

# Field effect transistor as heterodyne terahertz detector

B. Gershgorin, V.Yu. Kachorovskii, Y.V. Lvov and M.S. Shur

A theory of nonlinear response of the channel of a field effect transistor subjected to two terahertz beams (measured signal and local oscillator) with the close frequencies has been developed. It is shown that electric current flowing in the transistor channel drastically increases heterodyne efficiency. Also, it is demonstrated that such a heterodyne detector is capable of operating effectively with very high intermediate frequencies up to  $10 \div 100$  GHz.

**Introduction:** Plasma-wave terahertz emitters [1] and detectors [2] are promising candidates for bridging the famous terahertz gap moving from the electronic (i.e. low frequency) side. Terahertz detectors based on GaAs, GaN, and Si field effect transistors (FETs) have already achieved performances comparable to commercial detectors, demonstrating tunability and relatively low value of the noise equivalent power [3]. Such detectors can operate both at zero bias current, where shot noise is minimal, and in the regime of relatively large current. In the latter case, the detection efficiency can be significantly improved owing to current-driven increase of nonlinear properties of the channel [4, 5].

The performance characteristics of FET-based terahertz detectors can be further dramatically enhanced using the heterodyne detection principle. The idea of heterodyne detection is as follows. An incoming signal in question with a small amplitude  $U_a$  and frequency  $\omega$  is mixed with a strong signal from a local oscillator with the amplitude  $U_b$  ( $U_a \ll U_b$ ) and a close frequency  $\omega + \Omega$  ( $\Omega \ll \omega$ ), resulting in a signal with the slow intermediate (beat) frequency  $\Omega$  and the amplitude proportional to  $U_a U_b$ . Therefore, even for a weak incoming signal, the strength of the resulting signal can be enhanced by increasing the amplitude of the local oscillator signal. In this Letter, we discuss a possible realisation of such a detector based on two-dimensional (2D) plasma-wave-related effects. We show that the FET can effectively work as a heterodyne detector. One of the main advantages of the FET-based heterodyne detector is that the amplitude of the resulting beat signal can be further drastically increased by the electric current flowing in the transistor channel.

**Basic equations:** In this Letter, we restrict ourselves to the case of so-called nonresonant detection, since nonresonant detectors demonstrate the best detection characteristics even at room temperature. In this regime, the fundamental plasma frequency  $\omega_0$  is small compared to the inverse momentum relaxation time  $\tau$  ( $\omega_0 \tau \ll 1$ ), so that the plasma waves in the channel are overdamped. We assume that the condition  $\omega_c \ll 1$  is also satisfied. Under these assumptions, electron transport in the channel is described by the local Ohm's law  $v = -\mu \partial U / \partial x$  and the continuity equation  $\partial n / \partial t + \partial(nv) / \partial x = 0$  (see also discussion in [5]). Combining these equations we get

$$\frac{\partial U}{\partial t} = \frac{\mu}{2} \frac{\partial^2 U^2}{\partial x^2} \quad (1)$$

Here  $n = CU/e$  and  $v$  are the electron concentration and velocity, respectively,  $U$  is the local value of the gate-to-channel voltage,  $C = \varepsilon / 4\pi d$  is the gate-to-channel capacitance per unit area,  $d$  is gate-to-channel distance,  $\varepsilon$  is the dielectric constant,  $\mu$  is the electron mobility. Equation (1) requires two boundary conditions. We assume [1] that the voltage at the source is a sum of the fixed gate-to-channel swing and an oscillating part induced by two beams, while the drain-to-source current  $j_d = env = -CU\mu \partial U / \partial x$  is fixed at the drain, so that

$$U(0, t) = U_g + U_{osc}(t), \quad \left( \frac{\partial U^2}{\partial x} \right)_{x=L} = -\frac{2j_d}{\mu C} \quad (2)$$

Here  $U_{osc}(t) = U_a \cos(\omega t) + U_b \cos[(\omega + \Omega)t + \varphi]$ . The amplitudes  $U_a$  and  $U_b$  are proportional to the amplitude of the measured wave and the amplitude of the local oscillator signal, respectively. The measured signal is assumed to be weak, while the signal from the local oscillator is considered to be relatively large, so that  $U_a \ll U_b$ . The amplitude  $U_b$ , in turn, is assumed to be small compared to the voltage swing:  $U_b \ll U_g$ . We also assume that there is a certain phase shift  $\varphi$  between two signals.

We seek the solution for  $U(x, t)$  in the form

$$U(x, t) = U_0(x, t) + U_1(x, t) \quad (3)$$

where  $U_0(x, t)$  is a slow function of time changing on the characteristic time scale of the order of  $2\pi/\Omega$  and  $U_1(x, t)$  is the small ( $U_1(x, t) \ll U_0(x, t)$ ) rapidly oscillating function, containing both frequencies  $\omega$  and  $\omega + \Omega$ . Substituting (3) into (1) and (2), and separating fast and slowly oscillating terms, we obtain

$$\frac{\partial U_0}{\partial t} = \frac{\mu}{2} \frac{\partial^2 [U_0^2 + \langle U_1^2 \rangle_\omega]}{\partial x^2} \quad (4)$$

$$U_0(0) = U_g, \quad \frac{\partial}{\partial x} [U_0^2 + \langle U_1^2 \rangle_\omega]_{x=L} = -\frac{2j_d}{\mu C} \quad (5)$$

$$\frac{\partial U_1}{\partial t} = \mu \frac{\partial^2 (U_0 U_1)}{\partial x^2} \quad (6)$$

$$U_1(0) = U_{osc}(t), \quad \left[ \frac{\partial (U_0 U_1)}{\partial x} \right]_{x=L} = 0 \quad (7)$$

Here,  $\langle \dots \rangle_\omega$  stands for averaging over the fast oscillations with the frequencies  $\omega$  and  $\omega + \Omega$ . The averaged function  $\langle U_1^2 \rangle_\omega$  remains time-dependent and varies slowly with the characteristic frequency  $\Omega$ .

In this Letter, we consider only relatively long samples, i.e.  $qL \gg 1$ , where  $q = q(\omega) = \sqrt{2\omega/\mu U_g}$ . Then the solution of (6) and (7) is given by

$$U_1(x, t) \simeq e^{-qx/2} [U_a \cos(\omega t - qx/2) + U_b \cos[(\omega + \Omega)t + \varphi - qx/2]] \quad (8)$$

where we put  $q(\omega + \Omega) \simeq q(\omega)$  and replaced  $U_0$  by its value at  $x = 0$ , which is equal to  $U_g$  according to (5). The latter approximation is valid, since solution (8) is localised on a short distance  $\sim 1/q$  near  $x = 0$ . From (8), we find  $\langle U_1^2 \rangle_\omega = e^{-qx} g(t)$ , where  $g(t) = [U_a^2 + U_b^2 + 2U_a U_b \cos(\Omega t + \varphi)]/2$ . After substituting this equation into (5) and integrating twice over  $x$ , we obtain

$$U_0^2(x, t) = U_g^2 - \frac{2j_d}{\mu C} x + g(t)(1 - e^{-qx}) - \frac{2}{\mu} \int_0^x \int_{x'}^L \frac{\partial U_0(x'')}{\partial t} dx'' x' \quad (9)$$

Here, we neglected exponentially small terms of the order of  $\exp(-qL)$ . In the absence of both signals, i.e.  $U_a = U_b = 0$ , (9) leads to the stationary distribution of potential along the channel,  $U_0(x) = \sqrt{U_g^2 - 2j_d x / \mu C}$ . This equation yields the stationary  $I$ - $V$  characteristic of the channel:  $j_d = j_{sat}(2V/U_g - V^2/U_g^2)$ , where  $V = U_0(0) - U_0(L) = U_g - U(L)$  is the voltage drop across the channel and  $j_{sat} = \mu C U_g^2 / 2L$  is the saturation current in the Shockley model, which neglects velocity saturation and can be applied for  $j < j_{sat}$ . For nonzero signals, we search the solution of (9) treating a term containing  $\partial U_0 / \partial t$  as a small perturbation. First, we neglect this term and find

$$U_0(x, t) = \sqrt{U_g^2(1 - \lambda x/L) + g(t)(1 - e^{-qx})} \simeq U_g \sqrt{1 - \lambda x/L} + \frac{g(t)(1 - e^{-qx})}{2U_g \sqrt{1 - \lambda x/L}} \quad (10)$$

where  $\lambda = j_d / j_{sat}$ . In this approximation, the transistor response, which is defined as radiation induced change of voltage across the channel  $\delta U = U_0(L, t) - U_0(L) |_{U_{osc}=0}$ , is given by  $\delta U = g(t) / 2U_g \sqrt{1 - \lambda}$  (terms of the order of  $\exp(-qL)$  are neglected). Substituting now (10) back into (9) one can find response up to the first order with respect to the term  $\partial U_0 / \partial t$ :

$$\partial U(t) = \frac{g(t) - t_0 dg(t)/dt}{2U_g \sqrt{1 - \lambda}} \simeq \frac{g(t - t_0)}{2U_g \sqrt{1 - \lambda}} \quad (11)$$

where

$$t_0 = \frac{1}{\mu U_g} \int_0^L dx \int_x^L dx' (1 - \exp(-qx')) / \sqrt{1 - \lambda x'/L} \simeq \frac{2L^2}{\mu U_g} (\lambda + 3) / [6 + 3\sqrt{1 - \lambda}(2 + \lambda)] \sim \frac{L^2}{\mu U_g} \quad (12)$$

Here, we again took into account that  $qL \gg 1$ . As seen from (11), the part

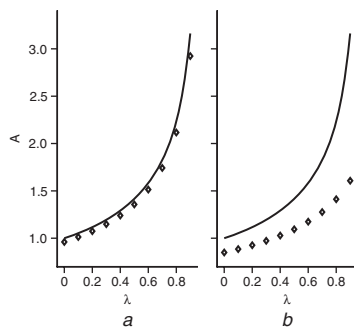
of the response alternating with the intermediate frequency  $\Omega$

$$\delta U_{\Omega}(t) = \frac{U_a U_b \cos(\Omega(t - t_0) + \varphi)}{2U_g \sqrt{1 - \lambda}} \quad (13)$$

This equation is valid for the case  $\Omega t_0 \ll 1$ , or, equivalently  $\Omega \ll \Omega_{\max} = \mu U_g / L^2$ . Provided this inequality is satisfied, the phase of the intermediate signal is equal to the phase difference of the signal in question and the phase of the local oscillator with a small correction  $\Omega t_0$ . One can show that in the opposite limit,  $\Omega \gg \Omega_{\max}$ , the amplitude of the intermediate signal acquires a factor  $\exp(-q(\Omega)L) = \exp(-\sqrt{2\Omega/\Omega_{\max}})$  and, therefore, becomes exponentially small.

As follows from (13), the amplitude of the beat signal measured in units  $U_a U_b / 2U_g$  is given by  $A(j_d) = 1/\sqrt{1 - \lambda} = 1/\sqrt{1 - j_d/j_{\text{sat}}}$ . This equation shows that the beat signal is dramatically enhanced when  $j_d \rightarrow j_{\text{sat}}$ . Physically, this occurs because the current depletes the channel close to the drain region, thus increasing the nonlinear properties of the channel [5].

The above equations were derived in a low-signal regime, under the assumption that  $U_a \ll U_b \ll U_g$ . In order to study the behaviour of the response at stronger signals, we performed direct numerical simulations of (1) with the boundary conditions (2). Fig. 1 compares the results of the numerical simulation with (13). We observe that for weak signals (Fig. 1a), the numerical simulation is in excellent agreement with the analytical prediction in the whole range of currents. On the other hand, for stronger incoming signals (Fig. 1b), the amplitude of the intermediate signal computed numerically is smaller than the analytical prediction. However, we observe that even for stronger incoming signals, the response increases when  $j_d$  approaches  $j_{\text{sat}}$ .



**Fig. 1** Amplitude of heterodyne signal (measured in units  $U_a U_b / 2U_g$ ) against drain-to-source current (measured in units of saturation current) for  $\omega/\Omega = 32$  (solid line obtained from (13), diamonds represent results of numerical simulations)

a For relatively small amplitudes of incoming signals,  $U_a/U_g = 0.02$ ,  $U_b/U_g = 0.1$   
b The same for higher amplitudes of incoming signals,  $U_a/U_g = 0.2$ ,  $U_b/U_g = 0.7$

Our theory can be easily extended to describe heterodyne detection deep in the subthreshold regime. In this regime the electron

concentration in the channel becomes exponentially small:  $n = n^* \exp(eU/\eta T)$  (below threshold  $U < 0$  and  $e|U|/\eta T > 1$  throughout the channel), where  $n^* = CT\eta/e^2$  and  $\eta \sim 1$  is the so-called ideality factor. One can show that for  $\Omega \ll \Omega_{\max} = \mu\eta T/eL^2$  and  $eU_a \ll eU_b \ll \eta T$ , the time dependent part of the response is given by  $\delta U_{\Omega} = eU_a U_b \cos(\Omega[t - t_0] + \varphi)/2\eta T(1 - \lambda)$  where  $t_0 = eL^2/2\mu\eta T$ ,  $\lambda = j_d/j_{\text{sat}}$  ( $\lambda < 1$ ) and  $j_{\text{sat}} = \mu C(\eta T)^2 \exp(eU_g/\eta T)/e^2 L$ .

**Conclusions:** It was demonstrated that heterodyne detection by FET is effective for beat frequencies smaller than a certain frequency  $\Omega_{\max}$  and that the amplitude of the intermediate signal can be drastically increased by passing a DC current between source and drain. Our estimates show that  $\Omega_{\max}$  can be rather high (of the order of 50–100 GHz for a 200 nm gate transistor operating at room temperatures in the above-threshold regime, and of the order of 5–10 GHz in the below-threshold regime).

**Acknowledgments:** The work at FTI was supported by RFBR, a grant of the RAS, a grant of the Russian Scientific School 2192.2003.2., and by a grant ‘Nanostructures’. Y.V. Lvov was supported by NSF CAREER DMS 0134955. V.Yu. Kachorovskii was supported by a grant of the fund ‘Dynasty’.

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15 March 2008

Electronics Letters online no: 20080737

doi: 10.1049/el:20080737

B. Gershgorin, V.Yu. Kachorovskii, Y.V. Lvov and M.S. Shur (Rensselaer Polytechnic Institute, 110, 8th Street, Troy, NY 12180, USA)

E-mail: kachor.valentin@gmail.com

V.Yu. Kachorovskii: Also with A.F. Ioffe Physical-Technical Institute, 26 Polytechnicheskaya Street, Saint Petersburg 194021, Russia

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