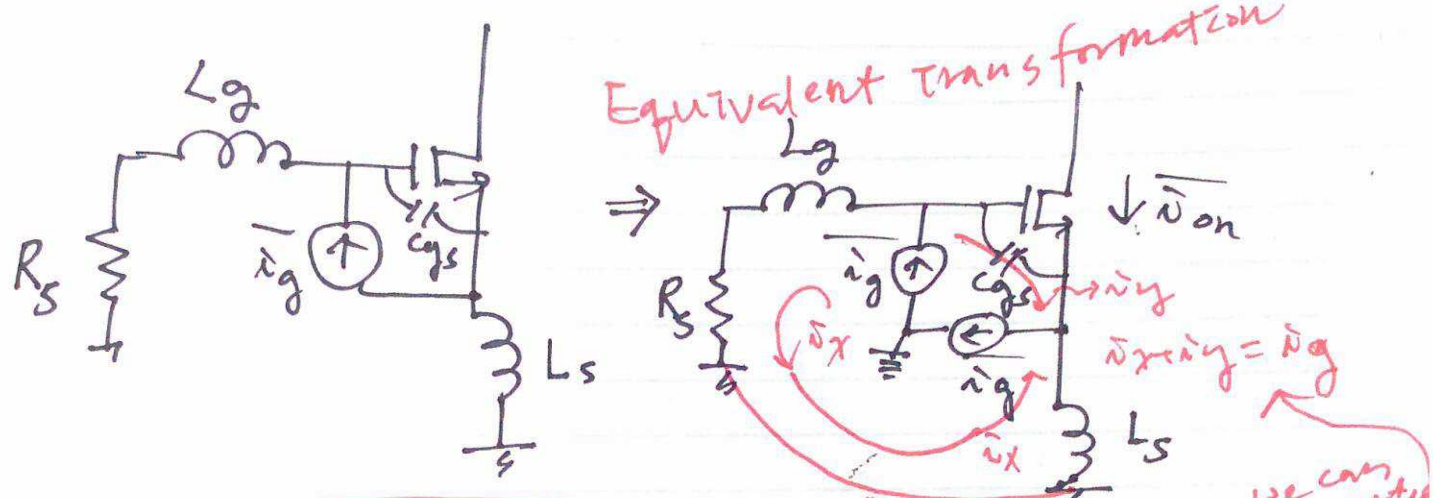
Separation of gate noise current

Equivalent transformation



if we tie together, we can guarantee that

$$\overline{i_{on}} = \underbrace{\frac{sL_g + R_s + sL_s}{sL_g + R_s + \frac{1}{sC_{gs}} + sL_s + \frac{g_m L_s}{C_{gs}}} \times \overline{i_g}}_{=\overline{i_y}} \times \frac{g_m}{sC_{gs}}$$

$$= \frac{sL_g + R_s + sL_s}{2R_s} \times \overline{i_g} \times \frac{\omega_T}{j\omega}$$

$$= \frac{1}{2R_s}\frac{\omega_T}{j\omega}\left(R_s + \frac{j}{\omega C_{gs}}\right)\overline{i_g}$$

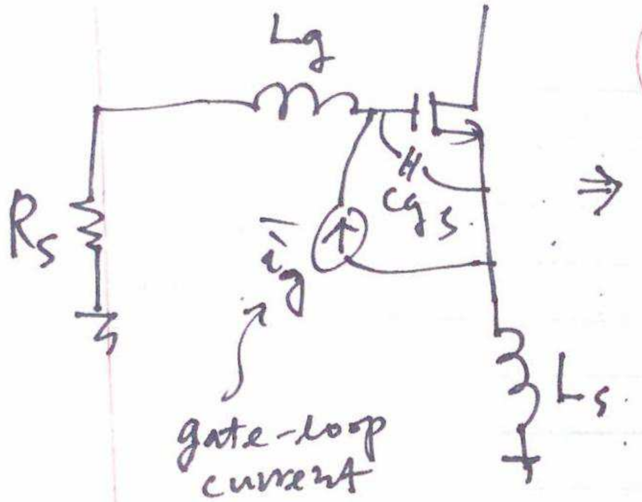
$\because s(L_g + L_s) = \frac{1}{sC_{gs}}$

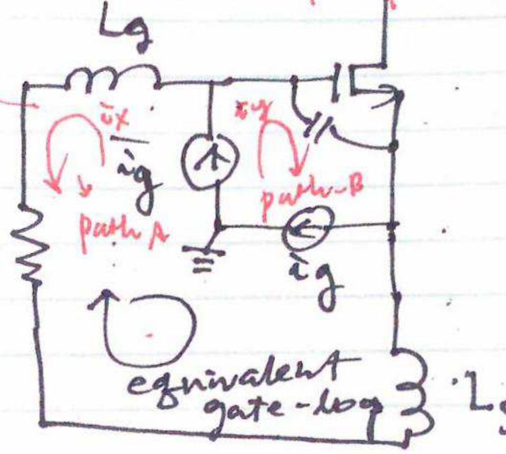
$\rightarrow$ at resonance.

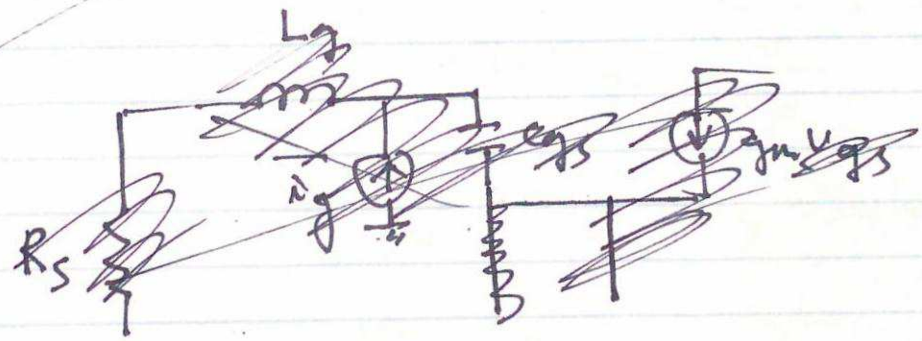
$$= \frac{1}{2}\frac{\omega_T}{j\omega}\left(1 + jQ\right)\overline{i_g}$$

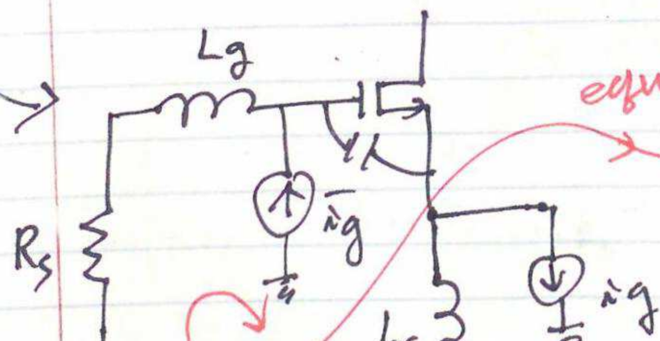
Separation of gate noise current (more)

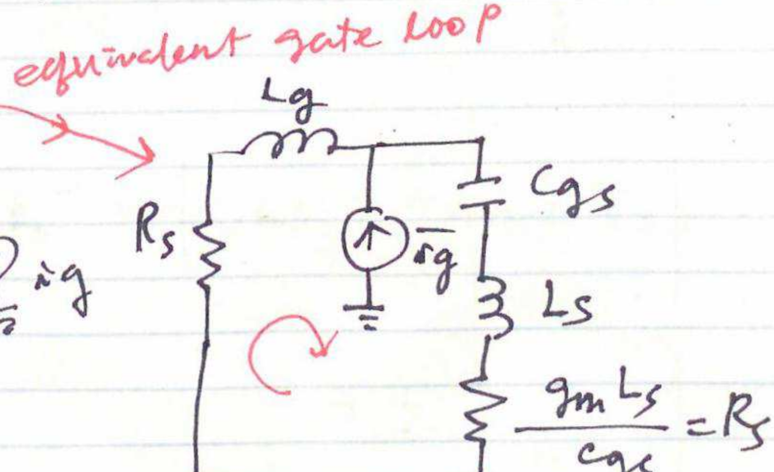
$$i_y = \frac{\text{loop impedance of path A}}{\text{total loop impedance}}$$



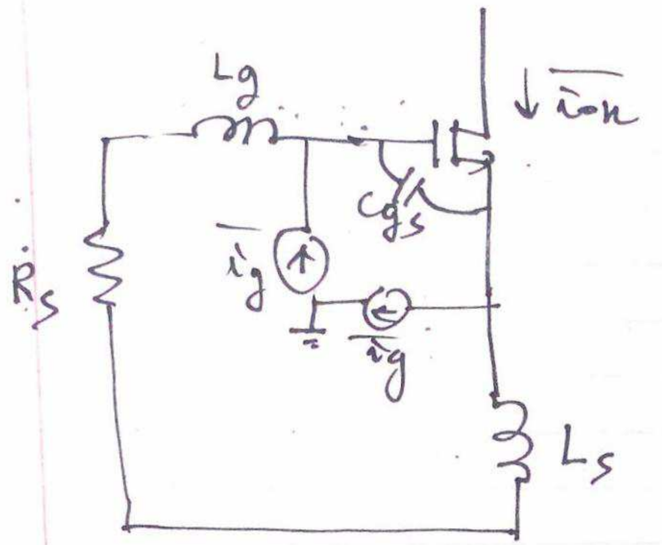








6) Output noise current due to Induced gate noise current



$$\overline{i_{on}}\Big|_{\text{due to } \overline{i_g}} = \frac{R_s + sL_g + sL_s}{R_s + sL_g + sL_s + \frac{1}{sC_{gs}} + \frac{g_m L_s}{C_{gs}}} \times i_g \times \frac{g_m}{sC_{gs}}$$

$$= \frac{1}{2R_s}\frac{\omega_T}{j\omega}\left(R_s + j\omega L_g + j\omega L_s\right)\overline{i_g}$$

$$\approx \frac{1}{2R_s}\frac{\omega_T}{j\omega}\left(R_s + j\omega L_g\right)\overline{i_g}$$

$$\therefore\ \overline{i_{on}}\Big|_{Total} = \overline{i_{on}}\Big|_{\text{due to } R_s} + \overline{i_{on}}\Big|_{\text{due to } R_g} + \overline{i_{on}}\Big|_{\text{due to } \overline{i_d}}$$

$$+ \overline{i_{on}}\Big|_{\text{due to } \overline{i_g}}$$

6) Output noise current due to induced gate noise current

(2)

$\overline{i_{on}}\Big|_{Total}$

$$= \frac{1}{2R_s}\frac{\omega_T}{j\omega}\left(\overline{v_s}+\overline{v_g}\right) + \frac{1}{2}\overline{i_d} + \frac{1}{2R_s}\frac{\omega_T}{j\omega}\left(R_s + j\omega L_g + j\omega L_s\right)\overline{i_g}$$

$$= \frac{1}{2R_s}\frac{\omega_T}{j\omega}\left(\overline{v_s}+\overline{v_g}+\left(R_s+j\omega L_g+j\omega L_s\right)\overline{i_g}\right) + \frac{1}{2}\overline{i_d}$$

overall noise factor

$$\Rightarrow F = \frac{\overline{i_{on}}\cdot\overline{i_{on}}^*}{\left(\frac{1}{2R_s}\cdot\frac{\omega_T}{j\omega}\overline{v_s}\right)\left(\frac{1}{2R_s}\frac{\omega_T}{j\omega}\overline{v_s}\right)^*}$$

$$= \frac{\begin{array}{l}\frac{1}{4R_s^2}\left(\frac{\omega_T}{\omega}\right)^2\left\{\overline{v_s}^2+\overline{v_g}^2+\left(R_s^2+\omega^2(L_g+L_s)^2\right)\overline{i_g}^2\right\}+\frac{1}{4}\overline{i_d}^2\\ +\frac{1}{4}\frac{1}{R_s}\frac{\omega_T}{j\omega}\left(R_s+j\omega L_g+j\omega L_s\right)\overline{i_g}\cdot\overline{i_d}^*\\ -\frac{1}{4}\frac{1}{R_s}\frac{\omega_T}{j\omega}\left(R_s-j\omega L_g-j\omega L_s\right)\overline{i_g}^*\cdot\overline{i_d}\end{array}}{\frac{1}{4R_s^2}\left(\frac{\omega_T}{\omega}\right)^2\overline{v_s}^2}$$

Numerator
~~Denominator~~

$$Q = \frac{\omega(L_g + L_s)}{2R_s} = \frac{1}{2R_s}\frac{1}{\omega C_{gs}}$$

$$= \frac{1}{4R_s^2}\left(\frac{\omega_T}{\omega}\right)^2 \left\{ \overline{v_s^2} + \overline{v_g^2} + R_s^2\left(1+4Q^2\right)\overline{i_g^2} \right\} + \frac{1}{4}\overline{i_d^2} + \frac{1}{4R_s}\left(\frac{\omega_T}{j\omega}\right) R_s \left(1 + j2Q\right)\overline{i_g \cdot i_d^*}$$

$$- \frac{1}{4R_s}\left(\frac{\omega_T}{j\omega}\right) R_s \left(1 - j2Q\right)\overline{i_g^* \cdot i_d}$$

$$\overline{i_g \cdot i_d^*} = C\sqrt{\overline{i_g^2}\cdot\overline{i_d^2}}$$

$$\overline{i_g^* \, i_d} = C^*\sqrt{\overline{i_g^2}\cdot\overline{i_d^2}}$$

$$C = j0.395$$

apply

$$= \frac{1}{4R_s^2}\left(\frac{\omega_T}{\omega}\right)^2 \left\{ \overline{V_s^2} + \overline{V_g^2} + R_s^2\left(1+4Q^2\right)\overline{i_g^2} \right\} + \frac{1}{4}\overline{i_d^2} + \frac{1}{4}\left(\frac{\omega_T}{j\omega}\right)\sqrt{\overline{i_g^2}\cdot\overline{i_d^2}}\left\{ C\left(1+j2Q\right) - C^*\left(1-j2Q\right) \right\}$$

$$= \frac{1}{4R_s^2}\left(\frac{\omega_T}{\omega}\right)^2 \left\{ \overline{V_s^2} + \overline{V_g^2} + R_s^2\left(1+4Q^2\right)\overline{i_g^2} \right\} + \frac{1}{4}\overline{i_d^2} + \left(\frac{\omega_T}{j\omega}\right)\sqrt{\overline{i_g^2}\cdot\overline{i_d^2}} \times \frac{C}{2}$$

uncorrelated noise

correlated noise

⊛ Components of F due to uncorrelated noise

From previous page, uncorrelated noise current power

$$= \frac{1}{4R_s^2}\left(\frac{\omega_T}{\omega}\right)^2 \left\{ \overline{v_s^2} + \overline{v_g^2} \right\} + \frac{1}{4}\overline{i_d^2} + \frac{1}{4}\left(1+\overset{4}{Q^2}\right)\left(\frac{\omega_T}{\omega}\right)^2 \overline{i_g^2}$$

$$F\Big|_{\substack{\text{due to}\\ \text{uncorrelated}\\ \text{noise}}} = \frac{\frac{1}{4R_s^2}\left(\frac{\omega_T}{\omega}\right)^2\left(\overline{v_s^2}+\overline{v_g^2}\right) + \frac{1}{4}\overline{i_d^2} + \frac{1}{4}\left(1+\overset{4}{Q^2}\right)\left(\frac{\omega_T}{\omega}\right)^2 \overline{i_g^2}}{\frac{1}{4R_s^2}\left(\frac{\omega_T}{\omega}\right)^2 \overline{v_s^2}}$$

From previous results

$$\overline{i_g^2} = 4kT\delta g_g = 4kT\delta \frac{\omega^2 C_{gs}^2}{5 g_{d0}} = 4kT\delta\alpha \frac{\omega^2 C_{gs}^2}{5 g_m}$$

$$= 1 + \frac{R_g}{R_s} + \frac{1}{2Q}\left(\frac{\omega}{\omega_T}\right)\frac{\gamma}{\alpha} + \frac{R_s^2\left(1+\overset{4}{Q^2}\right)\delta\frac{\alpha\,\omega^2 C_{gs}^2}{5\cdot g_m}}{R_s}$$

$$= \left(1+\overset{4}{Q^2}\right)\frac{\alpha\delta}{5}\,\frac{\omega^2 C_{gs}^2 \cdot R_s}{g_m}$$

$$= \boxed{\left(\frac{1+\overset{4}{Q^2}}{2Q}\right)\frac{\alpha\delta}{5}\left(\frac{\omega}{\omega_T}\right)}$$

$$\boxed{= 1 + \frac{R_g}{R_s} + \frac{1}{2Q}\left(\frac{\omega}{\omega_T}\right)\frac{\gamma}{\alpha} + \left(\frac{1+\overset{4}{Q^2}}{2Q}\right)\left(\frac{\omega}{\omega_T}\right)\frac{\alpha\delta}{5}}$$

⊛ component of F due to correlated noise of $\overline{i_g}$ to $\overline{i_d}$

$$\overline{v_s^2} = 4KTR_s \Delta f$$

$$\overline{v_g^2} = 4KTR_g \Delta f$$

$$\overline{i_d^2} = 4KT\frac{\gamma}{\alpha} g_m \Delta f$$

$$\overline{i_g^2} = 4KT\delta g_g \Delta f$$

$$g_g = \frac{\omega^2 C_{gs}^2}{5 g_{d0}} = \alpha \frac{\omega^2 C_{gs}^2}{5 \cdot g_m}$$

$$\sqrt{\overline{i_g^2} \cdot \overline{i_d^2}} = 4KT\sqrt{\frac{\delta\gamma}{\alpha} g_m \, \alpha \frac{1}{5 g_m}} \cdot \omega C_{gs} \Delta f$$

$$= 4KT\omega C_{gs}\sqrt{\frac{\delta \cdot \gamma}{5}} \cdot \Delta f$$

$\Rightarrow$ correlated noise

$$= \left(\frac{\omega_T}{j\omega}\right)\left(4KT\omega C_{gs}\sqrt{\frac{\delta\gamma}{5}}\right) \cdot \frac{c}{2} \Delta f$$

$\Rightarrow$ component of F due to correlate noise

$$= \frac{\left(\frac{\omega_T}{j\omega}\right) 4KT\omega C_{gs}\sqrt{\frac{\delta \cdot \gamma}{5}} \cdot \frac{c}{2} \Delta f}{\frac{1}{4R_s^2}\left(\frac{\omega_T}{\omega}\right)^2 \cdot 4KTR_s \cdot \Delta f}$$

$$= \left(\frac{\omega}{j\omega_T}\right) 2c\, R_s \omega C_{gs} \sqrt{\frac{\delta \cdot \gamma}{5}}$$

$$= \boxed{\left(\frac{\omega}{\omega_T}\right) \cancel{2} |c| \frac{1}{\cancel{2}Q} \sqrt{\frac{\delta \cdot \gamma}{5}}}$$

$$\boxed{F\Big|_{\text{due to correlated } \overline{i_g}} = \frac{|c|}{Q}\left(\frac{\omega}{\omega_T}\right)\sqrt{\frac{\delta \cdot \gamma}{5}}}$$

(7)

$2Q = 2 \cdot \frac{1}{2R_s \cdot \omega C_{gs}} = \frac{1}{R_s \omega C_{gs}}$

loaded-Q $\quad = Q_u \rightarrow$ unloaded Q

7) overall noise factor including gate noise

$$F = 1 + \underbrace{\frac{R_g}{R_s}}_{\text{due to } R_g} + \underbrace{\frac{1}{2Q}\frac{\gamma}{\alpha}\left(\frac{\omega}{\omega_T}\right)}_{\text{due to } \overline{i_d}} + \underbrace{\left(2Q + \frac{1}{2Q}\right)\left(\frac{\omega}{\omega_T}\right)^2\frac{\delta}{5}}_{\text{due to } \overline{i_g}} + \underbrace{\frac{1}{2Q}\cdot|c|\cdot\sqrt{\frac{\delta\gamma}{5}}\left(\frac{\omega}{\omega_T}\right)}_{\text{due to correlated component of } \overline{i_g}}$$

~~2Q → loaded Q~~

~~Q → unloaded Q~~

~~2Q → Q_s~~

due to $\overline{i_d}$:

$\propto \frac{1}{Q}$, $\propto \frac{1}{\omega_T}$

$\propto C_{gs}$, $\propto \frac{C_{gs}}{g_m}$

$\propto W$, $\propto W^{\frac{1}{2}}$

$\propto W^{\frac{3}{2}}$

due to $\overline{i_g}$:

$\propto Q$, $\propto \frac{1}{\omega_T}$

$\propto \frac{1}{C_{gs}}$, $\propto \frac{C_{gs}}{g_m}$

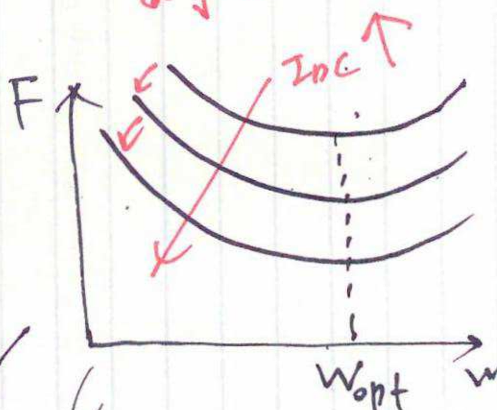
$\propto \frac{1}{W}$, $\propto W^{\frac{1}{2}}$

$\propto \frac{1}{\sqrt{W}}$

due to correlated component of $\overline{i_g}$:

$\propto W^{3/2}$

if I_{DC} = fixed



if W = fixed to W_{opt}
then increase g_m to increase f_T → increase I_{DC}

some remarks

- α, δ, γ & c are process parameters
- only design parameter is I_{DC} and device width,
- if we assume I_{DC} = constant there is an optimum W for the input transistor, where F will be minimum

9

$$Q = \frac{1}{2R_s\,\omega\, C_{gs}}$$

$$\omega_T = \frac{g_m}{C_{gs}}$$

$$\boxed{\begin{aligned} g_m &= \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th}) \\ C_{gs} &= \frac{2}{3}C_{ox}\cdot W\cdot L \end{aligned}}$$

$$\omega_T = \frac{\sqrt{2\mu_n C_{ox}\frac{W}{L}\cdot I_D}}{C_{gs}}$$

$$= \frac{\sqrt{\mu_n\left(\frac{2}{3}C_{ox}\cdot W\cdot L\right)\cdot 3\cdot\frac{1}{L^2}I_D}}{C_{gs}}$$

$$= \frac{1}{L}\cdot\frac{\sqrt{3\mu_n I_D}}{\sqrt{C_{gs}}}$$

$$= \frac{1}{L}\sqrt{3\cdot\mu_n\cdot I_D}\cdot\sqrt{2R_s\omega}\cdot\sqrt{Q}$$

$$= \frac{\sqrt{6\mu_n R_s\omega I_D}}{L}\sqrt{Q} \rightsquigarrow \omega_T \propto \sqrt{Q}$$

I_D will be set by "Design specification"

$$F = 1 + \frac{R_g}{R_s} + \frac{r}{\alpha}\left(\frac{\omega}{\omega_T}\right)\frac{1}{2}\frac{L}{\sqrt{6\mu_n \omega Z_0 R_s}\cdot Q^{\frac{3}{2}}}$$

$$F = 1 + \frac{R_g}{R_s} + \left(\frac{1}{2}\frac{r}{\alpha}\cdot\omega + \frac{1}{2}\frac{\alpha\delta}{5}\omega + |c|\sqrt{\frac{\delta r}{5}}\,\omega\right)\frac{1}{Q\cdot\omega_T}$$

$$+ 2\left(\frac{Q}{\omega_T}\right)\omega\frac{\alpha\delta}{5}$$

$$= 1 + \frac{R_g}{R_s} + \left(\frac{1}{2}\frac{r}{\alpha}\omega + \frac{1}{2}\frac{\alpha\delta}{5}\omega + |c|\sqrt{\frac{\delta r}{5}}\,\omega\right)$$

$$\times \frac{L}{\sqrt{6\mu_n Z_0 \omega\cdot R_s}}\, Q^{-3/2}$$

$$+ 2\omega\frac{\alpha\delta}{5}\frac{L}{\sqrt{6\mu_n Z_0\cdot\omega\cdot R_s}}\, Q^{\frac{1}{2}}$$

$$\frac{\partial F}{\partial Q} = 0$$

$$\Rightarrow \left(\frac{1}{2}\frac{r}{\alpha}\omega + \frac{1}{2}\frac{\alpha\delta}{5}\omega + |c|\sqrt{\frac{\delta r}{5}}\,\omega\right)\left(-\frac{3}{2}\right)Q^{-\frac{5}{2}}$$

$$+ \left(2\omega\frac{\alpha\delta}{5}\right)\frac{1}{2}Q^{-\frac{1}{2}} = 0$$

$$\Rightarrow Q^* = \sqrt{\frac{\frac{3}{2}\left(\frac{1}{2}\frac{r}{\alpha}\omega + \frac{1}{2}\frac{\alpha\delta}{5}\omega + |c|\sqrt{\frac{\delta r}{5}}\,\omega\right)}{\frac{1}{2}\left(2\omega\frac{\alpha\delta}{5}\right)}}$$

$$Q = \sqrt{3}\sqrt{\frac{1}{4} + \frac{5}{4}\frac{r}{\alpha^2\delta} + \frac{5}{2}|c|\sqrt{\frac{r}{5\alpha^2\delta}}}$$

$$= \frac{\sqrt{3}}{2}\sqrt{1 + 5\frac{r}{\alpha^2\delta} + 2|c|\sqrt{5\frac{r}{\alpha^2\delta}}}$$

if

$$\left.\begin{aligned} r &= 3 \\ \delta &= 2r = 6 \\ \alpha &= 0.8 \\ |c| &= 0.4 \end{aligned}\right\} \quad Q = 2.2$$

$$\longrightarrow Q_{\text{unloaded}} = \frac{1}{R_s \cdot \omega C_{gs}} = 4.4$$