to be stored by a field. Thus,
quivalent to that required to
ed weight. However, if a
creased in proportion to
energy density as

(J m^{-3}) (13)

ric medium

to the polarization of its mol-
a stretched spring.
ization and the energy density
$\frac{1}{2}PE$ to a total energy density

It will be shown later that a
oving electromagnetic field, or

citor of Fig. 4-10 is the integral
which the electric field **E** has a

$^2\,dv$ (14)

tes and that there is no fringing
on evaluating (14)

QV (J) (15)

throughout the volume be-
h the relation given by (5).

fair weather the earth's electric field
energy density and the total energy

ide with imaginary 1-m square con-
stitutes a capacitor of capacitance

$\frac{1^2}{1} = 8.85$ pF

$165^2 = 1.2 \times 10^{-7}$ J

ensity in the fair weather atmosphere

2×10^{-7} J m^{-3}

unit capacitance in coaxial cable
(Book "Electromagnetics"
by John D. Kraus)

Taking the electrosphere height $h = 25$ km and the earth's radius $r = 6.37$ Mm, the total energy in the earth-electrosphere capacitor is

$$W = wv = w4\pi r^2 h = 1.2 \times 10^{-7} \times 4\pi(6.37 \times 10^6)^2 \times 25 \times 10^3$$
$$= 1.5 \times 10^{12}\ \text{J}$$

Our calculation assumes that the fair weather field is uniform with height. Since it actually is not, the 1.5×10^{12} J result is significant only as an order of magnitude.

Example 4-7 Energy and energy density of three devices. Let us compare the energy storage and energy density of three commercially available devices:

a. A 1-kV, 1-μF power supply capacitor measuring $3 \times 4 \times 8$ cm
b. A 5-V, 0.1-F CMOS backup capacitor of 0.4 cm^3 volume
c. A 100-A-h, 12-V automobile storage battery measuring $15 \times 15 \times 20$ cm

Solution. (*a*) *Power supply capacitor* energy;

$$W = \tfrac{1}{2}CV^2 = \tfrac{1}{2} \times 10^{-6} \times (10^3)^2 = \tfrac{1}{2}\ \text{J}$$

and energy density:

$$w = \frac{W}{v} = \frac{1}{2}\,\frac{1}{3 \times 4 \times 8 \times 10^{-6}} = 5.2 \times 10^3\ \text{J m}^{-3}$$

(*b*) *CMOS backup capacitor* energy:

$$W = \tfrac{1}{2}CV^2 = \tfrac{1}{2}0.1 \times 5^2 = 1.25\ \text{J}$$

and energy density:

$$w = \frac{W}{v} = \frac{1.25}{0.4 \times 10^{-6}} = 3.1 \times 10^6\ \text{J m}^{-3}$$

(*c*) *12-V storage battery* energy:

$$W = V \times I \times t = 12 \times 100 \times 3600 = 4.3 \times 10^6\ \text{J}$$

and energy density:

$$w = \frac{W}{v} = \frac{4.3 \times 10^6}{15 \times 15 \times 20 \times 10^{-6}} = 9.6 \times 10^8\ \text{J m}^{-3}$$

The results of the above examples are summarized in Table 4-4, page 154.

Although the lead-acid battery achieves the highest energy density, capacitors can make their stored energy available much more rapidly.

4-12 COAXIAL TRANSMISSION LINE

The concentric cylindrical conductors arranged as in Fig. 4-12*a* form a useful current-carrying arrangement called a *coaxial transmission line*. Much can be learned about its properties from a consideration of its behavior under static conditions. Let a fixed potential difference be applied between the inner and outer

TABLE 4-4
Energies and energy densities of several storage devices

Device	Energy stored, J	Energy density, J m^{-3}
1-kV, 1-μF capacitor for power supply	0.5	5.2×10^3
5-V, 0.1-F capacitor for CMOS backup	1.25	3.1×10^6
Earth-electrosphere capacitor	1.5×10^{12}	1.2×10^{-7}
12-V, 100-ampere-hour battery for automobile	4.3×10^6	9.6×10^8

conductors of a long coaxial line so that the charge Q per unit length l of one conductor is ρ_L. The field is confined to the space between the two conductors. The field lines are radial, and the equipotential lines are concentric circles, as indicated in Fig. 4-12*b*. The magnitude of the field at a radius r is given by (2-10-1), where $a \leq r \leq b$, and where ρ_L is the charge per unit length on the inner conductor. The potential difference V between the conductors is, from (2-10-2),

$$V = \frac{\rho_L}{2\pi\epsilon} \ln \frac{b}{a} \tag{1}$$

Since capacitance is given by the ratio of charge to potential, $C = Q/V$. Dividing by length l we have $C/l = (Q/l)/V$. The ratio Q/l equals the linear charge density ρ_L (C m^{-1}). Hence, the *capacitance per unit length* C/l of the coaxial line

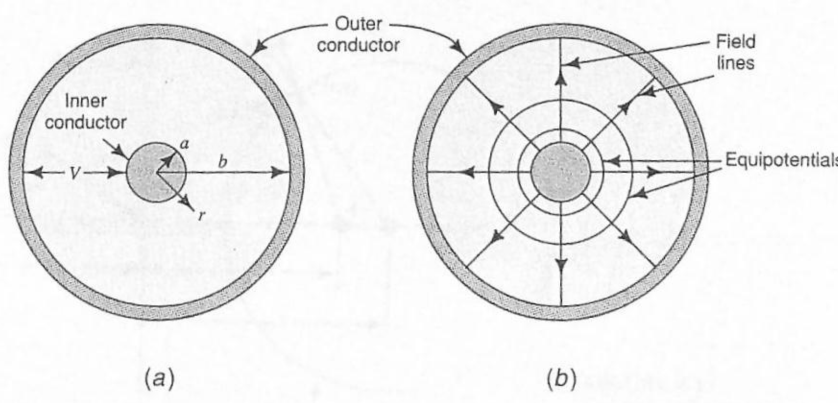


FIGURE 4-12
Coaxial transmission line.

'gy density, J m^{-3}

× 10^3

× 10^6

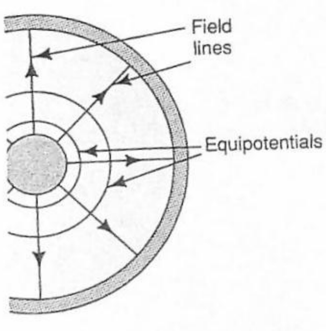
2 × 10^{-7}

.6 × 10^8

rge Q per unit length l of one con-
between the two conductors. The
are concentric circles, as indicated
adius r is given by (2-10-1), where
ength on the inner conductor. The
is, from (2-10-2),

(1)

of charge to potential, $C = Q/V$.
e ratio Q/l equals the linear charge
length C/l of the coaxial line



(b)

is

$$\boxed{\frac{C}{l} = \frac{\rho_L}{V} = \frac{2\pi\epsilon}{\ln (b/a)}} \qquad (\mathrm{F\,m^{-1}}) \tag{2}$$

where ϵ is the permittivity of the medium between conductors.

Since $\epsilon = \epsilon_0 \epsilon_r$, where $\epsilon_0 = 8.85\ \mathrm{pF\,m^{-1}}$, (2) can be expressed more conveniently as

$$\boxed{\frac{C}{l} = \frac{55.6\epsilon_r}{\ln (b/a)} = \frac{24.2\epsilon_r}{\log (b/a)} \qquad (\mathrm{pF\,m^{-1}})} \qquad \text{Coaxial line capacitance} \tag{3}$$

where ϵ_r = relative permittivity of medium between conductors
b = inside radius of outer conductor
a = radius of inner conductor (in same units as b)

4-13 TWO LINES OF CHARGE

Let two long parallel lines of charge be separated by a distance $2s$ as in Fig. 4-13. Assume that the linear charge density of the two lines is equal but of opposite sign. The resultant electric field **E** at a point P, distant r_1 from the negative line and r_2 from the positive line, is then the vector sum of the field of each line taken alone.

Let the origin of the coordinates in Fig. 4-13 be the reference for potential. Imagine that only the positively charged line is present. Then from (4-12-1) the potential difference between P and the origin is

$$V_+ = \frac{\rho_L}{2\pi\epsilon} \ln \frac{s}{r_2} \tag{1}$$

Similarly for the negatively charged line

$$V_- = -\frac{\rho_L}{2\pi\epsilon} \ln \frac{s}{r_1} \tag{2}$$

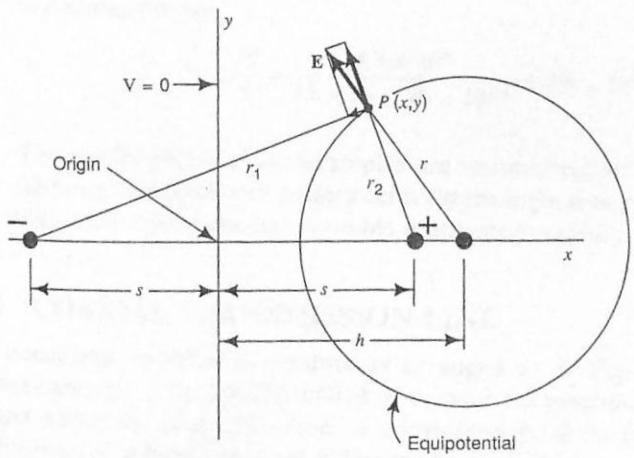


FIGURE 4-13
Two lines of charge separated by a distance $2s$.