

Since any point at a radius of 400 mm has an absolute potential of 5 V and any point at a radius of 100 mm an absolute potential of 20 V, we can draw circles at these radii and label them as 5 and 20 V equipotential contours. Also drawing circles at other potentials and radial field lines, we obtain the contour map of Fig. 2-16a. We observe that *field and potential lines contours always intersect at right angles*.

In Fig. 2-16b the absolute potential is plotted as a function of radius. In this presentation the slope of the potential versus radius is a measure of the electric field intensity: The steeper the slope of the potential "hill," the greater the field intensity. Thus, at $r = 100$ mm, $V = 20$ V and at $r = 200$ mm, $V = 10$ V so that the average field intensity for this distance interval is

$$|E| = \frac{\Delta V}{\Delta r} = \frac{20 - 10}{0.2 - 0.1} = \frac{10}{0.1} = 100 \text{ V m}^{-1}$$

Reducing the distance interval to an infinitesimal value, we can obtain the magnitude of the field intensity at a point as

$$|E| = \lim_{\Delta r \rightarrow 0} \frac{\Delta V}{\Delta r} = \frac{dV}{dr} \quad (6)$$

Thus, the *magnitude of the field equals the slope or gradient of the potential* which is the inverse of (5) in which we obtain the potential as the line integral of the field. We shall discuss gradient further in Sec. 2-14.

The work to move a test charge along an equipotential contour or surface is zero ($\theta = 90^\circ$). The maximum amount of work per unit distance is performed by moving normal to an equipotential surface. This coincides with the direction of the field.

The work to transport a charge around any *closed path* in a static field is zero since the path starts and ends at the same point. Thus, the upper and lower limits of the integrals in (5) become the same, and the result is zero. A property of the *static electric field* is, then, that *the line integral of this field around any closed path is zero*, that is,

$$\oint \mathbf{E} \cdot d\mathbf{L} = 0 \quad (7)$$

The circle on the integral sign of (7) signifies a closed path. A field for which (7) holds is called a *conservative or lamellar field*. It follows that the potential difference between any two points of a conservative field is independent of the path.

2-10 ELECTRIC FIELD STREAMLINES AND EQUIPOTENTIAL CONTOURS; ORTHOGONALITY

A field line indicates the direction of the force on a positive test charge introduced into the field. If the test charge is released, it accelerates in the direction of a field line.