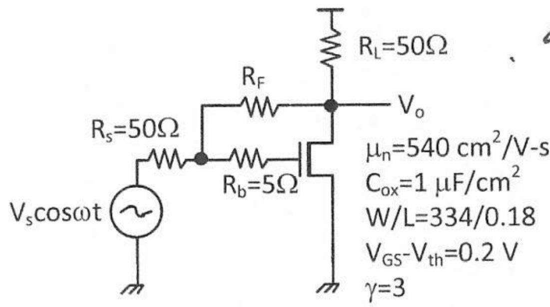
**ECE5220 RFIC Midterm, 03/19/2012** **Name:** Solution **Grade:**

1. In the amplifier shown below, DC characteristic of the NMOS is set by square-law characteristic, i.e.

$$I_{DS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2.$$

Assume that Rs << 1/ωCgs, **all resistors are noisy** and NMOS has drain thermal noise current. The drain thermal noise coefficient is γ.




- $g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th}) = 540\cdot\mu\cdot\frac{334}{0.18}\times 0.2 = 200\ m℧$
- $I_{DS} = \frac{1}{2}\cdot g_m(V_{GS}-V_{th}) = 20\ mA$

1) Determine feedback resistance, $R_F$, to match input and output impedance to $R_s$ and $R_L$, respectively (5pt).

$$Z_{in} = \frac{R_F+R_L}{1+g_mR_L} \simeq \frac{R_F}{g_mR_L} = R_s \rightarrow R_F = g_m\cdot R_L\cdot R_s = 500\,\Omega$$

$$Z_{out} = \frac{R_F+R_S}{1+g_mR_S} \simeq \frac{R_F}{g_mR_S} = 50\,\Omega = R_L$$

2) Express output noise voltages due to $R_s$, $R_F$, $R_b$, $R_L$ and drain current ($I_{DS}$), respectively, under matched condition (20pt). **You don't need to calculate them numerically**.

$$(V_{on})_{\text{due to } R_s} = \overline{V_{RS}}\cdot\frac{1}{2}\frac{R_F}{R_s} = \sqrt{4KTR_s\Delta f}\left(\frac{1}{2}\frac{R_F}{R_S}\right)$$

$$(V_{on})_{\text{due to } R_F} = \overline{V_{RF}}\cdot\frac{R_L}{Z_o+R_L} = \overline{V_{RF}}\,\frac{1}{2} = \sqrt{4KTR_F\Delta f}\left(\frac{1}{2}\right)$$

$$(V_{on})_{\text{due to } R_b} = \overline{V_{Rb}}\cdot g_m\frac{1}{2}R_L = \overline{V_{Rb}}\,\frac{1}{2}\frac{R_F}{R_s} = \sqrt{4KTR_b\Delta f}\left(\frac{1}{2}\frac{R_F}{R_s}\right)$$

$$(V_{on})_{\text{due to } R_L} = \overline{V_{RL}}\cdot\frac{1}{2} = \sqrt{4KTR_L\Delta f}\left(\frac{1}{2}\right)$$

$$(V_{on})_{\text{due to } I_{DS}} = \overline{i_{no}}\cdot\frac{1}{2}R_L = \sqrt{4KT\gamma g_m\Delta f}\left(\frac{1}{2}R_L\right)$$

3) Calculate noise factor (F) under the impedance matched condition (10pt).

$$F = 1 + \frac{4KTR_F\Delta f\left(\frac{1}{2}\right)^2 + 4KTR_D\Delta f\left(\frac{1}{2}\frac{R_F}{R_S}\right)^2 + 4KTR_L\Delta f\left(\frac{1}{2}\right)^2 + 4KT\gamma g_m \Delta f \left(\frac{1}{2}\right.}{4KTR_s\Delta f\left(\frac{1}{2}\frac{R_F}{R_s}\right)^2}$$

$$= 1 + \frac{R_S}{R_F} + \frac{R_D}{R_S} + \frac{R_L}{R_S}\left(\frac{R_S}{R_F}\right)^2 + \frac{\gamma g_m}{R_S}\left(\frac{R_S}{R_F}\right)^2 \cdot R_L^2$$

$$= 1 + \frac{R_S}{R_F} + \frac{R_D}{R_S} + \frac{R_L}{R_S}\left(\frac{R_S}{R_F}\right)^2 + \gamma\frac{R_S}{R_F}$$

$$= 1 + 0.1 + 0.1 + (0.1)^2 + 3\times 0.1 = 1.51 \rightarrow 1.79\,dB$$

4) Calculate THD without feedback ($R_F=\infty$). Assume that $V_s$=40 mV$_p$ (10pt).

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS}-V_{th}+V_s\cos\omega t\right)^2$$

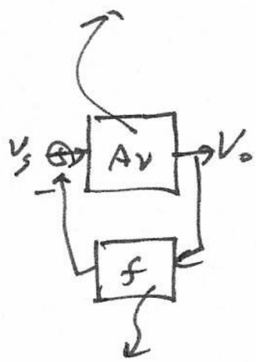
$$= I_{DC} + \underbrace{\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})}_{=g_m=\alpha_1}V_s\cos\omega t + \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}_{=\alpha_2}V_s^2\cos^2\omega t$$

$$\therefore THD = HD_2 = \frac{1}{2}\frac{\alpha_2}{\alpha_1}V_s = \frac{V_s}{4(V_{GS}-V_{th})} = \frac{40\,mV_p}{4\times 0.2}$$

$$= 0.05$$

$$= 5\%$$

$A_v = g_m R_L$




$f = R_s/R_F$

5) With $R_F$, determine feedback factor and loop gain (10pt).

- feedback factor. $f = \frac{R_s}{R_F+R_s} \simeq \frac{R_s}{R_F} = 0.1$
- loop gain $= (g_m \cdot R_L)\cdot f = \frac{R_F}{R_s}\cdot\frac{R_s}{R_F} = 1$

6) Calculate THD with feedback. Also assume that $V_s$=40 mV$_p$ (~~10~~ 15pt).

when applying feedback,

$$\alpha_1 \rightarrow \frac{\alpha_1}{1+\text{loop gain}}$$

$$\alpha_2 \rightarrow \frac{\alpha_2}{(1+\text{loop gain})^3}$$

$\Rightarrow (HD_2)_{\text{with feedback}}$

$$= (HD_2)_{\text{without feedback}} \times \frac{1}{(1+\text{loop gain})^2}$$

$$= 0.05 \times \frac{1}{4} = 0.0125 \rightarrow 1.25\%$$

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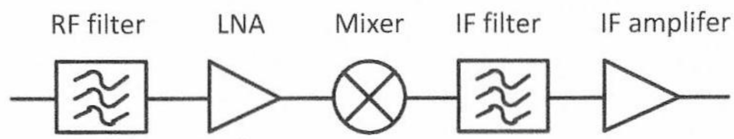
2. Consider the cascaded system shown below. All interfaces are matched to 50 Ω and signal bandwidth is 1MHz. **The required output SNR is 20 dB.**




| | RF filter | LNA | Mixer | IF filter | IF amplifer |
|---|---|---|---|---|---|
| Gain | 0.5<br>-3 dB | 31.6<br>15 dB | 0.2<br>-7 dB | 0.5<br>-3 dB | 31.6<br>15 dB |
| NF | 3 dB 2 | 2.5 dB 1.78 | 7 dB 5 | 3 dB 2 | 4 dB 2.5 |
| IIP3 | 43 dBm | -10 dBm | 12 dBm | 43 dBm | -9 dBm |

Linear scale

1) Determine input noise floor and input minimum detectable signal (MDS) of the system in dBm scale (10pt). Note, 10*log(KT)=-174 dBm @T=300K.

overall $F = 2 + \frac{1.78-1}{0.5} + \frac{5-1}{0.5\times 31.6} + \frac{2-1}{0.5\times 31.6\times 0.2} + \frac{2.5-1}{0.5\times 31.6\times 0.2\times 0.5}$

$= 5.078 \rightarrow 7.05\,dB$

∴ Input noise floor $= KT\Delta f\cdot F = -174\,dBm + 10\log 1MHz + 7.05\,dB$
$= -107\,dBm$

⇒ MDS = Input noise floor × SNRmin
$= -107\,dBm + 20\,dB = -87\,dBm$

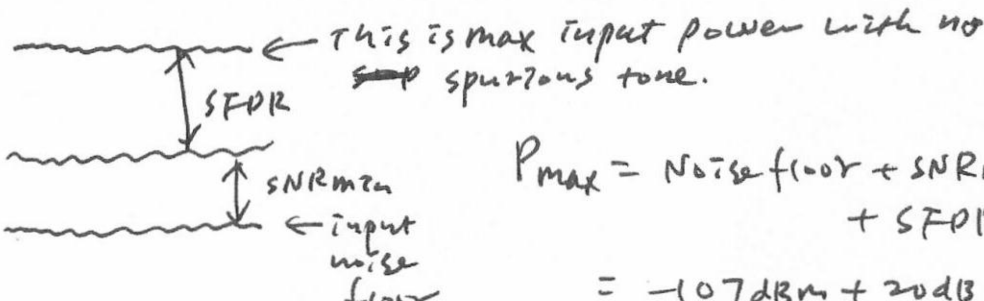
2) Determine spurious free dynamic range (SFDR) of the system (10pt).

$\frac{1}{\text{overall IIP3}} = \frac{1}{10^{4.3}} + \frac{0.5}{10^{-1}} + \frac{0.5\times 31.6}{10^{1.2}} + \frac{0.5\times 31.6\times 0.2}{10^{4.3}} + \frac{0.5\times 31.6\times 0.2\times 0.5}{10^{-0.9}}$

→ overall IIP3 $= 54\times 10^{-3}\,mW \rightarrow -12.7\,dBm$

∴ SFDR $= \frac{2}{3}(IIP_3 - \text{Noise floor}) - SNR_{min}$
$= 62.87\,dBm - 20\,dB = 42.87\,dB$

3) What is the maximum input power that allows no spurious tone? (10pt).




$P_{max}$ = Noise floor + SNRmin + SFDR

$= -107\,dBm + 20\,dB + 42.87\,dBm$

$= -44.13\,dBm$