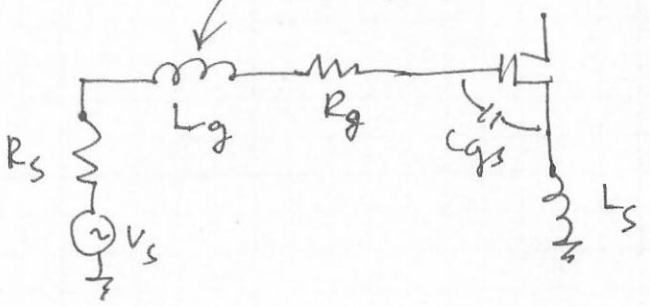
4) $Z_{in} = \underbrace{R_g + \frac{g_m L_s}{C_{gs}}}_{=R_s} + \underbrace{sL_s + \frac{1}{sC_{gs}}}$

Since $\omega_0 L_s < \frac{1}{\omega_0 C_{gs}}$, we need inductance to resonate the capacitance.



$$\omega_0 (L_g + L_s) = \frac{1}{\omega_0 C_{gs}}$$

$$\therefore L_g = \frac{1}{\omega_0^2 C_{gs}} - L_s$$

5) Under matched condition

$$i_g = \frac{V_s}{2R_s} \rightarrow V_{gs} = i_g \frac{1}{sC_{gs}} = \frac{1}{2R_s sC_{gs}} \cdot V_s$$

$$\rightarrow i_{out} = g_m V_{gs} = i_g \cdot \beta$$

$$= \frac{V_s}{2R_s} \cdot \frac{g_m}{sC_{gs}}$$

6) $A_V = \frac{V_{out}}{V_s} = \frac{-i_{out} \cdot R_L}{V_s} = -\frac{1}{2R_s} \cdot \frac{g_m}{sC_{gs}} \cdot R_L$

$\Rightarrow$ Under matched condition, gain will be increased by a factor of $\left(1 + \frac{sL_s + \frac{1}{sC_{gs}}}{2R_s}\right)$.

$\Rightarrow 1 + \frac{sL_s + \frac{1}{sC_{gs}}}{2R_s} = 1 - \frac{sL_g}{2R_s} = 1 - j\frac{1}{2}\frac{\omega_0 L_g}{R_s} \approx 1 - jQ$

$\therefore$ if Q is large enough, the gain factor is $\approx Q$, where Q is Quality factor of matching circuit.

7) L_s provides real impedance for matching.