1 mV (rms) dissipates the same power across a resistor as a dc voltage of 1 mV.

Similarly, for a noise current, $i_n(t)$, applied to a 1-Ω resistor,

$$P_{diss} = I^2_{n(rms)} \times 1\ \Omega = I^2_{n(rms)} \tag{4.4}$$

As a result, the square of the rms values, $V^2_{n(rms)}$ and $I^2_{n(rms)}$, are sometimes referred to as the normalized noise powers of these two signals.

SNR

The *signal-to-noise ratio (SNR)* value (in dB) of a signal node in a system is defined as

$$\mathrm{SNR} \equiv 10\log\left[\frac{\text{signal power}}{\text{noise power}}\right] \tag{4.5}$$

Thus, assuming a node in a circuit consists of a signal, $v_x(t)$, that has a normalized signal power of $V^2_{x(rms)}$ and a normalized noise power of $V^2_{n(rms)}$, the SNR is given by

$$\mathrm{SNR} = 10\log\left[\frac{V^2_{x(rms)}}{V^2_{n(rms)}}\right] = 20\log\left[\frac{V_{x(rms)}}{V_{n(rms)}}\right] \tag{4.6}$$

Clearly, when the mean-squared values of the noise and signal are the same, then $\mathrm{SNR} = 0$ dB.

Units of dBm

Although dB units relate the relative ratio of two power levels, it is often useful to know a signal's power in dB on an absolute scale. One common measure is that of dBm, where all power levels are referenced by 1 mW. In other words, a 1-mW signal corresponds to 0 dBm, whereas a 1-μW signal corresponds to −30 dBm. When voltage levels are measured, it is also common to reference the voltage level to the equivalent power dissipated if the voltage is applied across either a 50-Ω or a 75-Ω resistor.

EXAMPLE 4.1

Find the rms voltage of a 0-dBm signal referenced to a 50-Ω resistor. What is the level in dBm of a 2-volt rms signal?

Solution

A 0-dBm signal referenced to a 50-Ω resistor implies that the rms voltage level equals

$$V_{(rms)} = \sqrt{(50\ \Omega) \times 1\ \text{mW}} = 0.2236 \tag{4.7}$$

Thus, a 2-volt rms signal corresponds to

$$20\log\left(\frac{2.0}{0.2236}\right) = 19\ \text{dBm} \tag{4.8}$$

and would dissipate 80 mW across a 50-Ω resistor.

Note that the measured voltage may never be physically applied across any 50-Ω resistor. The measured voltage is referenced only to power levels that would occur if the voltage were applied. Similar results are obtained if the power is referenced to a 75-Ω resistor.

Noise Summation

Consider the case of two noise sources added together, as shown in Fig. 4.2. If the rms values of each individual noise source are known, what can be said about the rms value of the combined signal? We answer this question as follows. Define $v_{no}(t)$ as

$$v_{no}(t) = v_{n1}(t) + v_{n2}(t) \tag{4.9}$$

where $v_{n1}(t)$ and $v_{n2}(t)$ are two noise sources with known rms values $V_{n1(rms)}$ and $V_{n2(rms)}$, respectively. Then we can write

$$V^2_{no(rms)} = \frac{1}{T}\int_0^T [v_{n1}(t) + v_{n2}(t)]^2\,dt \tag{4.10}$$

which, when expanded, gives,

$$V^2_{no(rms)} = V^2_{n1(rms)} + V^2_{n2(rms)} + \frac{2}{T}\int_0^T v_{n1}(t)v_{n2}(t)\,dt \tag{4.11}$$

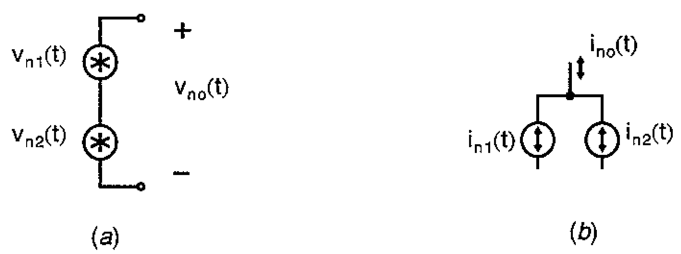


Fig. 4.2 Combining two noise sources, (*a*) voltage, and (*b*) current.

Note that the first two terms in the right-hand side of (4.11) are the individual mean-squared values of the noise sources. The last term shows the correlation between the two signal sources, $v_{n1}(t)$ and $v_{n2}(t)$. An alternate way to write (4.11) that better indicates the effects of signal correlation, is to define a correlation coefficient, C, as

$$C \equiv \frac{\frac{1}{T}\int_0^T v_{n1}(t)v_{n2}(t)\,dt}{V_{n1(rms)}V_{n2(rms)}} \tag{4.12}$$

With this definition, (4.11) can also be written as

$$V_{no(rms)}^2 = V_{n1(rms)}^2 + V_{n2(rms)}^2 + 2CV_{n1(rms)}V_{n2(rms)} \tag{4.13}$$

It can be shown that the correlation coefficient always satisfies the condition $-1 \le C \le 1$. Also, a value of $C = \pm 1$ implies the two signals are fully correlated, whereas $C = 0$ indicates the signals are uncorrelated. Values in between imply the signals are partially correlated. Fortunately, we have little reason to analyze partially correlated signals since different inherent noise sources are typically uncorrelated.

In the case of two uncorrelated signals, the mean-squared value of their sum is given by

$$V_{no(rms)}^2 = V_{n1(rms)}^2 + V_{n2(rms)}^2 \tag{4.14}$$

This relationship indicates that two rms values add as though they were vectors at right angles to each other (i.e., orthogonal) when signals are uncorrelated.

It is of interest to contrast this uncorrelated case with that for fully correlated signals. An example of two fully correlated (though deterministic) sources are two sinusoidal signals that have the same frequency and a phase of 0 or 180 degrees with each other. In this fully correlated case, the mean-squared value of their sum is given by

$$V_{no(rms)}^2 = [V_{n1(rms)} \pm V_{n2(rms)}]^2 \tag{4.15}$$

where the sign is determined by whether the signals are in or out of phase with each other. Note that, in this case where the signals are fully correlated, the rms values add linearly (similar to aligned vectors).

EXAMPLE 4.2

Given two uncorrelated noise sources that have $V_{n1(rms)} = 10\ \mu V$ and $V_{n2(rms)} = 5\ \mu V$, find their total output rms value when combined. If we are required to maintain the total rms value at $10\ \mu V$, how much should $V_{n1(rms)}$ be reduced while $V_{n2(rms)}$ remains unchanged?

Solution

Using (4.14) results in

$$V_{no(rms)}^2 = (10^2 + 5^2) = 125(\mu V)^2 \tag{4.16}$$

which results in $V_{no(rms)} = 11.2\ \mu V$.

To maintain $V_{no(rms)} = 10\ \mu V$ and $V_{n2(rms)} = 5\ \mu V$, we have

$$10^2 = V_{n1(rms)}^2 + 5^2 \tag{4.17}$$

which results in $V_{n1(rms)} = 8.7\ \mu V$. Therefore, reducing $V_{n1(rms)}$ by 13 percent is equivalent to eliminating $V_{n2(rms)}$ altogether!

The above example has an important moral. *To reduce overall noise, concentrate on large noise signals.*

4.2 FREQUENCY-DOMAIN ANALYSIS

As with deterministic signals, the frequency-domain techniques are useful for dealing with random signals such as noise. This section presents frequency-domain techniques for dealing with noise signals and other random signals. It should be noted that units of hertz (Hz) (rather than radians/second) are used throughout this chapter since, historically, such units have been commonly used in the measurement of continuous-time spectral densities.

Noise Spectral Density

Although periodic signals (such as a sinusoid) have power at distinct frequency locations, random signals have their power spread out over the frequency spectrum. For example, if the time-domain signal shown in Fig. 4.1 is applied to a spectrum analyzer, the resulting spectrum might look like that shown in Fig. 4.3(*a*). Note here that although the horizontal scale is the usual frequency axis, the vertical scale is in units of microvolts-squared/hertz. In other words, the vertical axis is a measure of the normalized noise power (mean-squared value) over a 1-Hz bandwidth at each frequency point. For example, the measurement at 100 Hz in Fig. 4.3(*a*) indicates that the normalized power between 99.5 Hz and 100.5 Hz is $10\ (\mu V)^2$.

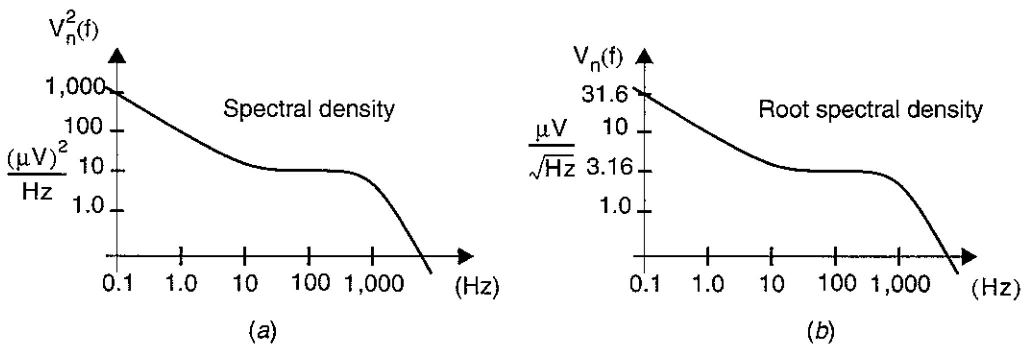


Fig. 4.3 Example of voltage spectral density (frequency domain), for (*a*) spectral density, and (*b*) root spectral density.

Thus, we define the noise spectral density, $V_n^2(f)$ (or, in the case of current, $I_n^2(f)$), as the average normalized noise power over a 1-Hz bandwidth. The units of $V_n^2(f)$ are volts-squared/hertz, whereas those of $I_n^2(f)$ are amps-squared/hertz. Also, $V_n^2(f)$ is a positive real-valued function.

It should be emphasized here that the *mean-squared value of a random noise signal at a single precise frequency is zero*. In other words, the mean-squared value of the signal shown in Fig. 4.3(*a*) measured at 100 Hz is directly proportional to the bandwidth of the bandpass filter used for the measurement. In a laboratory spectrum analyzer, the bandwidth of the bandpass filter is determined by the resolution-bandwidth control. Thus, as the resolution bandwidth goes to zero, the mean-squared value also becomes zero.[2] Conversely, as the resolution bandwidth increases, so does the measured mean-squared value. In either case, the measured mean-squared value should be normalized to the value that would be obtained for a bandwidth of 1 Hz when the noise spectral density is stated in units of $V^2/(Hz)$. For example, if the signal corresponding to Fig. 4.3(*a*) were measured at around $0.1\ Hz$ using a resolution bandwidth of $1\ mHz$, the mean-squared value measured would be $1\ (\mu V)^2$, which, when scaled to a 1-Hz bandwidth, equals $1{,}000\ (\mu V^2)/Hz$.

An intuitive explanation of how a random noise signal is measured using a spectrum analyzer is as follows. A random noise signal has a frequency that is continually changing through a broad continuum of frequencies. A spectrum analyzer is sensitive to only a narrow frequency range that is the passband of its filter, and it measures the mean-squared value in that range. The filter of the spectrum analyzer reacts with a time constant approximately given by

$$\tau \approx \frac{1}{\pi W} \tag{4.18}$$

where W is the bandwidth of the filter. For some of the time, the random signal has the same frequency as the spectrum analyzer's filter. Thus, the spectrum analyzer effectively measures what percentage of time the random signal is in the frequency range of its filter. The narrower the bandwidth of the filter, the less percentage of time the signal is within its frequency range, and therefore the smaller is the spectrum analyzer's output. However, if the signal is not a random signal wandering in and out of the frequency range of the spectrum analyzer's filter, but is a deterministic signal at the center frequency of the filter, then the spectrum analyzer's reading is independent of the filter's bandwidth.

It is often convenient to plot the square root of the noise spectral density when we deal with filtered noise. Taking a square root results in $V_n(f)$, as shown in Fig. 4.3(*b*). We will refer to $V_n(f)$ as the *root spectral density which is expressed in units of volts/root-hertz* (i.e., $V/\sqrt{Hz}$). In the case of current noise, the resulting units are *amps/root-hertz*. Note that the horizontal axis remains unchanged although there is a root-hertz factor in the vertical axis.

Since the spectral density measures the mean-squared value over a 1-Hz bandwidth, one can obtain the total mean-squared value by integrating the spectral density

2. Such a result would not occur when a 100-Hz sinusoidal waveform is measured.

over the entire frequency spectrum. Thus, the rms value of a noise signal can also be obtained in the frequency domain using the following relationship:

$$V_{n(rms)}^2 = \int_0^\infty V_n^2(f)\, df \tag{4.19}$$

and similarly for current noise,

$$I_{n(rms)}^2 = \int_0^\infty I_n^2(f)\, df \tag{4.20}$$

Finally, it should be mentioned here that the spectral density function, $V_n^2(f)$, is the Fourier transform of the autocorrelation function of the time-domain signal, $v_n(t)$. This relationship is known as the *Wiener–Khinchin theorem.* It should also be noted that the relationship just shown defines a one-sided spectral density function since the noise is integrated only over positive frequencies as opposed to both negative and positive frequencies, again primarily for historical reasons. A two-sided definition results in the spectral density being divided by two since, for real-valued signals, the spectral density is the same for positive and negative frequencies, and the total mean-squared value remains unchanged.

EXAMPLE 4.3

What mean-squared value would be measured on the signal shown in Fig. 4.3 at 100 Hz when a resolution bandwidth of 30 Hz is used on a spectrum analyzer? Answer the same question for a 0.1-Hz resolution bandwidth.

Solution

Since the portion of spectral density function is flat at about 100 Hz, the measured value should simply be proportional to the bandwidth. Since the noise spectral density is $10\ (\mu V)^2/Hz$ at $100\ Hz$, the output of a 30-Hz filter is $30\ Hz \times 10\ (\mu V)^2/Hz = 300\ (\mu V)^2$ (or an rms value of $\sqrt{300}\ \mu V$).

For a 0.1-Hz bandwidth, the measured value would be 10 times smaller than for a 1-Hz bandwidth, resulting in a value of $1\ (\mu V)^2$ (or an rms value of $1\ \mu V$).

White Noise

One common type of noise is *white noise.* A noise signal is said to be white if its spectral density is constant over a given frequency. In other words, a white noise signal would have a flat spectral density, as shown in Fig. 4.4, where $V_n(f)$ is given by

$$V_n(f) = V_{nw} \tag{4.21}$$

and V_{nw} is a constant value.

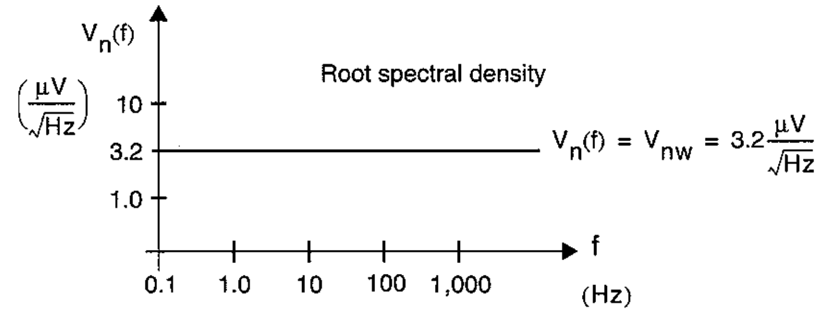


Fig. 4.4 An example of a white noise signal.

1/f, or Flicker, Noise

Another common noise shape is that of *1/f, or flicker,* noise.[3] The spectral density, $V_n^2(f)$, of 1/f noise is approximated by

$$V_n^2(f) = \frac{k_v^2}{f} \tag{4.22}$$

where k_v is a constant. Thus, the *spectral density is inversely proportional to frequency,* and hence the term "1/f noise." In terms of root spectral density, 1/f noise is given by

$$V_n(f) = \frac{k_v}{\sqrt{f}} \tag{4.23}$$

Note that it is inversely proportional to $\sqrt{f}$ (rather than f). An example of a signal having both 1/f and white noise is shown in Fig. 4.5. Note that the 1/f noise falls off at a rate of −10 dB/decade since it is inversely proportional to $\sqrt{f}$. The intersection of the 1/f and white noise curves is often referred to as the *1/f noise corner* (it occurs at 10 Hz in Fig. 4.5).

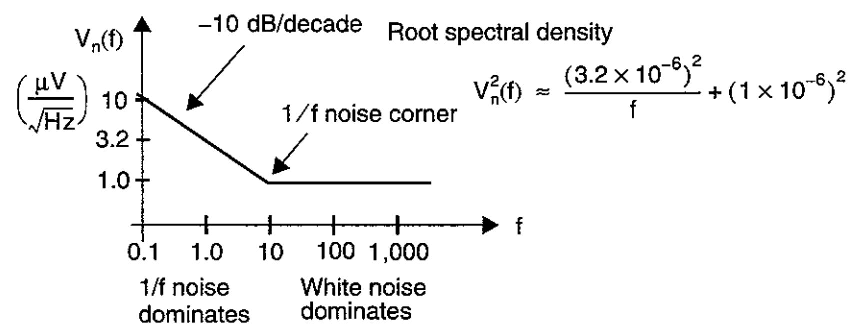


Fig. 4.5 A noise signal that has both 1/f and white noise.

3. 1/f, or flicker, noise is also referred to as *pink noise* since it has a large low-frequency content.

Filtered Noise

Consider the case of a noise signal, $V_{ni}(f)$, being filtered by the transfer function $A(s)$, as shown in Fig. 4.6. Here, $A(s)$ represents a linear transfer function as a result of some circuit amplification, filtering, or both. The following relationship between the input and output signals can be derived using the definition of the spectral density.

$$V_{no}^2(f) = |A(j2\pi f)|^2 V_{ni}^2(f) \tag{4.24}$$

The term $2\pi f$ arises here since, for physical frequencies, we replace s with $j\omega = j2\pi f$. Note that the output spectral density is a function only of the magnitude of the transfer function, and not its phase. As a result of (4.24), the total output mean-squared value is given by

$$V_{no(rms)}^2 = \int_0^\infty |A(j2\pi f)|^2 V_{ni}^2(f)\, df \tag{4.25}$$

If we wish to work with root spectral densities, we can take the square root of both sides of (4.24) resulting in

$$V_{no}(f) = |A(j2\pi f)| V_{ni}(f) \tag{4.26}$$

The relationship in (4.26) should make intuitive sense since it indicates that the transfer function simply shapes the root spectral density of the input noise signal. It is important to note here that the root spectral density is simply shaped by $|A(j2\pi f)|$, whereas the spectral density is shaped by $|A(j2\pi f)|^2$ (as seen by (4.24)). Hence, we see the benefit in dealing with the root spectral density rather than the spectral density. Specifically, straightforward transfer function analysis is applied when using root spectral densities, whereas squared terms are required to deal with spectral densities.

It is also of interest to consider the case of multiple uncorrelated noise sources that are each filtered and summed together. For example, consider the system shown in Fig. 4.7 in which three filtered, uncorrelated noise sources combine to form the total output noise, $V_{no}(f)$. In this case, one can show that if the input random signals are uncorrelated, the filter outputs are also uncorrelated. As a result, the output spectral density is given by

$$V_{no}^2(f) = \sum_{i=1,2,3} |A_i(j2\pi f)|^2 V_{ni}^2(f) \tag{4.27}$$

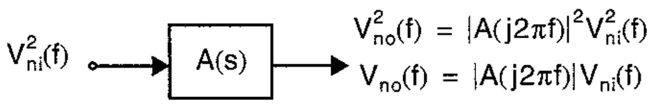


Fig. 4.6 Applying a transfer function (i.e., filter) to a noise signal.

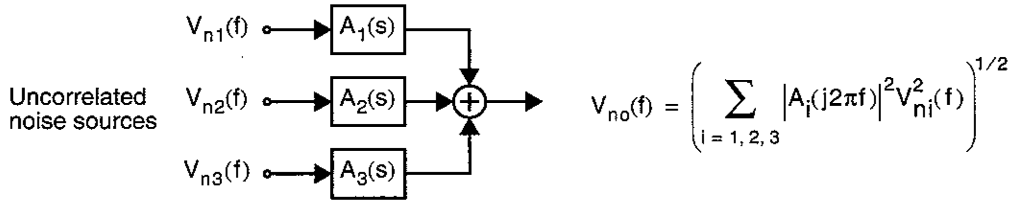


Fig. 4.7 Filtered uncorrelated noise sources contributing to total output noise.

EXAMPLE 4.4

Consider a noise signal, $V_{ni}(f)$, that has a white root spectral density of $20\ nV/\sqrt{Hz}$, as shown in Fig. 4.8(*a*). Find the total noise rms value between dc and 100 kHz. What is the total noise rms value if it is filtered by the RC filter shown in Fig. 4.8(*b*), where it is assumed the RC filter is noise free?

Solution

For the noise mean-square value from dc to 100 kHz of $V_{ni}(f)$, we have

$$V_{ni(rms)}^2 = \int_0^{10^5} 20^2\, df = 4 \times 10^7 (nV)^2 \qquad (4.28)$$

resulting in an rms value of $V_{ni(rms)} = 6.3\ \mu V$ rms. Note that, for this simple case, one could also obtain the rms value by multiplying $20\ nV/\sqrt{Hz}$ by the square root of the frequency span, or $\sqrt{100\ kHz}$, resulting in $6.3\ \mu V$ rms.

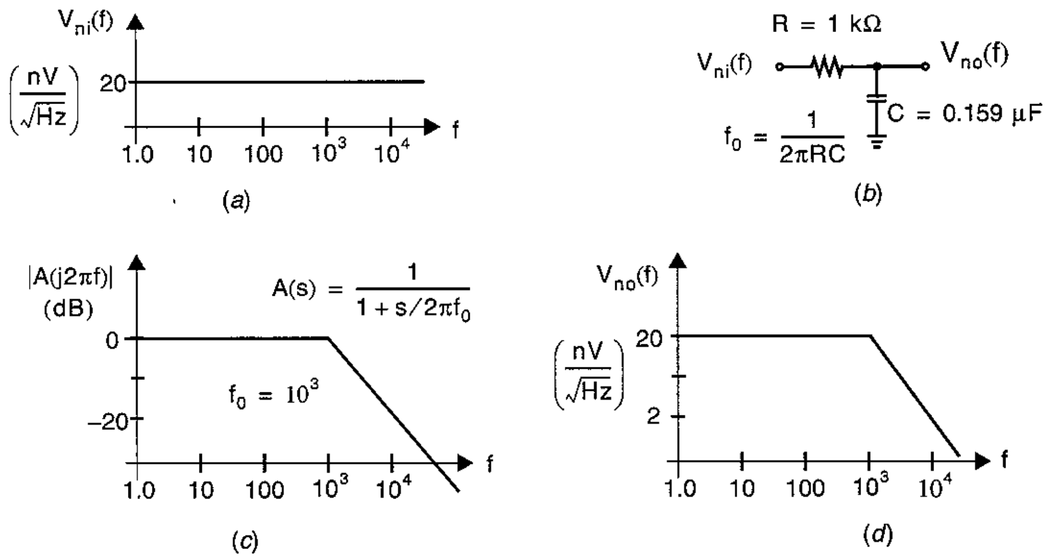


Fig. 4.8 (*a*) Spectral density for $V_{ni}(f)$. (*b*) RC filter to shape noise. (*c*) RC filter frequency response. (*d*) Spectral density for $V_{no}(f)$.

For the filtered signal, $V_{no}(f)$, we find that the RC filter has the frequency response shown in Fig. 4.8(*c*). Therefore, we can multiply the root spectral density of $V_{ni}(f)$ with the frequency response, $|A(j2\pi f)|$, to obtain the root spectral density of $V_{no}(f)$, as shown in Fig. 4.8(*d*). Mathematically, the output root spectral density is given by

$$V_{no}(f) = \frac{20 \times 10^{-9}}{\sqrt{1 + \left(\frac{f}{f_0}\right)^2}} \tag{4.29}$$

where $f_0 = 10^3$. Thus, the noise mean-squared value of $V_{no}(f)$ between dc and 100 kHz is given by

$$\begin{aligned} V_{no(rms)}^2 &= \int_0^{10^5} \frac{20^2}{1 + \left(\frac{f}{f_0}\right)^2} df = 20^2 f_0 \arctan(f/f_0) \Big|_0^{10^5} \\ &= 6.24 \times 10^5 (\text{nV})^2 \end{aligned} \tag{4.30}$$

which results in an rms value of $V_{no(rms)} = 0.79\ \mu\text{V rms}$. Note that the noise rms value of $V_{no}(f)$ is almost $1/10$ that of $V_{ni}(f)$ since high-frequency noise above 1 kHz was filtered. The lesson here is that you should not design circuits for larger bandwidths than your signal requires, otherwise noise performance suffers.

Noise Bandwidth

We just saw that the spectral density function is determined by the noise power within each 1-Hz bandwidth. Thus, in theory, one could measure the spectral density function by filtering a noise signal with a brick-wall bandpass filter having a 1-Hz bandwidth. The term brick wall implies here that the 1-Hz bandwidth of the filter is passed with a gain of one, whereas all other frequencies are entirely eliminated. However, practical filters can only approach a brick-wall response as their complexity (i.e., filter order) is increased. Thus, for lower-order filters with a 1-Hz passband, more noise power is passed than what is simply in a 1-Hz bandwidth. To account for the fact that practical filters have more gradual stopband characteristics, the term *noise bandwidth* is defined. *The noise bandwidth of a given filter is equal to the frequency span of a brick-wall filter that has the same output noise rms value that the given filter has when white noise is applied to both filters (peak gains are the same for the given and brick-wall filters).* In other words, given a filter response with peak gain A_0, the noise bandwidth is the width of a rectangular filter that has the same area and peak gain, A_0, as the original filter.

For example, consider a first-order, low-pass response with a 3-dB bandwidth of f_0, as shown in Fig. 4.9(*a*). Such a response would occur from the RC filter shown in Fig. 4.8(*b*) with $f_0 = (1/2\pi RC)$. The transfer function of $A(s)$ is given by

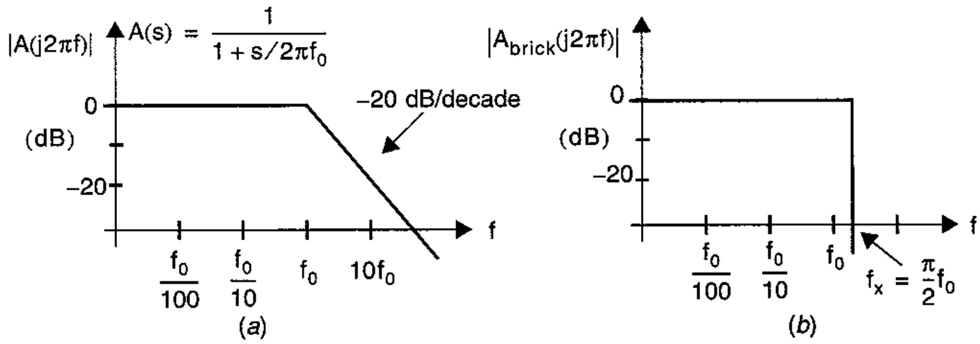


Fig. 4.9 (*a*) A first-order, low-pass response, and (*b*) a brick-wall filter that has the same peak gain and area as the first-order response.

$$A(s) = \frac{1}{1 + \frac{s}{2\pi f_0}} \tag{4.31}$$

This results in the magnitude response of $A(s)$ being equal to

$$|A(jf)| = \left(\frac{1}{1 + \left(\frac{f}{f_0}\right)^2}\right)^{1/2} \tag{4.32}$$

An input signal, $V_{ni}(f)$, is a white noise source given by

$$V_{ni}(f) = V_{nw} \tag{4.33}$$

where V_{nw} is a constant. The total output noise rms value of $V_{no}(f)$ is equal to

$$V^2_{no(rms)} = \int_0^{\infty} \frac{V^2_{nw}}{1 + \left(\frac{f}{f_0}\right)^2} df = V^2_{nw} f_0 \arctan\left(\frac{f}{f_0}\right)\Bigg|_0^{\infty} = \frac{V^2_{nw}\pi f_0}{2} \tag{4.34}$$

If this same input signal, $V_{ni}(f)$, is applied to the filter shown in Fig. 4.9(*b*), then the total output noise rms value equals,

$$V^2_{brick(rms)} = \int_0^{f_x} V^2_{nw}\, df = V^2_{nw} f_x \tag{4.35}$$

Finally, equating the two output noise rms values, $V_{no(rms)} = V_{brick(rms)}$, results in

$$f_x = \frac{\pi f_0}{2} \tag{4.36}$$

Thus, the noise bandwidth of a first-order, low-pass filter with a 3-dB bandwidth of f_0 equals $\pi(f_0/2)$. Note that, for the common case in which a first-order circuit is realized by a capacitor, C, and the resistance seen by that capacitor, R_{eq}, then

$$f_0 = \frac{1}{2\pi R_{eq} C} \tag{4.37}$$

and the noise bandwidth is given by

$$f_x = \frac{1}{4R_{eq}C} \tag{4.38}$$

The advantage of knowing the noise bandwidth of a filter is that, when white noise is applied to the filter input, the total output noise mean-squared value is easily calculated by multiplying the spectral density by the noise bandwidth. Specifically, in the first-order case just described, the total output noise mean-squared value, $V^2_{no(rms)}$, is equal to

$$V^2_{no(rms)} = V^2_{nw}f_x = V^2_{nw}\left(\frac{\pi}{2}\right)f_0 \tag{4.39}$$

Similar results for noise-bandwidth relationships can be obtained for higher-order and bandpass filters.

Piecewise Integration of Noise

Although simulation and computer techniques can analyze circuits or systems quite precisely (depending on their modelling accuracy), it is often convenient to make approximations that are useful in the early design stages. Such approximations are particularly useful in noise analysis, where large inaccuracies occur naturally due to such things as unknown parasitic effects and incomplete noise models. One such approximation is the estimation of total noise by integrating over frequency with the assumption that piecewise-linear Bode diagrams are exact. With such an approximation, integration formulas become much simpler when one needs only to integrate under linear functions and add together the resulting portions. The following example demonstrates this approach.

EXAMPLE 4.5

Consider an input noise signal, $V_{ni}(f)$, being applied to the amplifier $A(s)$, as shown in Fig. 4.10. Find the output noise rms value of $V_{no}(f)$ above 1 Hz.

Solution

As shown, the output root spectral density, $V_{no}(f)$, is determined by the addition of the Bode diagrams for $V_{ni}(f)$ and $A(s)$. To perform piecewise integration of $V_{no}(f)$, the frequency range is broken into four regions, N_1 through N_4, as shown.

For the region N_1, the total mean-square noise is given by

$$N_1^2 = \int_1^{100} \frac{200^2}{f}\, df = 200^2 \ln(f)\Big|_1^{100} = 1.84 \times 10^5 (\text{nV})^2 \tag{4.40}$$

In the region N_2, we have

$$N_2^2 = \int_{100}^{10^3} 20^2\, df = 20^2 f\Big|_{100}^{10^3} = 3.6 \times 10^5 (\text{nV})^2 \tag{4.41}$$

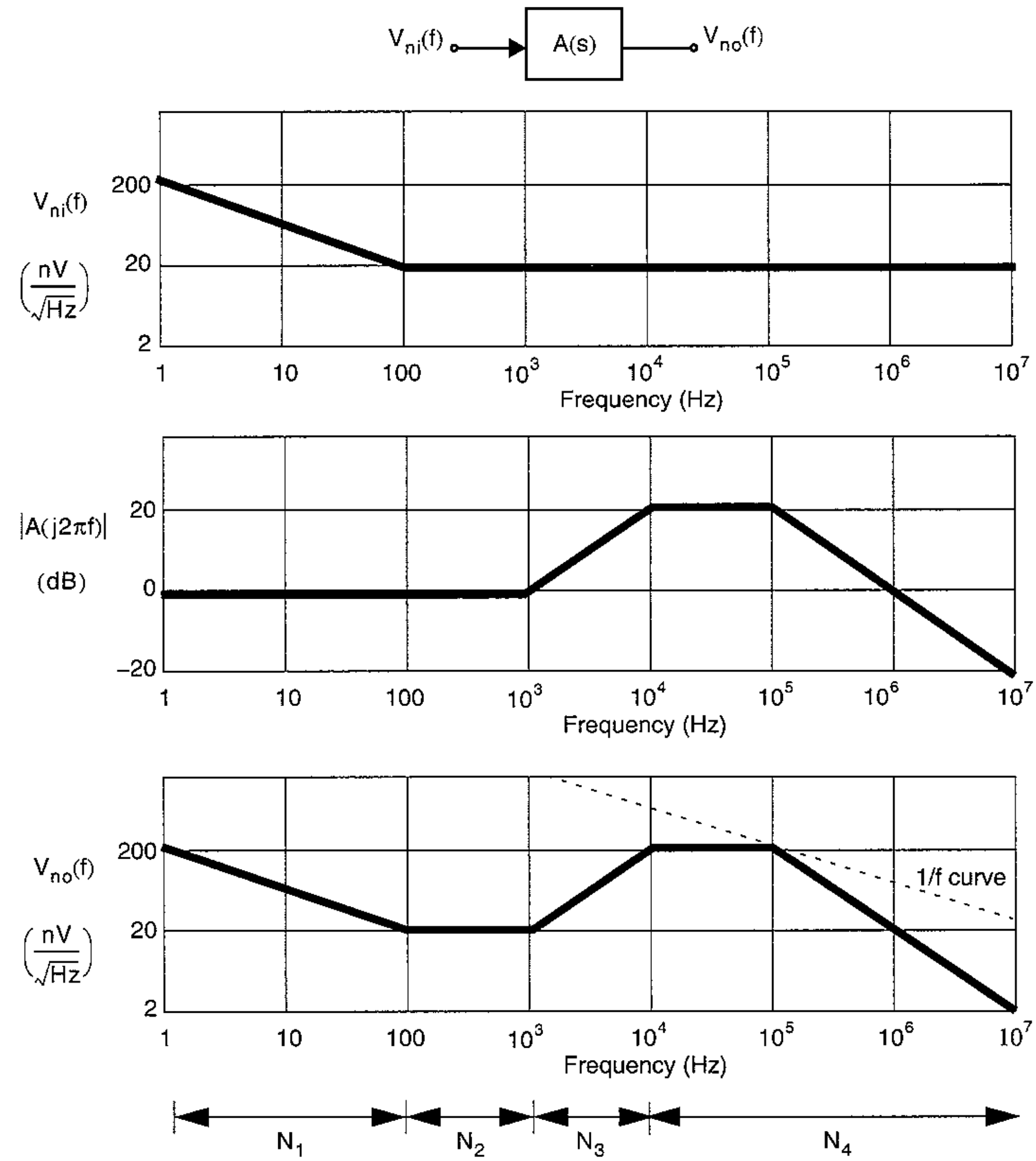


Fig. 4.10 Root spectral densities and amplifier curve example. $V_{no}(f)$ is the output noise that results from applying an input signal with noise $V_{ni}(f)$ to the amplifier $A(s)$.

Region N_3 ramps up rather than down, resulting in

$$N_3^2 = \int_{10^3}^{10^4} \frac{20^2 f^2}{(10^3)^2} df = \left(\frac{20}{10^3}\right)^2 \left[\frac{1}{3} f^3 \Big|_{10^3}^{10^4}\right] = 1.33 \times 10^8 (\mathrm{nV})^2 \tag{4.42}$$

Finally, for region N_4, we can use the noise bandwidth result of a first-order, low-pass response and simply remove the noise portion resulting from under

10^4 Hz. Specifically, we have,

$$N_4^2 = \int_{10^4}^{\infty} \frac{200^2}{1+\left(\frac{f}{10^5}\right)^2} df = \int_0^{\infty} \frac{200^2}{1+\left(\frac{f}{10^5}\right)^2} df - \int_0^{10^4} 200^2 df \tag{4.43}$$

$$= 200^2\left(\frac{\pi}{2}\right)10^5 - (200^2)(10^4) = 5.88 \times 10^9 (\text{nV})^2$$

Thus, the total output noise can be estimated to be

$$V_{no(rms)} = (N_1^2 + N_2^2 + N_3^2 + N_4^2)^{1/2} = 77.5\ \mu\text{V rms} \tag{4.44}$$

An interesting point to note here is that in the preceding example, $N_4 =$ $76.7\ \mu\text{V rms}$ is quite close to the total noise value of $77.5\ \mu\text{V rms}$. Thus, in practice, there is little need to find the noise contributions in the regions N_1, N_2, and N_3. Such an observation leads us to the 1/f noise tangent principle.

1/f Noise Tangent Principle

The *1/f noise tangent principle* is as follows: To determine the frequency region or regions that contribute the dominant noise, *lower a 1/f noise line until it touches the spectral density curve—the total noise can be approximated by the noise in the vicinity of the 1/f line* [Kennedy, 1988]. For example, lowering a 1/f line toward the root spectral density of $V_{no}(f)$ in Fig. 4.10 indicates that the noise around 10^5 dominates. The reason this simple rule works is that a curve proportional to $1/x$ results in equal power over each decade of frequency. Therefore, by lowering this constant power/frequency curve, the largest power contribution will touch it first. However, because the integration of $1/x$ approaches infinity if either the upper bound is infinity or the lower bound is zero, one must be careful in cases where the spectral density curve runs parallel to a 1/f tangent line for an appreciable frequency span. For example, consider once again Fig. 4.10, where the region N_1 runs parallel to the 1/f tangent line. However, in this example, region N_1 does not contribute much noise since the noise was only integrated above 1 Hz. If a much lower frequency bound is used, this region can also contribute appreciable noise power.

4.3 NOISE MODELS FOR CIRCUIT ELEMENTS

There are three main fundamental noise mechanisms—thermal, shot, and flicker. In this section, we discuss noise models for popular circuit elements where all three mechanisms occur. However, first we briefly describe these noise phenomena.

Thermal noise is due to the thermal excitation of charge carriers in a conductor. This noise has a white spectral density and is proportional to absolute temperature. It

is not dependent on bias conditions (dc bias current) and it occurs in all resistors (including conductors) above absolute zero. Thus, thermal noise places fundamental limits on the dynamic range achievable in electronic circuits. It should be mentioned here that thermal noise is also referred to as Johnson or Nyquist noise since it was first observed by J. B. Johnson [Johnson, 1928] and analyzed using the second law of thermodynamics by H. Nyquist [Nyquist, 1928].

Shot noise was first studied by W. Schottky using vacuum-tube diodes [Schottky, 1918], but shot noise also occurs in pn junctions. This noise occurs because the dc bias current is not continuous and smooth but instead is a result of pulses of current caused by the individual flow of carriers. As such, shot noise is dependent on the dc bias current. It can also be modelled as a white noise source. Shot noise is also typically larger than thermal noise and is sometimes used to create white noise generators.

Flicker noise is the least understood of the three noise phenomena. It is found in all active devices as well as in carbon resistors,[4] but it occurs only when a dc current is flowing. Flicker noise usually arises due to traps in the semiconductor, where carriers that would normally constitute dc current flow are held for some time period and then released. Flicker noise is also commonly referred to as 1/f noise since it is well modelled as having a $1/f^{\alpha}$ spectral density, where α is between 0.8 and 1.3. Although both bipolar and MOSFET transistors have flicker noise, it is a significant noise source in MOS transistors, whereas it can often be ignored in bipolar transistors.

Resistors

The major source of noise in resistors is thermal noise. As just discussed, it appears as white noise and can be modelled as a voltage source, $V_R(f)$, in series with a noiseless resistor. With such an approach, the spectral density function, $V_R^2(f)$, is found to be given by

$$V_R^2(f) = 4kTR \tag{4.45}$$

where k is Boltzmann's constant ($1.38 \times 10^{-23}\ \mathrm{JK^{-1}}$), T is the temperature in Kelvins, and R is the resistance size.

An alternate way to write (4.45) is to note that a $1\text{-k}\Omega$ resistor exhibits a root spectral density of $4.06\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$ in thermal noise at room temperature (300 °K). Since the root spectral density is proportional to the square root of the resistance, we can also write

$$V_R(f) = \sqrt{\frac{R}{1k}} \times 4.06\ \mathrm{nV}/\sqrt{\mathrm{Hz}} \quad \text{for } 27\ ^\circ\mathrm{C} \tag{4.46}$$

Note that, to reduce the thermal noise due to resistors, one must either lower the temperature or use lower resistance values. The fact that lower resistance values cause less thermal noise becomes much more apparent when we look at kT/C noise in capacitors later in this section.

4. Carbon resistors are not used in integrated-circuit design but are available as discrete elements.

An alternate model can be derived by finding the Norton equivalent circuit. Specifically, the series voltage noise source, $V_R(f)$, can be replaced with a parallel current noise source, $I_R(f)$, given by

$$I_R^2(f) = \frac{V_R^2(f)}{R^2} = \frac{4kT}{R} \tag{4.47}$$

Both resistor models are summarized in Fig. 4.11.

Diodes

Shot noise is typically the dominant noise in diodes and can be modelled with a current source in parallel with the small-signal resistance of the diode, as Fig. 4.11 shows. The spectral density function of the current source is found to be given by

$$I_d^2(f) = 2qI_D \tag{4.48}$$

where q is one electronic charge (1.6×10^{-19} C) and I_D is the dc bias current flowing through the diode. The small-signal resistance of the diode, r_d, is given by the usual relationship,

$$r_d = \frac{kT}{qI_D} \tag{4.49}$$

Note that the Thévenin equivalent circuit can also be used, as shown in Fig. 4.11. It should be noted here that the small-signal resistance, r_d, is used for modelling and is not a physical resistor; hence, r_d does not contribute any thermal noise.

Bipolar Transistors

The noise in bipolar transistors is due to the shot noise of both the collector and base currents, the flicker noise of the base current, and the thermal noise of the base resistance. A common practice is to combine all these noise sources into two equivalent noise sources at the base of the transistor, as shown in Fig. 4.11. Here, the equivalent input voltage noise, $V_i(f)$, is given by

$$V_i^2(f) = 4kT\left(r_b + \frac{1}{2g_m}\right) \tag{4.50}$$

where the r_b term is due to the thermal noise of the base resistance and the g_m term is due to collector-current shot noise referred back to the input. The equivalent input current noise, $I_i(f)$, equals

$$I_i^2(f) = 2q\left(I_B + \frac{KI_B}{f} + \frac{I_C}{|\beta(f)|^2}\right) \tag{4.51}$$

where the $2qI_B$ term is a result of base-current shot noise, the KI_B/f term models 1/f noise (K is a constant dependent on device properties), and the I_C term is the input-referred collector-current shot noise (it is often ignored).

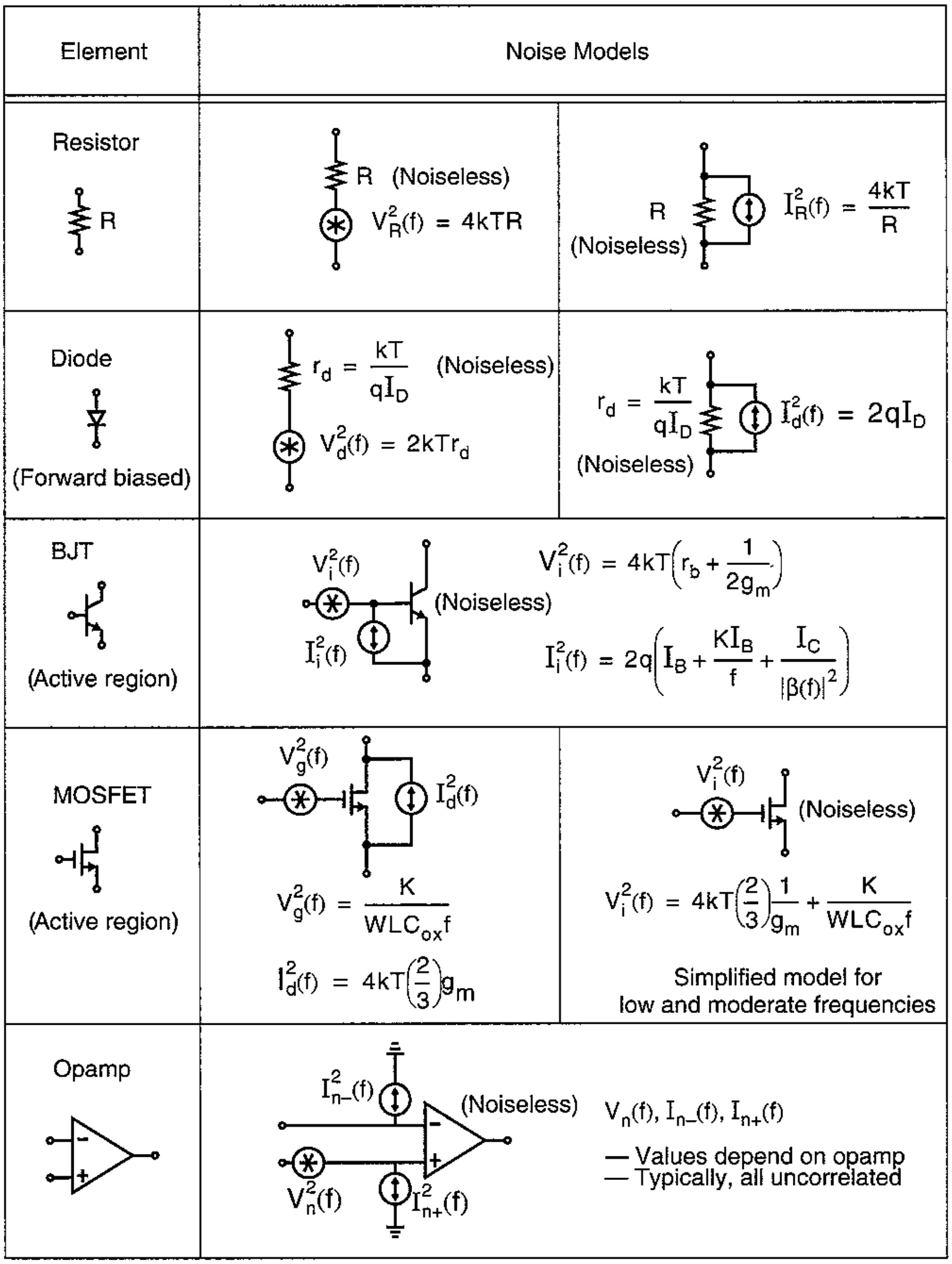


Fig. 4.11 Circuit elements and their noise models. Note that capacitors and inductors do not generate noise.

A couple of comments here are that the noise of r_b typically dominates in $V_i(f)$, and the base-current shot noise often dominates in the input-current noise, $I_i^2(f)$. Thus, the equivalent voltage and current noise are not derived from the same noise sources. As a result, it is common practice to assume that the input voltage and current noise sources in a bipolar transistor are uncorrelated.[5]

MOSFETS

The dominant noise sources for active MOSFET transistors are flicker and thermal noise, as shown in Fig. 4.11. The flicker noise is modelled as a voltage source in series with the gate of value

$$V_g^2(f) = \frac{K}{WLC_{ox}f} \tag{4.52}$$

where the constant K is dependent on device characteristics and can vary widely for different devices in the same process. The variables W, L, and C_{ox} represent the transistor's width, length, and gate capacitance per unit area, respectively. *An important point to note here is that the 1/f noise is inversely proportional to the transistor area,* WL. In other words, larger devices have less 1/f noise. 1/f noise is extremely important in MOSFET circuits, because it typically dominates at low frequencies unless switching-circuit techniques are used to reduce its effect. Also, typically p-channel transistors have less noise than their n-channel counterparts since their majority carriers (holes) are less likely to be trapped.

The derivation of the thermal noise term is straightforward and is due to the resistive channel of a MOS transistor in the active region. If the transistor was in triode, the thermal noise current in the drain due to the channel resistance would simply be given by $I_d^2(f) = (4kT)/r_{ds}$, where r_{ds} is the channel resistance. However, when the transistor is in the active region, the channel cannot be considered homogeneous, and thus, the total noise is found by integrating over small portions of the channel. Such an integration results in the noise current in the drain being given by

$$I_d^2(f) = 4kT\left(\frac{2}{3}\right)g_m \tag{4.53}$$

for the case $V_{DS} = V_{GS} - V_T$.

Often, noise analyses are done just by including this noise source between the transistor drain and source. Sometimes, however, analysis may be simplified if it is replaced by an equivalent input noise source. To find the equivalent noise voltage that would cause this drain current, we note that the drain current is equal to the gate voltage times the transconductance of the device, or, mathematically, $I_d(f) = g_m V_i(f)$.

5. An exception to this is high-frequency designs in which the collector shot noise becomes more important because, at high frequencies, β becomes small. In this case, the input current noise source is partially correlated with the input voltage noise source. If neither noise source dominates, then the correct analysis is more difficult and beyond the scope of this text. Fortunately, this case is not often encountered in practice.

Thus, dividing (4.53) by g_m^2 results in the simplified MOSFET model, also shown in Fig. 4.11, where there is now only one voltage noise source. However, one should be aware that this simplified model assumes the gate current is zero. Although this assumption is valid at low and moderate frequencies, an appreciable amount of current would flow through the gate-source capacitance, C_{gs}, at higher frequencies. In summary, although most noise analysis can be performed using the simplified model, if in doubt, one should use the model with the thermal noise placed as a current source in parallel with the drain-source channel.

Finally, it should be noted that no gate leakage noise terms have been included in this noise model since, in modern process, the gate leakage is so small that its noise contribution is rarely significant.

EXAMPLE 4.6

A large MOS transistor consists of ten individual transistors connected in parallel. Considering 1/f noise only, what is the equivalent input voltage noise spectral density for the ten transistors compared to that of a single transistor?

Solution

From (4.52), the 1/f noise can be modelled as a current source going from the drain to the source with a noise spectral density of

$$I_d^2(f) = \frac{Kg_m^2}{WLC_{ox}f} \tag{4.54}$$

where g_m is the transconductance of a single transistor. When ten transistors are connected in parallel, the drain-current noise spectral density is ten times larger, so we have

$$I_{d=10}^2(f) = \frac{10Kg_m^2}{WLC_{ox}f} \tag{4.55}$$

When this noise is referred back to the input of the equivalent transistor, we have

$$V_{i=10}^2(f) = \frac{10Kg_m^2}{WLC_{ox}fg_{m=10}^2} = \frac{K}{10WLC_{ox}f} \tag{4.56}$$

since the transconductance of ten transistors in parallel, $g_{m=10}$, is equal to $10\,g_m$. Thus, the drain current noise spectral density of the equivalent transistor is ten times larger, but the input voltage noise spectral density is ten times smaller. This result is expected because the input voltage noise source due to 1/f noise is inversely proportional to the equivalent transistor area, which in this case is $10\,WL$.

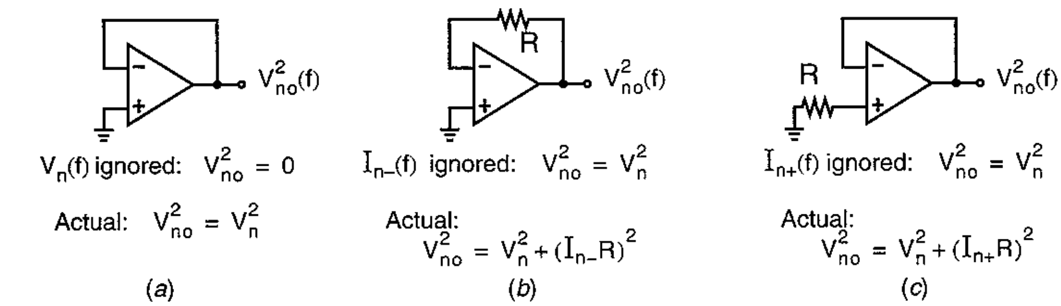


Fig. 4.12 Opamp circuits showing the need for three noise sources in an opamp noise model. Assume the resistance, R, is noiseless. Also, notation is simplified from $V_n(f)$ to V_n, and so on.

Opamps

Noise in opamps is modelled using three uncorrelated input-referred noise sources, as shown in Fig. 4.11. With an opamp that has a MOSFET input stage, the current noises can often be ignored at low frequencies since their values are small. However, for bipolar input stages, all three noise sources are typically required, as shown in Fig. 4.12. In Fig. 4.12(*a*), if $V_n(f)$ is not included in the model, a unity-gain buffer with no resistors indicates that the circuit output is noiseless. In fact, the voltage noise source, $V_n(f)$, dominates. If $I_{n-}(f)$ is not included in an opamp model, the circuit shown in Fig. 4.12(*b*) indicates that the output noise voltage equals $V_n(f)$. Here, the current noise may dominate if the resistance, R, is large. A similar conclusion can be drawn with $I_{n+}(f)$, as shown in Fig. 4.12(*c*).

Capacitors and Inductors

Capacitors and inductors do not generate any noise. However, they do accumulate noise generated by other noise sources. Here, we will see that the capacitor noise mean-squared value equals kT/C when it is connected to an arbitrary resistor value.

Consider a capacitance, C, in parallel with a resistor of arbitrary size, R, as shown in Fig. 4.13(*a*). The equivalent circuit for noise analysis is shown in Fig. 4.13(*b*). To determine the total noise mean-squared value across the capacitor, we note that $V_{no}(f)$ is simply a first-order, low-pass, filtered signal with $V_R(f)$ as the

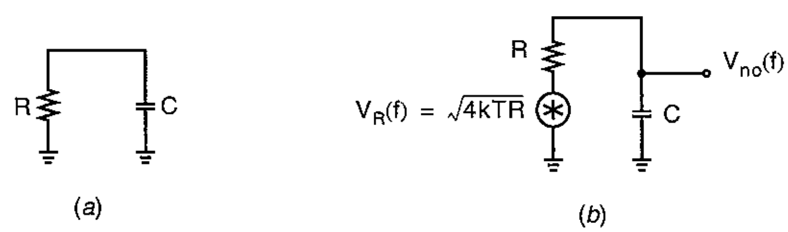


Fig. 4.13 (*a*) Capacitor, C, in parallel with a resistor, and (*b*) equivalent noise model circuit.

input. Therefore, we recognize that the noise bandwidth is given by $(\pi/2)f_0$ (see Section 4.2, Eq. (4.36), and since the input has a white spectral density, the total output mean-squared value is calculated as

$$V_{no(rms)}^2 = V_R^2(f)\left(\frac{\pi}{2}\right)f_0 = (4kTR)\left(\frac{\pi}{2}\right)\left(\frac{1}{2\pi RC}\right)$$
$$V_{no(rms)}^2 = \frac{kT}{C} \tag{4.57}$$

In other words, the rms voltage value across a capacitor is equal to $\sqrt{kT/C}$, regardless of the resistance seen across it. Such a result is due to fact that small resistances have less noise spectral density but result in a wide bandwidth, compared to large resistances, which have reduced bandwidth but larger noise spectral density.

Finally, it should be stated that this noise property for capacitors gives a fundamental limit on the minimum noise level across a capacitor.[6] Thus, to lower the noise level, either the temperature must be lowered or the capacitance value must be increased.

EXAMPLE 4.7

At a room temperature of 300 °K, what capacitor size is needed to achieve a 96-dB dynamic range in an analog circuit with maximum signal levels of 1 V rms?

Solution

The value of noise that can be tolerated here is 96 dB down from 1 V rms, which is

$$V_{n(rms)} = \frac{1V}{10^{96/20}} = 15.8\ \mu V \text{ rms} \tag{4.58}$$

Using (4.57), we have

$$C = \frac{kT}{V_{n(rms)}^2} = 16.6\ pF \tag{4.59}$$

Thus, the minimum capacitor size that can be used (without oversampling) is 16.6 pF. The use of this minimum capacitor size determines the maximum resistance size allowed to achieve a given time constant.

Finally, it should be mentioned that the equivalent noise current mean-squared value in an inductor of value L is given by (see Problem 4.9)

$$I_{no(rms)}^2 = \frac{kT}{L} \tag{4.60}$$

6. Some feedback circuits can make the noise smaller but signal levels are also typically smaller (see Problem 4.8).

Sampled Signal Noise

In many cases, one should obtain a sampled value of an analog voltage. For example, switched-capacitor circuits make extensive use of sampling, and sample-and-hold circuits are commonly used in analog-to-digital and digital-to-analog conversion.

Consider a basic sample-and-hold circuit, as shown in Fig. 4.14. When ϕ_{clk} drops, the transistor turns off and, in an ideal noiseless world, the input voltage signal at that instance would be held on capacitance C. However, when thermal noise is present, the resistance when the transistor is switched on causes the capacitance voltage noise to be equal to $\sqrt{kT/C}$. Thus, when the switch is turned off, the noise as well as the desired signal is held on C. As a result, a fundamental limit occurs for sampled signals using a capacitance C—an rms noise voltage of $\sqrt{kT/C}$.

It should be noted that this noise voltage will not depend on the sampling rate and is independent from sample to sample. This fact suggests a method to reduce the noise level of a signal measurement. Specifically, in the case where V_{in} is a dc (or low-frequency) signal, taking only one sample results in a noise voltage of $\sqrt{kT/C}$. However, if many samples are taken (say, 1,000) and all samples are averaged, the averaged value will have a reduced noise level. The reason this technique improves the measurement accuracy is that, when individual sampled values are summed together, their signal values add linearly, whereas their noise values add as the root of the sum of squares. This technique is known as oversampling and will be discussed at length with respect to oversampling converters in Chapter 14.

4.4 NOISE ANALYSIS EXAMPLES

In this section, a variety of circuits are analyzed from a noise perspective. Although some useful design techniques for reducing noise are presented, the main purpose of this section is to give the reader some experience in analyzing circuits from a noise perspective.

Opamp Example

Consider an inverting amplifier and its equivalent noise model, as shown in Fig. 4.15. Here, $V_n(f)$, $I_{n-}(f)$, and $I_{n+}(f)$ represent the opamp's equivalent input noise, and the remaining noise sources are resistor thermal noise sources. Note that current noise sources are used in the models for R_1 and R_f, whereas a voltage noise source is used for R_2. As we will see, these choices of noise sources simplify the circuit analysis.

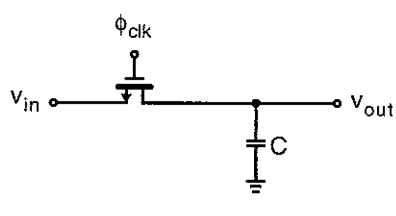


Fig. 4.14 A sample-and-hold circuit.

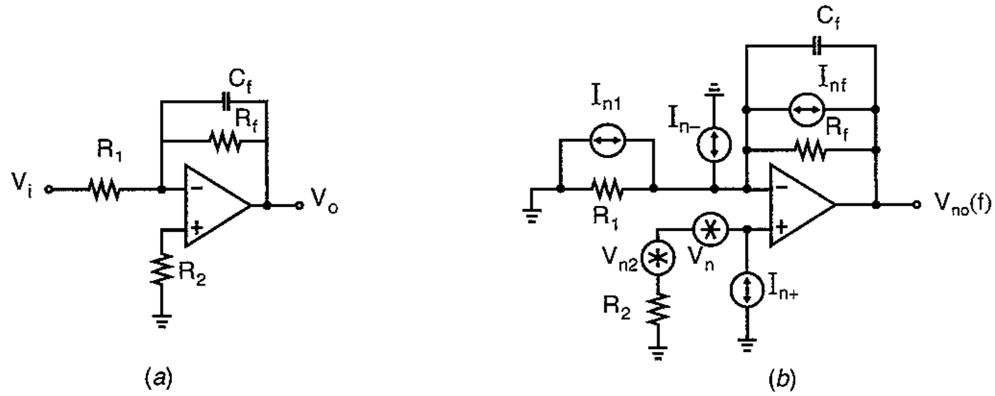


Fig. 4.15 (*a*) Low-pass filter, and (*b*) equivalent noise model.

First, using superposition and assuming all noise sources are uncorrelated, consider the output noise voltage, $V_{no1}^2(f)$, due only to I_{n1}, I_{nf}, and I_{n-}. These three noise currents add together, and their total current sum is fed into the parallel combination of C_f and R_f. Note that no current passes through R_1 since the voltage across it is zero due to the virtual ground at the negative opamp terminal (assuming a high-gain opamp). Thus, the output noise mean-squared value due to these three noise currents is given by

$$V_{no1}^2(f) = [I_{n1}^2(f) + I_{nf}^2(f) + I_{n-}^2(f)]\left|\frac{R_f}{1 + j2\pi f R_f C_f}\right|^2 \tag{4.61}$$

This equation indicates that this part of the output noise value equals the sum of the noise currents multiplied by R_f^2. This noise portion is then shaped by a low-pass filter with a 3-dB frequency equal to $f_0 = 1/(2\pi R_f C_f)$.

Using superposition further, the output mean-squared value, $V_{no2}^2(f)$, due to the three noise sources at the positive opamp terminal can be found as follows: By converting I_{n+} to a voltage source (by multiplying it by R_2), we see that the three noise voltages are summed and are applied to the positive opamp terminal. Since the gain from the positive opamp terminal to the output signal is easily found, the output noise mean-squared value due to these three noise sources is given by

$$V_{no2}^2(f) = [I_{n+}^2(f)R_2^2 + V_{n2}^2(f) + V_n^2(f)]\left|1 + \frac{R_f/R_1}{1 + j2\pi f C_f R_f}\right|^2 \tag{4.62}$$

This equation indicates that this part of the output noise mean-squared value equals the sum of the noise voltages, and this noise portion is then shaped by the shown transfer function. For this transfer function, if $R_f \ll R_1$, then its gain approximately equals unity for all frequencies. Thus, for an ideal opamp, the noise would exist up to infinite frequency, resulting in an infinite amount of mean-squared volts. However, for practical circuits, the gain drops off above the unity-gain frequency of the opamp, and thus the noise is effectively low-pass filtered. In the case where $R_f \gg R_1$, the low-frequency gain is roughly R_f/R_1, and its 3-dB frequency is the same as in the

case of the noise sources at the negative opamp terminal (i.e., $f_0 = 1/(2\pi R_f C_f)$). However, in this case, the gain only decreases to unity and then remains at that level. Treating the Bode plot as exact and noting that, above f_0, the gain drops off at –20 dB/decade, this transfer function reaches unity around $f_1 = (R_f/R_1)f_0$. Thus, one should also include the opamp's positive input noise (with a gain of one) integrated over the region between f_1 and the unity-gain frequency of the opamp.

Finally, the total output noise mean-squared value is simply the sum

$$V_{no}^2(f) = V_{no1}^2(f) + V_{no2}^2(f) \tag{4.63}$$

or, if rms values are found,

$$V_{no(rms)}^2 = V_{no1(rms)}^2 + V_{no2(rms)}^2 \tag{4.64}$$

EXAMPLE 4.8

Estimate the total output noise rms value for a 10-kHz low-pass filter, as shown in Fig. 4.15, when $C_f = 160\ \text{pF}$, $R_f = 100\ \text{k}$, $R_1 = 10\ \text{k}$, and $R_2 = 9.1\ \text{k}$. Also, find the SNR for an input signal equal to 100 mV rms. Assume that the noise voltage of the opamp is given by $V_n(f) = 20\ \text{nV}/\sqrt{\text{Hz}}$, both its noise currents are $I_n(f) = 0.6\ \text{pA}/\sqrt{\text{Hz}}$, and that its unity-gain frequency equals 5 MHz.

Solution

Assuming the device is at room temperature, the resistor noise sources are equal to

$$I_{nf} = 0.406\ \text{pA}/\sqrt{\text{Hz}} \tag{4.65}$$

$$I_{n1} = 1.28\ \text{pA}/\sqrt{\text{Hz}} \tag{4.66}$$

$$V_{n2} = 12.2\ \text{nV}/\sqrt{\text{Hz}} \tag{4.67}$$

The low-frequency value of $V_{no1}^2(f)$ is found by letting $f = 0$ in (4.61).

$$\begin{aligned} V_{no1}^2(0) &= [I_{n1}^2(0) + I_{nf}^2(0) + I_{n-}^2(0)]R_f^2 \\ &= (0.406^2 + 1.28^2 + 0.6^2)(1 \times 10^{-12})^2(100\ \text{k})^2 \\ &= (147\ \text{nV}/\sqrt{\text{Hz}})^2 \end{aligned} \tag{4.68}$$

Since (4.61) also indicates that this noise is low-pass filtered, the rms output noise value due to these three sources can be found using the concept of a noise equivalent bandwidth. Specifically, we multiply the spectral density by $(\pi f_0)/2$, where $f_0 = 1/(2\pi R_f C_f)$. Thus,

$$\begin{aligned} V_{no1(rms)}^2 &= (147\ \text{nV}/\sqrt{\text{Hz}})^2 \times \frac{1}{4(100\ \text{k}\Omega)(160\ \text{pF})} \\ &= (18.4\ \mu\text{V})^2 \end{aligned} \tag{4.69}$$

To estimate the output noise due to the sources at the positive opamp terminal, we find $V_{no2}^2(0)$ to be given by

$$\begin{aligned} V_{no2}^2(0) &= [I_{n+}^2(f)R_2^2 + V_{n2}^2(f) + V_n^2(f)](1 + R_f/R_1)^2 \\ &= (24.1\ \mathrm{nV}/\sqrt{\mathrm{Hz}})^2 \times 11^2 \\ &= (265\ \mathrm{nV}/\sqrt{\mathrm{Hz}})^2 \end{aligned} \tag{4.70}$$

This noise is also low-pass filtered at f_0 until $f_1 = (R_f/R_1)f_0$, where the noise gain reaches unity and it remains until $f_t = 5$ MHz (i.e, the unity-gain frequency of the opamp). Thus, breaking this noise into two portions, and using (4.38) to calculate the first portion, we have

$$\begin{aligned} V_{no2(rms)}^2 &= (265 \times 10^{-9})^2\left(\frac{1}{4R_fC_f}\right) + (24.1 \times 10^{-9})^2\left(\frac{\pi}{2}\right)(f_t - f_1) \\ &= (74.6\ \mu\mathrm{V})^2 \end{aligned} \tag{4.71}$$

Thus, the total output noise is estimated to be

$$V_{no(rms)} = \sqrt{V_{no1(rms)}^2 + V_{no2(rms)}^2} = 77\ \mu\mathrm{V\ rms} \tag{4.72}$$

It should be noted here that the major source of noise at low frequencies is due to the opamp's voltage noise, $V_n(f)$.

To obtain the SNR for this circuit with a 100-mV rms input level, one can find the output signal mean-squared value and relate that to the output noise value. Alternatively, one can find the equivalent input noise value by dividing the output noise value by the circuit's gain and then relate the input signal and noise mean-squared values. Taking the first approach results in an output signal level of 1 V rms, which gives an SNR of

$$\mathrm{SNR} = 20\log\left(\frac{1\ \mathrm{V}}{77\ \mu\mathrm{V}}\right) = 82\ \mathrm{dB} \tag{4.73}$$

Note here that using a lower-speed opamp would have reduced the total output noise, as would choosing an opamp with a lower noise voltage. Also note that R_2 contributes to the output noise through both its thermal noise and the noise current of the opamp's positive input. Since its only purpose is to improve the dc offset performance, R_2 should be eliminated in a low-noise circuit (assuming dc offset can be tolerated).

Bipolar Common-Emitter Example

In this example, we consider a bipolar common-emitter amplifier, as shown in Fig. 4.16. Here, we wish to find the optimum bias current to minimize the equivalent input noise of this amplifier (i.e., maximize the signal-to-noise ratio of the amplifier).

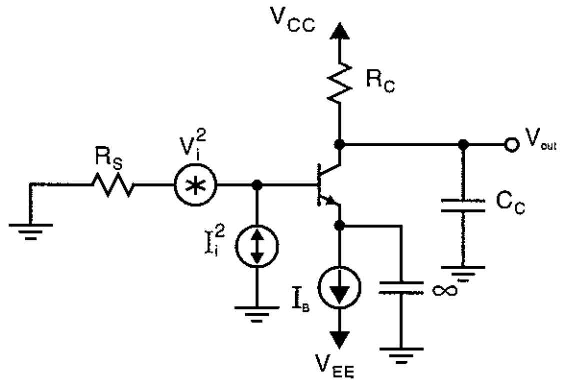


Fig. 4.16 A bipolar common-emitter amplifier.

We should state that it is assumed here that the collector-current shot noise dominates in the input voltage noise source and the base-current shot noise dominates in the input current noise source. Therefore, the input voltage noise due to the transistor alone is given by

$$V_i^2(f) = 2kT/g_m \tag{4.74}$$

and the input current noise due to the transistor alone is given by

$$I_i^2(f) = 2qI_B \tag{4.75}$$

These assumptions are reasonable since usually 1/f noise is not important for bipolar transistors, and the collector shot noise is usually not a major component of the input current noise source for wideband examples. Also, although the base resistance has temporarily been ignored, it will be taken into account shortly by simply modifying the size of the source resistance.

The first step is to replace the input current noise source by an input voltage noise source. First, note that the gain from the base to the collector (ignoring the transistor output impedance, r_o) is equal to $g_m R_C$, and the impedance looking into the base is r_π. Therefore, the output noise due to $I_i^2(f)$ is given by

$$V_{oi}^2(f) = I_i^2(f)[(R_S \parallel r_\pi)g_m R_C]^2 \tag{4.76}$$

Now the gain from the input voltage noise source to the output is given by

$$A_V = \frac{V_o}{V_i} = \frac{r_\pi}{r_\pi + R_S} g_m R_C \tag{4.77}$$

To represent the output noise due to the input current source by an equivalent input voltage source, we have

$$V_{ieq}^2(f) = \frac{V_{oi}^2(f)}{A_V^2} = \left(\frac{\dfrac{r_\pi R_S}{(r_\pi + R_S)} g_m R_C}{\dfrac{r_\pi}{(r_\pi + R_S)} g_m R_C}\right)^2 I_i^2(f) = I_i^2(f) R_S^2 \tag{4.78}$$

We can now replace all noise sources by a single voltage noise source, which includes the noise due to the input voltage noise source and the input current noise source of the transistor, as well as the noise of the source resistor. We have

$$V_{i=total}^2(f) = V_i^2(f) + V_{ieq}^2(f) + 4kTR_S \tag{4.79}$$

The first two terms represent the noise of the transistor and the third term represents the noise of the source resistance. Using (4.74), (4.75), and (4.78), we have

$$V_{i=total}^2(f) = \frac{2kT}{g_m} + 2qI_BR_S^2 + 4kTR_S \tag{4.80}$$

Substituting $g_m = qI_C/(kT)$ and $I_B = I_C/\beta$, we find

$$V_{i=total}^2(f) = \frac{2(kT)^2}{qI_C} + \frac{2qI_CR_S^2}{\beta} + 4kTR_S \tag{4.81}$$

The noise of the transistor base resistor can now be included by replacing R_S with $R_S + r_b$ to obtain

$$V_{i=total}^2(f) = \frac{2(kT)^2}{qI_C} + \frac{2qI_C(R_S + r_b)^2}{\beta} + 4kT(R_S + r_b) \tag{4.82}$$

Alternatively, (4.82) can be expressed in terms of $g_m = qI_C/(kT)$ as in [Buchwald, 1995]

$$V_{i=total}^2(f) = 4kT\left[R_s + r_b + \frac{1}{2g_m} + \frac{g_m(R_s + r_b)^2}{2\beta}\right] \tag{4.83}$$

These two equations are good approximations for most applications of bipolar common-emitter amplifiers. Notice in (4.82), the first term models the transistor base-current shot noise and increases with increased bias current. However, the second term (which models the transistor collector shot noise) decreases with increasing collector bias current. Finally, the terms modelling the noise of the source resistor and the transistor base resistor are independent of the transistor bias current. To find the optimum bias current, we differentiate (4.82) with respect to I_C and set the result to zero. This yields the rather simple result of

$$I_{C=opt} = \frac{kT}{q}\frac{\sqrt{\beta}}{R_S + r_b} \tag{4.84}$$

or, equivalently,

$$g_{m=opt} = \frac{\sqrt{\beta}}{R_S + r_b} \tag{4.85}$$

EXAMPLE 4.9

Consider the common-emitter amplifier shown in Fig. 4.16, where $R_C = 10\ k\Omega$, $C_C = 1\ pF$, $R_S = 500\ \Omega$, $\beta = 100$, and the base resistance is given

by $r_b = 300\ \Omega$. At room temperature, (i.e. 300 °K), find the optimum bias current and the total equivalent input noise.

Solution

Using (4.84), we find

$$I_{C=opt} = 0.026 \times \frac{\sqrt{100}}{500+300} = 0.325 \text{ mA} \quad (4.86)$$

implying that

$$g_m = \frac{I_C}{V_T} = 12.5 \text{ mA/V} \quad (4.87)$$

To find the output noise spectral density, we now use (4.83) to find

$$V_{i=total}^2(f) = 4kT(500+300+40+40) = 1.46 \times 10^{-17}\ \text{V}^2/\text{Hz} \quad (4.88)$$

Notice that the noise due to the source resistance dominates, even though the source resistance is moderately small. Even for no source resistance, the thermal noise due to the base resistance would still dominate. Thus, for a very low source resistance, the major way to improve the noise is to minimize the base resistance by using larger transistors (or to combine a number of parallel transistors).

Assuming the RC time constant at the collector dominates the frequency response, the noise bandwidth of the amplifier is given by

$$f_x = \frac{1}{4R_C C_C} = 25 \text{ MHz} \quad (4.89)$$

resulting in the total input-referred voltage noise given by

$$V_{ni(rms)} = [f_x V_{i=total}^2(f)]^{1/2} = 19.1\ \mu\text{V} \quad (4.90)$$

CMOS Example

In this example, we look at the input circuitry of a traditional two-stage CMOS opamp, as shown in Fig. 4.17. Note that each of the transistors have been modelled using an equivalent voltage noise source, as presented in Fig. 4.11. Voltage noise sources are used here since we will be addressing the low-frequency noise performance of this stage. It should be mentioned here that in the following derivations, we have assumed matching between transistor pair Q_1 and Q_2 as well as in pair Q_3 and Q_4.

We start by finding the gains from each noise source to the output node, V_{no}. The gains from V_{n1} and V_{n2} are the same as the gains from the input signals, resulting in

$$\left|\frac{V_{no}}{V_{n1}}\right| = \left|\frac{V_{no}}{V_{n2}}\right| = g_{m1}R_o \quad (4.91)$$

where R_o is the output impedance seen at V_{no}.

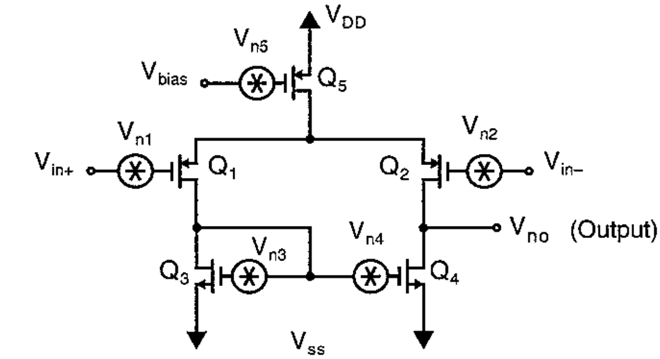


Fig. 4.17 A CMOS input stage for a traditional opamp with MOSFET noise sources shown.

Next, for V_{n3}, notice that the current through this voltage source must be zero because one side is connected to the gate of Q_3 only. Therefore, assuming all other sources are zero, the drain current is unaffected by V_{n3}. This implies that the gate voltage of Q_3 is also unaffected by V_{n3}. Therefore, in the small-signal model, V_{n3} is equal to v_{gs4}, and the gain from V_{n3} to the output is the same as the gain from V_{n4} to the output. Thus, we have

$$\left|\frac{V_{no}}{V_{n3}}\right| = \left|\frac{V_{no}}{V_{n4}}\right| = g_{m3}R_o \tag{4.92}$$

Finally, the noise gain from V_{n5} to the output can be found by noting that it modulates the bias current and the fact that, due to the symmetry in the circuit, the drain of Q_2 will track that of Q_1. As a result, the last gain factor is given by

$$\left|\frac{V_{no}}{V_{n5}}\right| = \frac{g_{m5}}{2g_{m3}} \tag{4.93}$$

Since this last gain factor is relatively small compared to the others, it will be ignored from here on.

Using the gain factors just shown, the output noise value is seen to be given by

$$V_{no}^2(f) = 2(g_{m1}R_o)^2V_{n1}^2(f) + 2(g_{m3}R_o)^2V_{n3}^2(f) \tag{4.94}$$

This output noise value can be related back to an equivalent input noise value, $V_{neq}(f)$, by dividing it by the gain, $g_{m1}R_o$, which results in

$$V_{neq}^2(f) = 2V_{n1}^2(f) + 2V_{n3}^2(f)\left(\frac{g_{m3}}{g_{m1}}\right)^2 \tag{4.95}$$

Thus, for the white noise portion of $V_{n1}(f)$ and $V_{n3}(f)$, we make the substitution

$$V_{ni}^2(f) = 4kT\left(\frac{2}{3}\right)\left(\frac{1}{g_{mi}}\right) \tag{4.96}$$

resulting in

$$V_{neq}^2(f) = \left(\frac{16}{3}\right)kT\left(\frac{1}{g_{m1}}\right) + \left(\frac{16}{3}\right)kT\left(\frac{g_{m3}}{g_{m1}}\right)^2\left(\frac{1}{g_{m3}}\right) \tag{4.97}$$

Assuming g_{m3}/g_{m1} is not far from unity, we see here that the two pairs of transistors contribute an approximately equal amount of noise, and this noise is inversely proportional to the transconductance of g_{m1}. *In other words,* g_{m1} *should be made as large as possible to minimize thermal noise contribution.*

However, to look at the effects of 1/f, or flicker, noise, which normally greatly dominates at low frequencies, we make the following substitution into (4.95),

$$g_{mi} = \sqrt{2\mu_i C_{ox}\left(\frac{W}{L}\right)_i I_{Di}} \tag{4.98}$$

resulting in

$$V_{ni}^2(f) = 2V_{n1}^2(f) + 2V_{n3}^2(f)\left[\frac{(W/L)_3\mu_n}{(W/L)_1\mu_p}\right] \tag{4.99}$$

Now, letting each of the noise sources have a spectral density given by

$$V_{ni}^2(f) = \frac{K_i}{W_i L_i C_{ox} f} \tag{4.100}$$

we have [Bertails, 1979]

$$V_{ni}^2(f) = \frac{2}{C_{ox}f}\left[\frac{K_1}{W_1 L_1} + \left(\frac{\mu_n}{\mu_p}\right)\left(\frac{K_3 L_1}{W_1 L_3^2}\right)\right] \tag{4.101}$$

Recall that the first term in (4.101) is due to the p-channel input transistors, Q_1 and Q_2, and the second term is due to the n-channel loads, Q_3 and Q_4. We note some points for 1/f noise here:

1. For $L_1 = L_3$, the noise of the n-channel loads dominate since $\mu_n > \mu_p$ and typically n-channel transistors have larger 1/f noise than p-channel transistors (i.e., $K_3 > K_1$).
2. Taking L_3 longer greatly helps due to the inverse squared relationship in the second term of (4.101). This limits the signal swings somewhat, but it may be a reasonable trade-off where low noise is important.
3. The input noise is independent of W_3, and therefore we can make it large to maximize signal swing at the output.
4. Taking W_1 wider also helps to minimize 1/f noise. (Recall that it helps white noise, as well.)
5. Taking L_1 longer increases the noise because the second term in (4.101) is dominant. Specifically, this decreases the input-referred noise of the p-channel drive transistors, which are not the dominant noise sources, but it also increases

the input-referred noise of the n-channel load transistors, which are the dominant noise sources!

Finally, we can integrate (4.101) from f_1 to f_2 to find the equivalent input noise value given by

$$V_{ni(rms)}^2 = 2\left[\frac{a_p}{W_1L_1} + a_n\left(\frac{\mu_n}{\mu_p}\right)\left(\frac{L_1}{W_1L_3^2}\right)\right] \tag{4.102}$$

where

$$a_i = \frac{K_i}{C_{ox}}\ln\left(\frac{f_2}{f_1}\right) \tag{4.103}$$

Fiber-Optic Preamp Example

The most popular means of detecting light from a fiber-optic cable is to use a transresistance amplifier, as shown in Fig. 4.18. The light from the fiber cable hits a photodetector, such as a reverse-biased diode. The light produces electron hole carriers in the depletion region of the reverse-biased diode, causing current to flow through the resistor, and therefore the output of the amplifier becomes positive. A popular choice for the first stage of the amplifier is to use a common-source amplifier with a resistor load. In low-noise applications, an active load would be too noisy. Assuming a CMOS transistor is used,[7] the preamp can be modelled as shown in Fig. 4.19. The photodetector is modelled as an input current source along with a parasitic capacitance, C_{in}. Also shown in Fig. 4.19 are the two major noise sources, namely the thermal current noise at the drain of Q_1 and the thermal noise from the feedback resistor, R_F. The 1/f noise of the transistor is ignored because it is assumed that the circuit is high speed, such that thermal noise dominates. The second (and perhaps subsequent) stage is modelled by an amplifier that has a gain of A_2. The noise due to this second stage is also ignored since the noise sources in the second stage are not amplified by the gain of the first stage.

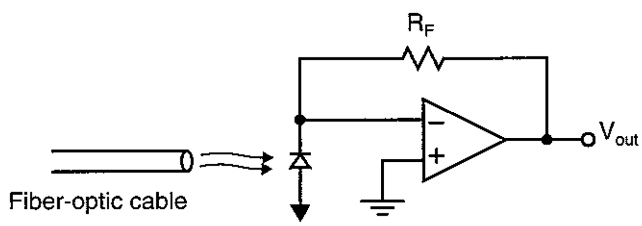


Fig. 4.18 A fiber-optic transresistance preamp.

7. Fiber-optic preamps are often realized using JFET transistors as well, but since their thermal noise model is identical to the noise model of the CMOS transistor, the analysis would be almost unchanged.

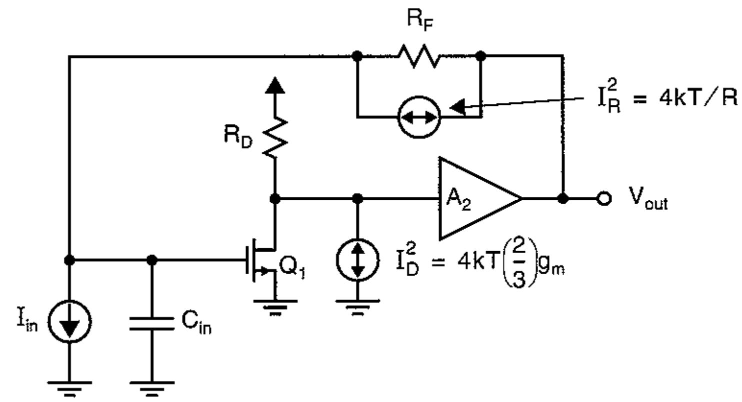


Fig. 4.19 A simplified model for a CMOS fiber-optic preamp.

A simplified small-signal model of this preamplifier, used for noise analysis, is shown in Fig. 4.20. The only parasitic capacitance considered in the transistor model is C_{gs}, since the gain of the first stage is only moderate, due to the resistor load, and therefore we can assume that C_{gd} and C_{db} can be ignored. The transfer function from i_{in} to the output is found by using nodal analysis to be

$$\frac{V_{out}(s)}{I_{in}(s)} = R_F\left(\frac{A_V}{1+A_V}\right)\frac{1}{1+s\left(\frac{R_F C_T}{1+A_V}\right)} \tag{4.104}$$

where $A_V = A_1 A_2$ is the total voltage gain of the preamp and $C_T = C_{gs} + C_{in}$. It is also found that this is the same transfer function from I_R^2 to the output. Thus, the noise current source I_R^2 can be replaced by an input current noise source having a spectral density of $4kT/R_F$. In a typical design, one would choose R_F as large as possible to limit this noise. However, this choice is constrained by the bandwidth requirements of passing the signal frequencies. From (4.104), we have the –3-dB frequency of the amplifier, given by

$$\omega_{-3\text{ dB}} = \frac{1+A_V}{R_F C_T} \tag{4.105}$$

For a given amplifier gain and detector capacitance, R_F is chosen using (4.105) to place the –3-dB frequency as small as possible without substantially attenuating the signals. This makes the dominant node for determining stability the input node. The bandwidth of the second amplifier must be substantially greater than that of the input node to guarantee stability. Unfortunately, this constraint greatly amplifies the thermal noise due to input transistor Q_1, as we will see next.

The gain from the noise source I_D^2 to the output is found by using nodal analysis to be

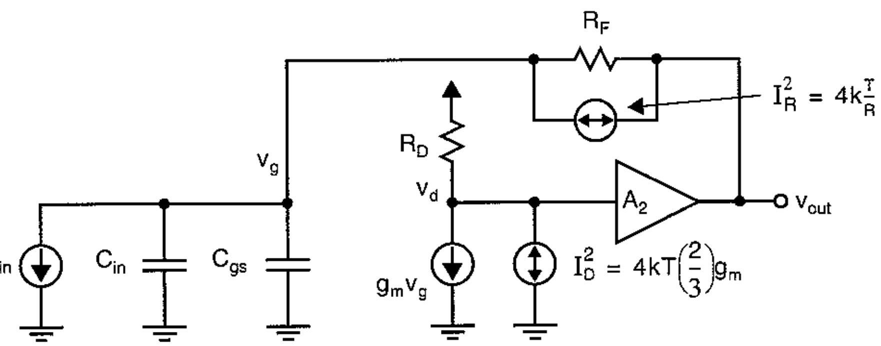


Fig. 4.20 A simplified small-signal model used for noise analysis.

$$\frac{V_{out}(s)}{I_D(s)} = \frac{1}{g_m}\frac{A_V}{1+A_V}\frac{1+sR_FC_T}{1+s\left[\frac{R_FC_T}{(1+A_V)}\right]} \tag{4.106}$$

At low frequencies, this gain is approximately given by $1/g_m$, whereas at high frequencies it is as much as $1 + A_V$ times greater. Thus, the high-frequency output noise due to Q_1 is much greater than the low-frequency noise due to Q_1. Furthermore, the only bandwidth limitation of this noise at high frequencies is due to the finite bandwidth of the second amplifier, A_2. From the discussion on compensation in Chapter 5, we know the second pole frequency of the amplifier must be almost four times greater than the closed-loop –3-dB frequency when lead compensation is not used.[8]

Continuing, in order to refer the noise due to Q_1 back to the input, we use (4.104) and (4.106) to obtain

$$I_{iD}^2(f) = I_D^2(f)\left|\frac{V_{out}(j\omega)}{I_D(jw)}\right|^2\left|\frac{I_{in}(j\omega)}{V_{out}(jw)}\right|^2 = I_D^2(f)\frac{1+\omega^2(R_FC_T)^2}{(g_mR_F)^2} \tag{4.107}$$

We use

$$I_D^2(f) = 4kT\left(\frac{2}{3}\right)g_m \tag{4.108}$$

and we add to this the input noise source that models the noise of the feedback resistor to obtain the total input-referred noise, given by

$$I_i^2(f) = \frac{4kT}{R_F} + 4kT\left(\frac{2}{3}\right)\frac{1+\omega^2(R_FC_T)^2}{g_mR_F^2} \tag{4.109}$$

Notice that the second term starts to quickly increase at a frequency of

---

8. Lead compensation could possibly be achieved by placing a capacitor in parallel with R_F. This is beyond the scope of this text.

$$\omega_Z = \frac{1}{R_F C_T} = \frac{\omega_{-3\text{ dB}}}{1 + A_V} \tag{4.110}$$

which is a relatively low frequency.

Continuing, normally $2/(3g_m R_F) \ll 1$, and we have

$$\begin{aligned} I_i^2(f) &= \frac{4kT}{R_F}\left(1 + \frac{2}{3g_m R_F}\right) + \frac{8kT}{3}\frac{\omega^2 C_T^2}{g_m} \\ &\cong \frac{4kT}{R_F} + \frac{8kT}{3}\frac{\omega^2 C_T^2}{g_m} \end{aligned} \tag{4.111}$$

Using the facts that

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{eff} \tag{4.112}$$

and that

$$C_{gs} = \frac{2}{3} C_{ox} WL \tag{4.113}$$

we can write (4.111) as

$$I_i^2(f) \cong \frac{4kT}{R_F} + \frac{16}{9} kT \frac{L^2}{\mu_n V_{eff}} \omega^2 \frac{(C_{gs} + C_{in})^2}{C_{gs}} \tag{4.114}$$

Normally, V_{eff} is taken as large as possible given power-supply-voltage and power-dissipation constraints. The only parameter left for the designer to choose is the width of the input transistor and, therefore, C_{gs}. It can be shown (by differentiating the second term of (4.114) with respect to C_{gs} and then setting the result to zero) that the second term of (4.114) is minimized by the choice $C_{gs} = C_{in}$, in which case we have

$$I_i^2(f) \cong 4kT\left(\frac{1}{R_F} + \frac{4}{9}\frac{L^2}{\mu_n V_{eff}} \omega^2 C_{in}\right) \tag{4.115}$$

4.5 REFERENCES

J. C. Bertails, "Low-Frequency Noise Considerations for MOS Amplifiers Design," *IEEE J. of Solid-State Circuits*, Vol. SC-14, No. 4, pp. 773–776, August, 1979.

A. Buchwald and K. Martin. *Integrated Fiber-Optic Receivers*. Kluwer Academic Publishers, Norwell, Massachusetts, 1995.

P. R. Gray and R. G. Meyer. *Analysis and Design of Analog Integrated Circuits*. John Wiley & Sons, New York, 1993.

J. B. Johnson, *Phys. Rev.*, Vol. 32, pp. 97–109, 1928.

E. J. Kennedy, *Operational Amplifier Circuits: Theory and Applications.* Holt, Rinehart, & Winston, New York, 1988.

A. Leon-Garcia, *Probability and Random Processes for Electrical Engineering*. Addison-Wesley, Reading, Massachusetts, 1989.

C. D. Motchenbacher and J. A. Connelly. *Low Noise Electronic System Design*. John Wiley & Sons, New York, 1993.

H. Nyquist, *Phys. Rev.*, Vol. 32, pp. 110–113, 1928.

W. Schottky, *Ann. Phys. (Leipzig)*, Vol. 57, pp. 541–567, 1918.

M. Steyaert, Z. Y. Chang, and W. Sansen. "Low-Noise Monolithic Amplifier Design: Bipolar versus CMOS," *Analog Integrated Circuits and Signal Processing 1*. Kluwer Academic Publishers, Boston, pp. 9–19, September 1991.

A. Van der Ziel, *Noise in Solid State Devices and Circuits*. John Wiley & Sons, New York, 1986.

4.6 PROBLEMS

Unless otherwise stated, assume dBm values are referenced to $50\ \Omega$.

4.1 If a signal is measured to have $V_{(rms)}$ volts, what is the difference in db if it is expressed in dBm referenced to $50\ \Omega$ as opposed to being referenced to $75\ \Omega$?

4.2 Consider the sum of two noise sources of values -20 dBm and -23 dBm. Find the total noise power in dBm for the cases in which the two noise sources are (*a*) uncorrelated, (*b*) $C = 0.3$, (*c*) $C = +1$, and (*d*) $C = -1$.

4.3 The output noise of a circuit is measured to be -40 dBm around 100 kHz when a resolution bandwidth of 30 Hz is used. What is the expected dBm measurement if a resolution bandwidth of 10 Hz is used? Find the root spectral density in $\mathrm{V}/\sqrt{\mathrm{Hz}}$.

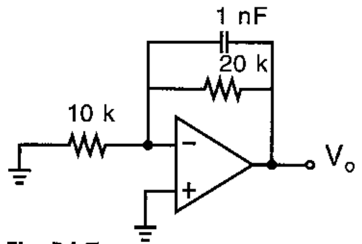
4.4 At 0.1 Hz, a low-frequency measurement has a noise value of -60 dBm when a resolution bandwidth of 1 mHz is used. Assuming 1/f noise dominates, what would be the expected noise value (in dBm) over the band from 1 mHz to 1 Hz?

4.5 Show that, when two resistors of values R_1 and R_2 are in series, their noise model is the same as a single resistance of value $R_1 + R_2$. Repeat the problem for parallel resistances.

4.6 Sketch the spectral density of voltage noise across a 100-pF capacitor when it is in parallel with a 1-kΩ resistor. Make another sketch for the same capacitor but with a 1-MΩ resistance in parallel. What can you say about the area under the curves of the two sketches?

4.7 Consider the circuit shown in Fig. P4.7 on p. 218, where the opamp has a unity-gain frequency of 1 MHz and equivalent input voltage and current noise sources of $V_n(f) = 20\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$ and $I_n(f) = 10\ \mathrm{pA}/\sqrt{\mathrm{Hz}}$, respectively. Estimate the total rms noise voltage at V_o.

4.8 Assuming an ideal noiseless opamp, find the noise level at V_x and V_o for the opamp circuit shown in Fig. P4.8. By what factor is the noise value at V_o larger (or smaller) than $kT/1\ \mathrm{nF}$? How do you account for this increase (or decrease)? Also explain why the noise value at V_x is smaller than $kT/1\ \mathrm{nF}$.



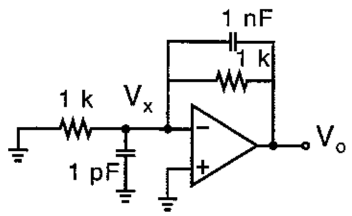
Opamp

$V_n(f) = 20\ nV/\sqrt{Hz}$

$I_{n+}(f) = I_{n-}(f) = 10\ pA/\sqrt{Hz}$

$f_t = 1\ MHz$

Fig. P4.7



Ideal noiseless opamp

Fig. P4.8

4.9 Consider an inductor of value L and an arbitrary resistor in parallel, as shown in Fig. P4.9. Show that the current noise, $i_{no}(t)$, has a noise value given by

$$I^2_{no(rms)} = \frac{kT}{L}$$

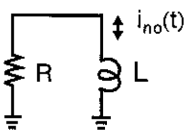


Fig. P4.9

4.10 The two circuits shown in Fig. P4.10 realize a first-order, low-pass filter. Assume the opamps are ideal and noiseless.

(*a*) Show that the two circuits have the same input-output transfer function.

(*b*) Estimate the total output noise for circuit I, using only dominant noise sources.

(*c*) Repeat (*b*) for circuit II.

4.11 Modify circuit I in Fig. P4.10 such that the new circuit has the same transfer function but uses an 80-pF capacitor instead of an 80-nF capacitor. If the opamp is ideal and noiseless, what is the new total output noise?

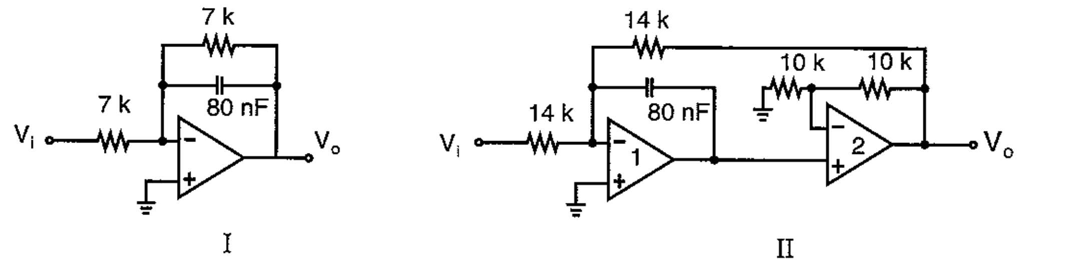


Fig. P4.10

4.12 Using the 1/f tangent principle, estimate the total noise above 0.1 Hz for the spectral density shown in Fig. 4.3.

4.13 Consider a bandpass amplifier that has equivalent input noise root spectral density and amplifier response, as shown in Fig. P4.13. Sketch the root spectral density for the output signal. Estimate the total output rms noise value by applying the 1/f tangent principle.

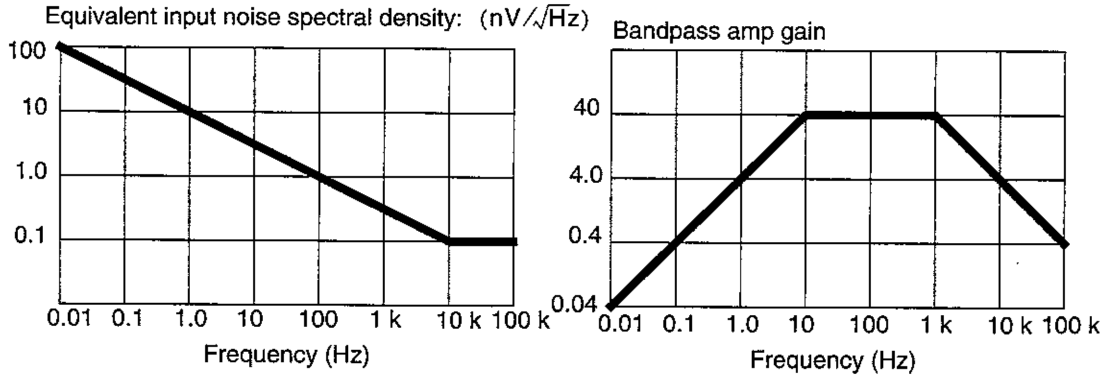


Fig. P4.13

4.14 Consider the noise root spectral density of the signal shown in Fig. P4.14. Find the total rms noise value using a graphical approach for 0.01 to ∞ Hz. Compare your result with that obtained when using the 1/f tangent principle.

4.15 We saw on page 193, Eq. (4.36), that the noise bandwidth of a first-order, low-pass filter is $(\pi/2)f_0$. Show that the noise bandwidth of a second-order, low-pass filter given by

$$A(s) = \frac{A_o}{\left(1 + \frac{s}{(2\pi f_0)}\right)^2}$$

is equal to $(\pi/4)f_o$.

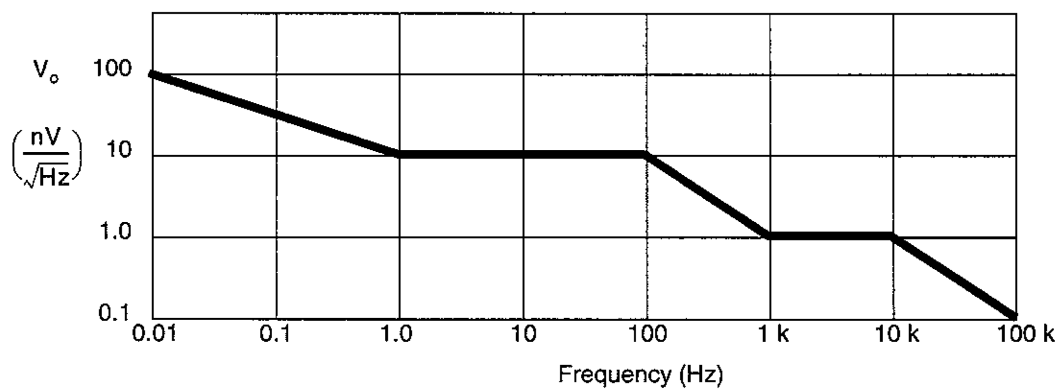


Fig. P4.14

4.16 Consider the two bipolar current mirrors shown in Fig. P4.16, where I_{in} is a 1-mA bias current plus a 100-μA(rms) signal current. Assuming that the base resistance for each transistor is $r_b = 330\ \Omega$ and dominates the output noise, estimate the resulting SNR (in dB) for the two current mirrors over a 50-MHz bandwidth (also assume the output noise is white).

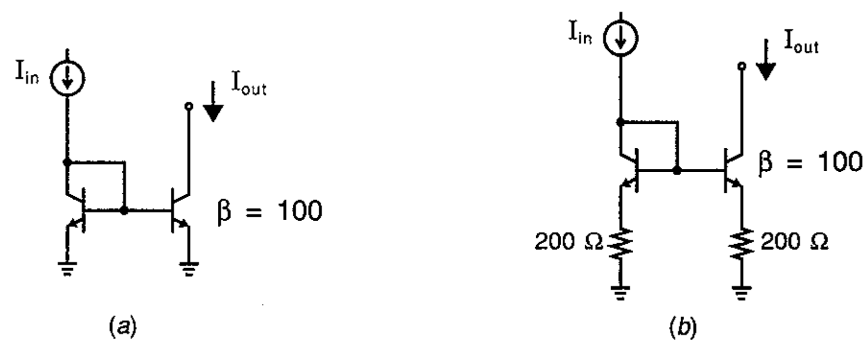


Fig. P4.16

4.17 Estimate the total output noise rms value for a low-pass filter, as shown in Fig. 4.15, when $C_f = 1\ \text{nF}$, $R_f = 16\ \text{k}$, $R_1 = 1.6\ \text{k}$, and $R_2 = 0$. Also, find the SNR for an input signal equal to 100 mV rms. Assume that the noise voltage of the opamp is given by $V_n(f) = 20\ \text{nV}/\sqrt{\text{Hz}}$, that both its noise currents are $I_n(f) = 0.6\ \text{pA}/\sqrt{\text{Hz}}$, and that its unity-gain frequency equals 2 MHz.

4.18 Consider the CMOS differential input stage shown in Fig. 4.17, where Q_5 supplies a bias current of 100 μA, resulting in $g_{m1} = g_{m2} = 1\ \text{mA/V}$ and $g_{m3} = g_{m4} = 0.5\ \text{mA/V}$. Find the equivalent input noise spectral density associated with thermal noise. If the bias current is doubled, how does the equivalent input noise density change?