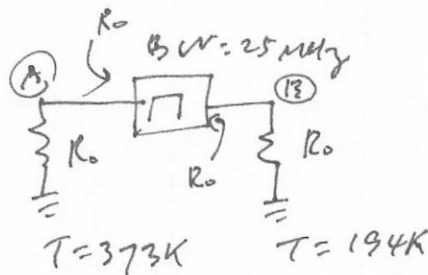


$$\begin{aligned}
 1) \quad \overline{V_o}^2 &= \frac{kT}{C} \\
 &= \frac{1.38 \times 10^{-23} \text{ J/K} \cdot 300 \text{ K}}{1.6 \times 10^{-12} \text{ F}} \\
 &= 2.5875 \times 10^{-9} \text{ V}_{\text{rms}}^2 \\
 \therefore \overline{V_o} &= 50.87 \mu\text{V}_{\text{rms}}
 \end{aligned}$$

3)



~~KTaf~~ at (A)

at (A)

noise power

$$\begin{aligned}
 &= kTaf \\
 &= 1.38 \times 10^{-23} \times 373 \times 25 \times 10^6
 \end{aligned}$$

$$= 128.69 \text{ fW}$$

at (B)

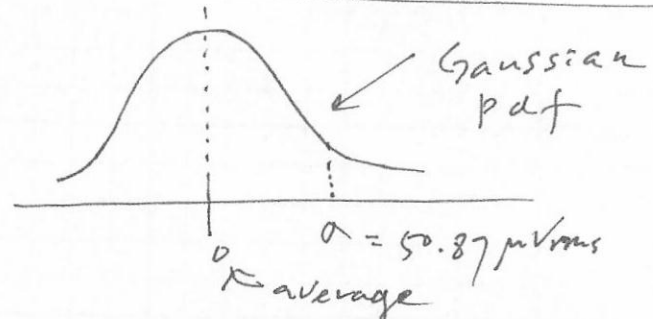
noise power

$$\begin{aligned}
 &= kTaf \\
 &= 1.38 \times 10^{-23} \times 194 \times 25 \times 10^6
 \end{aligned}$$

$$= 66.93 \text{ fW}$$

\therefore 61.76 fW of net power flow from (A) to (B).

2)



normalized random variable for $88 \mu\text{V}_{\text{rms}}$

$$Z = \frac{88 \mu\text{V}_{\text{rms}} - 0}{50.87 \mu\text{V}_{\text{rms}}}$$

$$= 1.7299 \approx 1.73$$

\rightarrow probability ($Z \leq 1.73$)

$$= 0.9582$$

\rightarrow probability ($Z > 1.73$)

$$= 1 - 0.9582 = 0.0418$$

\rightarrow probability ($Z < -1.73$)

$$= 0.0418$$

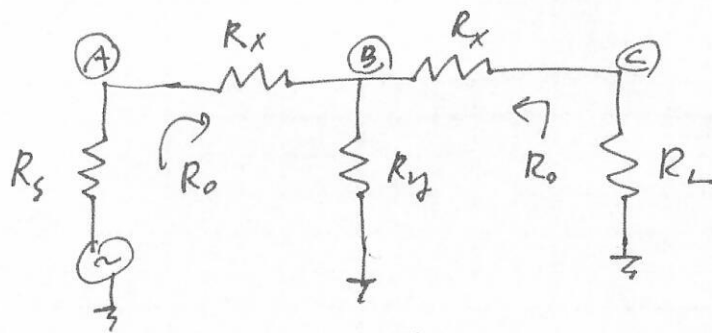
\therefore probability for $\overline{V_o} > 88 \mu\text{V}$

= probability ($Z > 1.73$)

+ probability ($Z < -1.73$)

$$= 0.0836$$

$$= 8.36\%$$



$$R_s = R_L = R_o$$

$$1) R_s \rightarrow \overline{V_{R_s}} = \sqrt{4kTR_s \Delta f}$$

$$\Rightarrow V_A = \frac{1}{2} \overline{V_{R_s}}$$

$$\Rightarrow V_B = \frac{R_o - R_x}{R_x + (R_o - R_x)} \cdot V_A = \frac{R_o - R_x}{R_o} \cdot \frac{1}{2} \overline{V_{R_s}}$$

$$\Rightarrow V_C = \frac{R_L}{R_x + R_L} V_B = \frac{R_o - R_x}{R_o + R_x} \cdot \frac{1}{2} \overline{V_{R_s}}$$

\therefore Noise power due to R_s across R_L is

$$P_L \Big|_{\text{due to } R_s} = \frac{V_C^2}{R_L} = \frac{\overline{V_{R_s}}^2}{4R_L} \left(\frac{R_o - R_x}{R_o + R_x} \right)^2 = kT \Delta f \frac{1}{2}$$

$$2) \text{ ① } R_x \text{ at left side} \rightarrow \overline{V_{R_x}} = \sqrt{4kTR_x \Delta f}$$

\Rightarrow Follow the same steps in 1)

\Rightarrow Noise power across R_L due to R_x at left side is

$$P_L \Big|_{\text{due to } R_x} = \frac{\overline{V_{R_x}}^2}{4R_o} \left(\frac{R_o - R_x}{R_o + R_x} \right)^2 = kT \Delta f \frac{R_x}{R_o} \cdot \frac{1}{2}$$

② R_x @ right side $\rightarrow \overline{V_{R_x}} = \sqrt{4kTR_x \Delta f}$

$\Rightarrow V_{(C)}$ due to R_x at right side

$$= \frac{R_L}{R_0 + R_L} \overline{V_{R_x}} = \frac{1}{2} \overline{V_{R_x}}$$

\Rightarrow noise power across R_L due to R_x at right side

$$P_L |_{\text{due to } R_x} = \frac{1}{R_L} \left(\frac{R_L}{R_0 + R_L} \right)^2 \overline{V_{R_x}}^2$$

$$= \frac{1}{4R_0} \overline{V_{R_x}}^2 = \cancel{\frac{R_x}{R_0}} \frac{R_x}{R_0} = kT \Delta f \frac{R_x}{R_0}$$

③ $R_y \rightarrow \overline{V_{R_y}} = \sqrt{4kTR_y \Delta f}$

$$\Rightarrow V_{(B)} = \frac{\frac{1}{2}(R_0 + R_x)}{R_y + \frac{1}{2}(R_0 + R_x)} \overline{V_{R_y}}$$

$$\begin{aligned} \Rightarrow V_{(C)} &= \frac{R_L \times V_{(B)}}{R_x + R_L} = \frac{R_0}{R_x + R_0} \frac{R_0 + R_x}{2R_y + R_0 + R_x} \overline{V_{R_y}} \\ &= \frac{R_0}{2R_y + R_0 + R_x} \overline{V_{R_y}} \end{aligned}$$

\Rightarrow noise power across R_L due to R_y

$$P_L |_{\text{due to } R_y} = \frac{1}{R_0} \left(\frac{R_0}{2R_y + R_0 + R_x} \right)^2 \overline{V_{R_y}}^2$$

$$= \frac{\overline{V_{R_y}}^2}{4R_0} \left(\frac{2R_0}{2R_y + R_0 + R_x} \right)^2 = kT \Delta f \frac{R_y}{R_0} \left(\frac{2R_0}{2R_y + R_0 + R_x} \right)^2$$

$$= \cancel{kT \Delta f \frac{R_y}{R_0} \left(\frac{2R_0}{2R_y + R_0 + R_x} \right)^2}$$



(3)

(3) Noise power at input is

$$P_{N, in} = K T \Delta f \quad (\text{Because attenuator is matched to source})$$

From (2)

$$P_{N, out} = \frac{1}{R_0} \left\{ \frac{V_s^2}{4} \frac{(R_0 - R_x)^2}{(R_0 + R_x)^2} + \frac{V_{N_1}^2}{4} \frac{(R_0 - R_x)^2}{(R_0 + R_x)^2} + \frac{V_{N_2}^2}{4} + \frac{V_y^2}{4} \frac{R_0^2}{\left(\frac{R_0 + R_x}{2} + R_y\right)^2} \right\}$$

$$= \frac{1}{R_0} \left\{ K T \Delta f \frac{(R_0 - R_x)^2}{(R_0 + R_x)^2} + K T \Delta f R_x + K T \Delta f R_y \frac{R_0^2}{\left(\frac{R_0 + R_x}{2} + R_y\right)^2} \right\}$$

$$\Rightarrow P_{N, out} = \frac{K T \Delta f}{R_0} \left[\frac{(R_0 - R_x)^2}{R_0 + R_x} + R_x + \frac{R_y R_0^2}{\left(\frac{R_0 + R_x}{2} + R_y\right)^2} \right]$$

$$\text{From } R_0 = R_x + \frac{R_y (R_x + R_0)}{R_y + R_x + R_0} \Rightarrow R_0 = R_x \sqrt{1 + \frac{2 R_y}{R_x}}$$

$$\text{Let } R_z = \sqrt{1 + \frac{2 R_y}{R_x}} \Rightarrow R_0 = R_z R_x$$

$$\Rightarrow \left[\frac{(R_0 - R_x)^2}{R_0 + R_x} + R_x + \frac{4 R_y R_0^2}{(R_0 + R_x + 2 R_y)^2} \right]$$

$$= \frac{R_x (R_z - 1)^2}{R_z + 1} + R_x + \frac{R_x^2 4 R_y R_z^2}{R_x^2 \left(R_z + 1 + \frac{2 R_y}{R_x}\right)^2}$$

$\underbrace{\left(R_z + 1 + \frac{2 R_y}{R_x}\right)^2}_{R_z^2}$

$$= \frac{R_x (R_z - 1)^2}{R_z + 1} + R_x + \frac{4 R_y}{(1 + R_z)^2}$$

$$= \frac{R_x (R_z - 1)^2}{R_z + 1} + R_x + \frac{4R_y}{(1 + R_z)^2}$$

$$= \frac{R_x (R_z - 1)^2 (R_z + 1) + R_x (R_z + 1)^2 + 4R_y}{(1 + R_z)^2}$$

$$= \frac{R_x}{(1 + R_z)^2} \left\{ R_z^3 - R_z - R_z^2 + 1 + 1 + R_z^2 + 2R_z + \frac{4R_y}{R_x} \right\}$$

$$\text{Since } 1 + 1 + \frac{4R_y}{R_x} = 2R_z^2$$

$$= \frac{R_x}{(1 + R_z)^2} \left\{ R_z^3 + 2R_z^2 + R_z \right\}$$

$$= \frac{R_x R_z}{(1 + R_z)^2} (R_z^2 + 2R_z + 1) = R_x R_z$$

$$= R_0$$

$$\Rightarrow P_{N, \text{out}} = \frac{K T \Delta f}{R_0} \cdot R_0 = \underline{K T \Delta f}$$

$$\Rightarrow P_{N, \text{in}} = P_{N, \text{out}}$$

4) Noise factor

$$= \frac{\text{Total Noise power delivered to load}}{\text{output Noise power due to source noise only}}$$

→ From 1), P_L | due to R_S

$$= \frac{\overline{V_{R_S}}^2}{4R_L} \left(\frac{R_0 - R_X}{R_0 + R_X} \right)^2$$

$$= KT\Delta f \frac{1}{L}$$

$$\therefore F = \frac{KT\Delta f}{KT\Delta f \frac{1}{L}} = L$$

$$5) P_L | \text{due to } R_S = KT\Delta f \left(\frac{R_0 - R_X}{R_0 + R_X} \right)^2$$

$$= 1.38 \times 10^{-23} \times 300 \left(\frac{50 - 8.55}{50 + 8.55} \right)^2$$

$$= 2.07 \times 10^{-21} \text{ W/Hz} \quad \text{--- (A)}$$

$$P_L | \text{due to left } R_X = KT\Delta f \cdot \frac{R_X}{R_0} \left(\frac{R_0 - R_X}{R_0 + R_X} \right)^2 = 2.07 \times 10^{-21} \times \frac{R_X}{R_0}$$

$$= 354.8 \times 10^{-24} \text{ W/Hz} \quad \text{--- (B)}$$

$$P_L | \text{due to right } R_X = KT\Delta f \frac{R_X}{R_0}$$

$$= 1.38 \times 10^{-23} \times 300 \times \frac{8.55}{50}$$

$$= 707.94 \times 10^{-24} \text{ W/Hz} \quad \text{--- (C)}$$

$$P_L | \text{due to } R_{Y_1} = KT\Delta f \frac{R_{Y_1}}{R_0} \left(\frac{2R_0}{2R_{Y_1} + R_0 + R_X} \right)^2$$

$$= 1.38 \times 10^{-23} \times 300 \cdot \frac{141.9}{50} \left(\frac{2 \times 50}{2 \times 141.9 + 50 + 8.55} \right)^2$$

$$= 1 \times 10^{-21} \text{ W/Hz} \quad \text{--- (D)}$$

- Total noise power

$$= \textcircled{A} + \textcircled{B} + \textcircled{C} + \textcircled{D} = \underline{4.13 \times 10^{-21} \text{ W/Hz}}$$

- ~~available~~ available source noise power

$$= kT \Delta f = 1.38 \times 10^{-23} \times 300$$
$$= \underline{4.14 \times 10^{-21} \text{ W/Hz}}$$

These results are same.

\Rightarrow Noise power keeps constant.

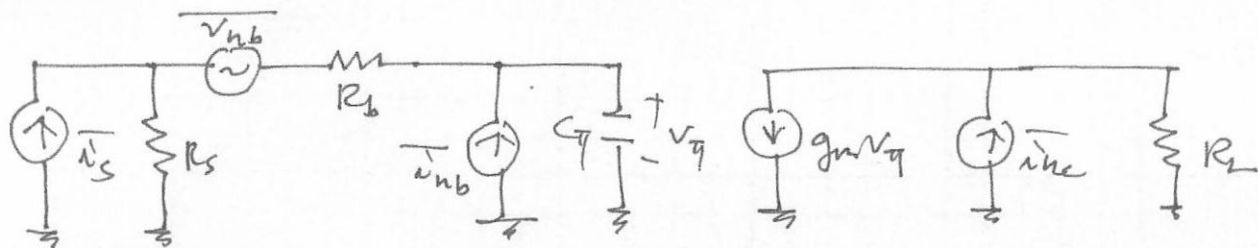
$$F = \frac{4.13 \times 10^{-21}}{2.07 \times 10^{-21}} \approx 2 \rightarrow 3 \text{ dB}$$

~~$$L = \left(\frac{R_o - R_x}{R_o + R_x} \right)^2 = \left(\frac{50 - 8.55}{50 + 8.55} \right)^2 = \dots$$~~

~~\therefore~~

$$L = \left(\frac{50 + 8.55}{50 - 8.55} \right)^2 \approx 2 \rightarrow 3 \text{ dB}$$

$$\Rightarrow F = L$$



1) input referred noise voltage ($= \overline{v_{in}}$)

$$\overline{v_{in}} = \overline{v_{nb}} + \overline{i_{nb}} (R_s + R_b) \parallel \frac{1}{sC_{\pi}} + \frac{\overline{i_{nc}}}{g_m} \frac{1}{\frac{1}{sC_{\pi}} + \frac{1}{R_s + R_b + \frac{1}{sC_{\pi}}}}$$

$$\approx \overline{v_{nb}} + \overline{i_{nb}} (R_s + R_b) + \frac{\overline{i_{nc}}}{g_m}$$

$$\left. \begin{aligned} \overline{v_{nb}} &= 4kTR_b \Delta f \\ \overline{i_{nb}} &= 2qI_B \Delta f \\ \overline{i_{nc}} &= 2qI_C \Delta f \\ g_m &= \frac{I_C}{V_T} \end{aligned} \right\} \text{apply}$$

\Rightarrow input referred noise power

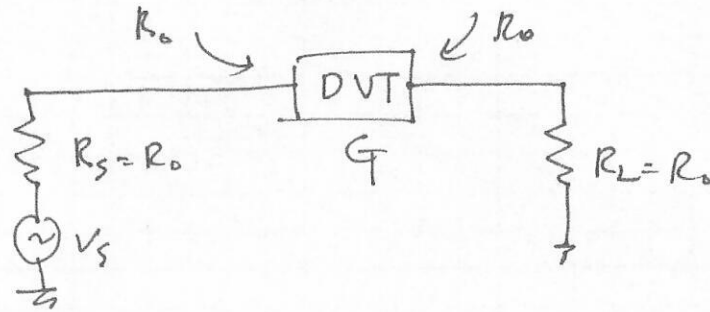
$$\overline{v_{in}}^2 = 4kTR_b \Delta f + 2qI_B (R_s + R_b)^2 \Delta f + 2qI_C \left(\frac{V_T}{I_C} \right)^2 \Delta f$$

2) Noise factor

$$F = 1 + \frac{4kTR_b \Delta f + 2qI_B (R_s + R_b)^2 \Delta f + 2qI_C \left(\frac{V_T}{I_C} \right)^2 \Delta f}{4kTR_s \Delta f}$$

$$= 1 + \frac{R_b}{R_s} + \frac{(R_s + R_b)^2}{R_s} \frac{1}{2} \frac{I_B}{V_T} + \frac{1}{R_s} \frac{1}{2} \frac{V_T}{I_C}$$

NF Measurement (1)



— step 1) $V_s = 0$

\Rightarrow output noise power ($= P_{on}$)

$$P_{on} = K\Delta f \cdot G + \text{DUT noise power}$$

$$= K\Delta f \cdot G + P_{on, out}$$

— step 2) Increase V_s until out power becomes $2P_{on}$.

$$\Rightarrow 2P_{on} = \underbrace{\frac{V_s^2}{4R_o} \cdot G}_{\text{output power due to applied source, } V_s} + \underbrace{K\Delta f \cdot G}_{\text{output power due to noise in } R_s} + \underbrace{P_{on, out}}_{\text{DUT output noise power}}$$

$$= \frac{V_s^2}{4R_o} \cdot G + P_{on}$$

$$\therefore P_{on} = \frac{V_s^2}{4R_o} \cdot G \Rightarrow P_{on, out} = P_{on} - K\Delta f \cdot G$$

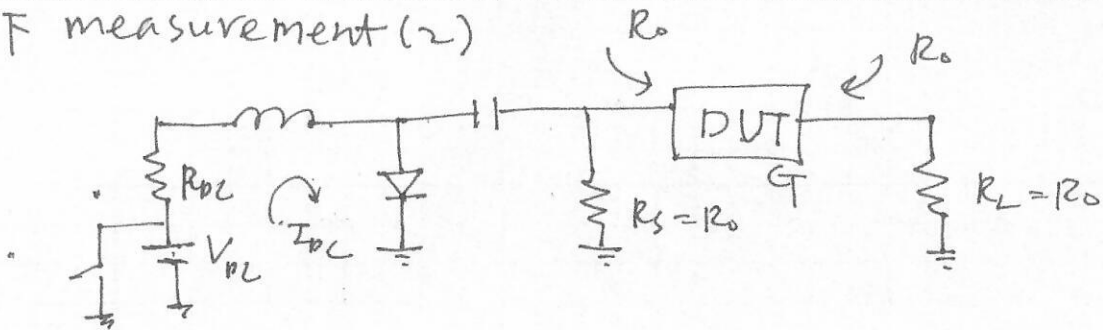
$$= \frac{V_s^2}{4R_o} \cdot G - K\Delta f \cdot G$$

From step-1) and step-2),

$$F = 1 + \frac{P_{on, out}}{K\Delta f \cdot G} = \frac{\frac{V_s^2}{4R_o} \cdot G}{K\Delta f \cdot G} = \frac{\frac{V_s^2}{4R_o}}{K\Delta f}$$

Note: you need to know Δf in this method.

NF measurement (2)



- Step-1) Turn-on switch $\rightarrow I_{DC} = 0$
 \Rightarrow output noise power ($= P_{on}$)

$$P_{on} = \underbrace{KT\Delta f \cdot G}_{\text{noise power from } R_s} + \underbrace{P_{on, DUT}}_{\text{noise power from DUT}}$$

- Step-2) Turn-off switch and adjust V_{DC}
 Until output power becomes $2 \cdot P_{on}$

$$\Rightarrow 2P_{on} = \underbrace{2qI_{DC}\Delta f (R_s \parallel R_o)^2 \cdot G}_{\text{output noise power due to diode noise}}$$

$$+ KT\Delta f \cdot G + P_{on, DUT}$$

$$= \frac{1}{2} q I_{DC} \Delta f R_s^2 G + P_{on}$$

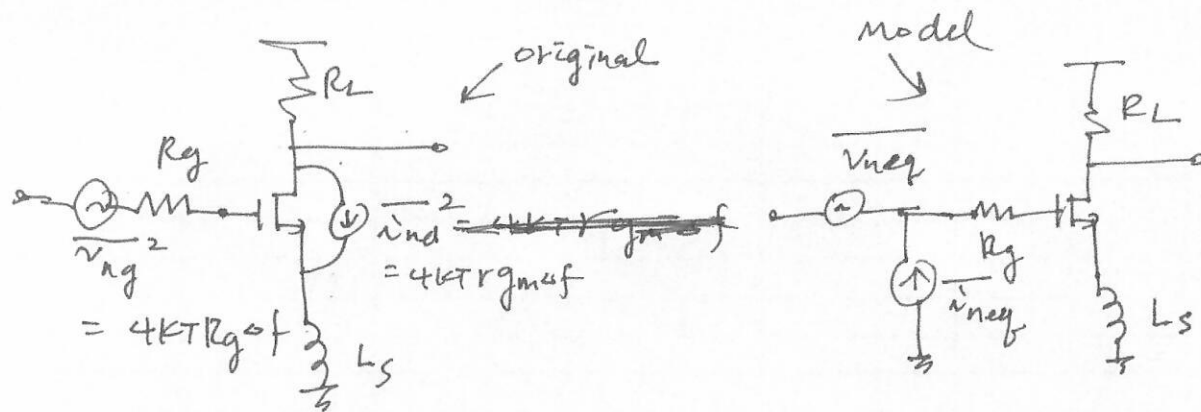
$$\therefore P_{on} = \frac{1}{2} q I_{DC} \Delta f R_s^2 G$$

$$\Rightarrow P_{on, DUT} = q I_{DC} \Delta f R_s^2 G - KT\Delta f \cdot G$$

From Step-1) and Step-2),

$$\text{Noise factor, } F = 1 + \frac{P_{on, DUT}}{KT\Delta f \cdot G} = \frac{\frac{1}{2} q I_{DC} R_s^2}{2KT} = \frac{I_{DC}}{2V_T} \cdot R_s^2$$

* Note: In this method, you don't need to know bandwidth of the DUT.



1) Equivalent input noise generators.

① Computing $\overline{v_{neq}} \Rightarrow$ short input to ground.

From original circuit, output noise ~~voltage~~ ^{current} is

$$\begin{aligned} \overline{i_{on}} &= \overline{v_{ng}} \cdot \frac{g_m}{1 + g_m S L_s} + \overline{i_{nd}} \cdot \frac{\frac{1}{g_m}}{\frac{1}{g_m} + S L_s} \\ &= \frac{1}{1 + g_m S L_s} (\overline{v_{ng}} g_m + \overline{i_{nd}}) \quad \text{--- (A)} \end{aligned}$$

From model, output noise current is

$$\overline{i_{on}} = \overline{v_{neq}} \frac{g_m}{1 + g_m S L_s} \quad \text{--- (B)}$$

\Rightarrow From (A) and (B),

$$\overline{v_{neq}} = \overline{v_{ng}} + \frac{\overline{i_{nd}}}{g_m}$$

② Computing $\overline{i_{n\text{eq}}} \Rightarrow$ open input node.

From original circuit, output noise current is

$$\overline{i_{on}} = \overline{i_{nd}} \quad \text{--- (A)}$$

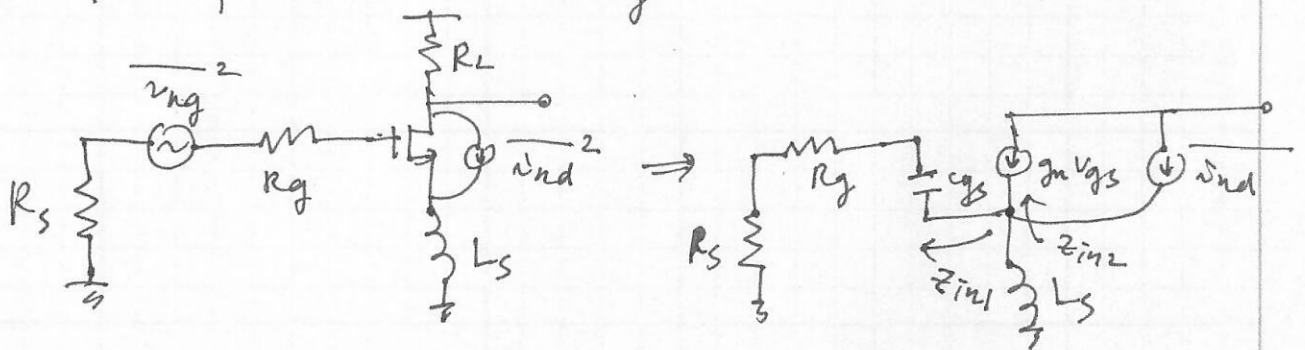
From model, output noise current is

$$\overline{i_{on}} = \overline{i_{n\text{eq}}} \cdot \frac{1}{sC_{gs}} \cdot g_m \quad \text{--- (B)}$$

\Rightarrow From (A) and (B)

$$\overline{i_{n\text{eq}}} = \overline{i_{nd}} \cdot \frac{sC_{gs}}{g_m} = \overline{i_{nd}} \frac{j\omega C_{gs}}{g_m}$$

2) Input referred noise voltage



From equivalent circuit shown in ~~the~~ right side,

$$Z_{in1} = \frac{1}{sC_{gs}} + R_g + R_S \approx \frac{1}{sC_{gs}} + R_S$$

$$Z_{in2} = \frac{\frac{1}{sC_{gs}} + R_S + R_g}{\frac{g_m}{sC_{gs}}} \approx \frac{1 + sC_{gs}R_S}{g_m}$$

\Rightarrow Output ^{noise} Current due to $\overline{i_{nd}}$ is

$$\overline{i_{no}} \Big|_{\text{due to } \overline{i_{nd}}} = \frac{Z_{in2}}{Z_{in1} \parallel sL_S + Z_{in2}} \cdot \overline{i_{nd}}$$

Therefore equivalent input noise voltage to create this noise current is

$$\overline{V_{n_{eq}}} \cdot \frac{\frac{g_m}{s C_{gs}}}{R_s + R_g + \frac{1}{s C_{gs}} + \left(1 + \frac{g_m}{s C_{gs}}\right) \cdot S L_s} = \hat{i}_{no} \Big|_{\text{due to } \overline{i_{nd}}}$$

$$\Rightarrow \overline{V_{n_{eq}}} = \frac{\left(\frac{\overline{z_{in2}}}{\overline{z_{in1}} \parallel S L_s + \overline{z_{in2}}} \cdot \overline{i_{nd}} \right)}{\left(\frac{\frac{g_m}{s C_{gs}}}{R_s + R_g + \frac{1}{s C_{gs}} + \left(1 + \frac{g_m}{s C_{gs}}\right) \cdot S L_s} \right)}$$



if $|\overline{z_{in1}}| \gg \omega L_s$

$$\Rightarrow = \frac{\frac{1 + s C_{gs} R_s}{g_m S L_s + 1 + s C_{gs} R_s} \cdot \overline{i_{nd}}}{\frac{g_m}{s C_{gs}}}$$

$$\downarrow \text{if } |1 + j \omega C_{gs} \cdot R_s| \gg g_m \cdot \omega L_s$$

$$\approx \frac{1}{g_m} \left(1 + s C_{gs} \left(R_s + \frac{g_m L_s}{C_{gs}} \right) \right) \overline{i_{nd}}$$

\therefore Input referred noise voltage, $\overline{V_{n, in}}$

$$\overline{V_{n, in}} = \overline{V_{n_g}} + \overline{V_{n_{eq}}}$$

$$\approx \overline{V_{n_g}} + \overline{i_{nd}} \cdot \frac{1}{g_m} \left(1 + s C_{gs} \left(R_s + \frac{g_m L_s}{C_{gs}} \right) \right)$$

3) correlation admittance

$$Y_c = \frac{\overline{V_{neg}}^* \overline{i_{neg}}}{\overline{V_{neg}}^2}$$

$$\approx j\omega C_{gs} \leftarrow \text{same as without } L_s \text{ (see lecture note, page-65)}$$

4) Uncorrelated noise current

$$\overline{i_{nu}} = \overline{i_{neg}} - Y_c \overline{V_{neg}}$$

$$= -j\omega C_{gs} \overline{V_{ng}} \leftarrow \text{see lecture note, page-65}$$

correlated noise current

$$\overline{i_{nc}} = \overline{i_{neg}} - \overline{i_{nu}}$$

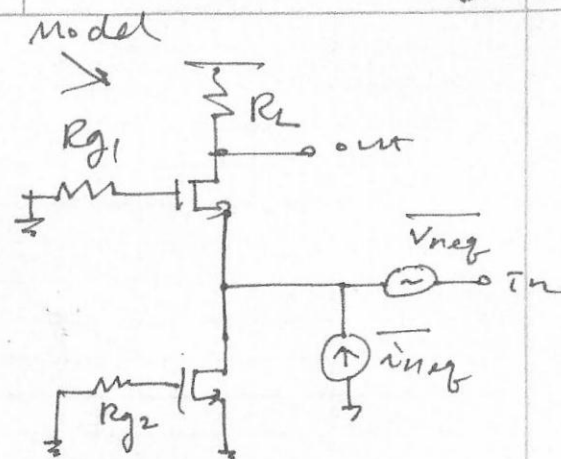
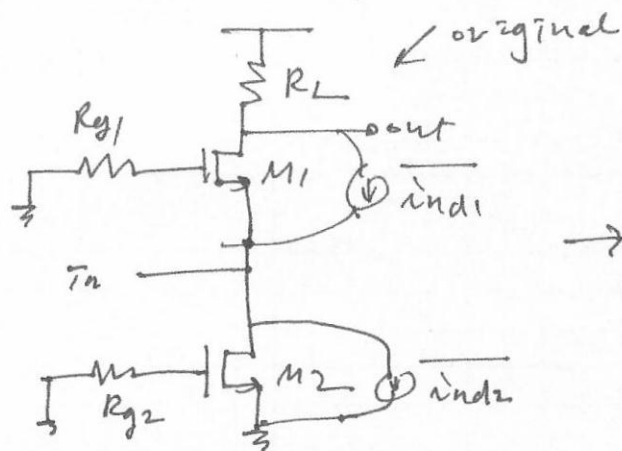
4) Optimum admittance

$$Y_{opt} = \omega C_{gs} \sqrt{\frac{g_m R_g}{r}} - j\omega C_{gs} \leftarrow \text{see lecture note page-66}$$

5) F_{min}

$$F_{min} = 1 + 2\omega C_{gs} \sqrt{r \frac{R_g}{g_m}} \leftarrow \text{see lecture note page-67.}$$

Think about why these results are same as in the case of without source inductor



1) Input noise generators

① computing $\overline{V_{neq}} \Rightarrow$ short input to ground

~~$\overline{i_{on}}$~~ For original circuit, output noise current is

$$\overline{i_{on}} = \overline{i_{nd1}} + \cancel{g_{m1} \overline{V_{ng1}}} \quad \text{--- (A)}$$

For model circuit, output noise current is

$$\overline{i_{on}} = g_{m1} \overline{V_{neq}} \quad \text{--- (B)}$$

\Rightarrow From (A) and (B),

$$\overline{V_{neq}} = \frac{1}{g_{m1}} \overline{i_{nd1}} + \overline{V_{ng1}}$$

② computing $\overline{i_{neq}} \Rightarrow$ open input node

For original circuit,

$$\overline{i_{on}} = \overline{i_{nd2}} + g_{m2} \overline{V_{ng2}} \quad \text{--- (A)}$$

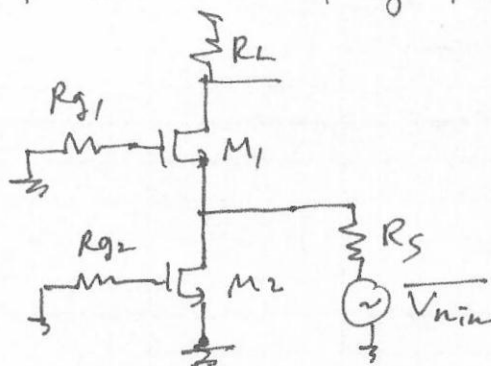
For model circuit,

$$\overline{i_{on}} = \overline{i_{neq}} \quad \text{--- (B)}$$

\Rightarrow From (A) and (B)

$$\overline{i_{neq}} = \overline{i_{nd2}} + g_{m2} \overline{V_{ng2}}$$

2) Input referred noise voltage, $\overline{V_{nin}}$



$$\overline{V_{nin}} \cdot \frac{\frac{1}{g_{m1}}}{R_S + \frac{1}{g_{m1}}} \cdot g_{m1} = \overline{i_{nd1}} \cdot \frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}} + R_S} + \overline{v_{ng1}} \cdot \frac{g_{m1}}{1 + g_{m1} R_S}$$

voltage across cgs of M_1

equivalent noise current in model circuit

effective output noise current in original circuit

$$\therefore \overline{V_{nin}} = \overline{i_{nd1}} \cdot \frac{1}{g_{m1}} + \overline{v_{ng1}}$$

3) correlation admittance

$$\overline{v_{neq}} = \frac{1}{g_{m1}} \overline{i_{nd1}} + \overline{v_{ng1}}$$

$$\overline{i_{neq}} = \overline{i_{nd2}} + g_{m2} \overline{v_{ng2}}$$

$$Y_c = \frac{\overline{v_{neq}}^* \overline{i_{neq}}}{\overline{v_{neq}}^2} = \frac{0}{\left(\frac{1}{g_{m1}}\right)^2 \overline{i_{nd1}}^2} = 0$$

$$\therefore \overline{v_{nc}} = 0$$

$$\therefore \overline{i_{nw}} = \overline{i_{neq}} = \overline{i_{nd2}} + g_{m2} \overline{v_{ng2}}$$

4) optimum admittance

$$R_n = \frac{\overline{V_{nq}}^2}{4kT\Delta f} = \frac{\left(\frac{1}{g_{m1}}\right)^2 4kTRg_{m1}\Delta f + 4kTRg_{m1}\Delta f}{4kT\Delta f}$$

$$= \frac{r}{g_{m1}} + R_g$$

$$G_u = \frac{\overline{i_{nu}}^2}{4kT\Delta f} = \frac{4kTRg_{m2}\Delta f + g_{m2}^2 \cdot 4kTRg_2\Delta f}{4kT\Delta f}$$

$$= r g_{m2} + g_{m2}^2 R_{g2}$$

$$= g_{m2} (r + g_{m2} R_{g2})$$

$$\therefore G_{s,opt} = \sqrt{\frac{G_u}{R_n}} = \sqrt{\frac{g_{m2} (r + g_{m2} R_{g2})}{\frac{r}{g_{m1}} + R_{g1}}}$$

$$= \sqrt{\frac{g_{m1} g_{m2} \left(1 + \frac{1}{r} g_{m2} R_{g2}\right)}{1 + \frac{1}{r} g_{m1} R_{g1}}}$$

$$B_{s,opt} = 0$$

$$\Rightarrow Y_{s,opt} = G_{s,opt}$$

$$5) F_{min} = 1 + 2 R_n \sqrt{\frac{G_u}{R_n}}$$

$$= 1 + 2 \frac{r}{g_{m1}} \sqrt{\frac{g_{m1} g_{m2} \left(1 + \frac{1}{r} g_{m2} R_{g2}\right)}{1 + \frac{1}{r} g_{m1} R_{g1}}}$$

if, $g_{m1} = g_{m2}$ and $R_{g2} \approx R_{g1}$, then $F_{min} \approx 1 + 2r \cdot g_m$