EECS 142

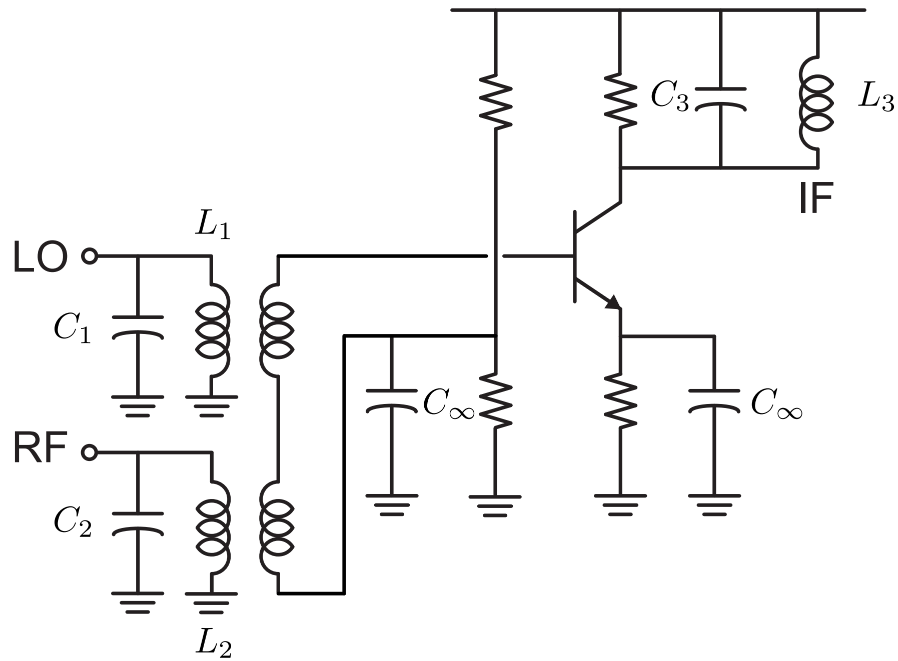
Integrated Circuits for Communication

Lecture 17: BJT/FET Mixers/Mixer Noise

Prof. Ali M. Niknejad

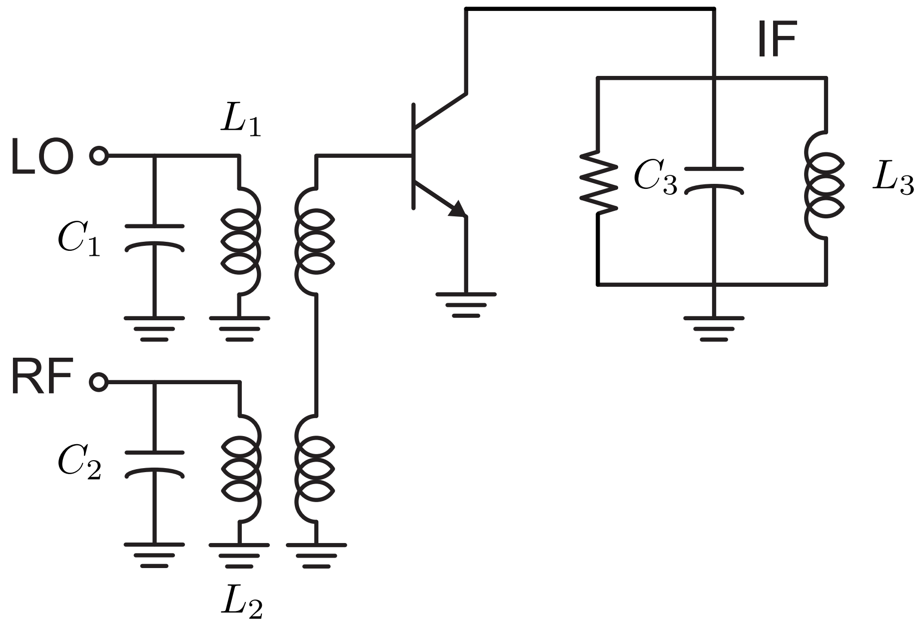
University of California, Berkeley

A BJT Mixer



- The transformer is used to sum the LO and RF signals at the input. The winding inductance is used to form resonant tanks at the LO and RF frequencies.
- The output tank is tuned to the IF frequency.
- Large capacitors are used to form AC grounds.

AC Eq. Circuit



- The AC equivalent circuit is shown above.

BJT Mixer Analysis

- When we apply the LO alone, the collector current of the mixer is given by

$$I_C = I_Q \left(1 + \frac{2I_1(b)}{I_0(b)} \cos \omega t + \frac{2I_2(b)}{I_0(b)} \cos 2\omega t + \cdots \right)$$

- We can therefore define a time-varying $g_m(t)$ by

$$g_m(t) = \frac{I_C(t)}{V_t} = \frac{qI_C(t)}{kT}$$

- The output current when the RF is also applied is therefore given by $i_C(t) = g_m(t) v_s$

$$i_C = \frac{qI_Q}{kT} \left(1 + \frac{2I_1(b)}{I_0(b)} \cos \omega t + \frac{2I_2(b)}{I_0(b)} \cos 2\omega t + \cdots \right) \times \hat{V}_s \cos \omega_s t$$

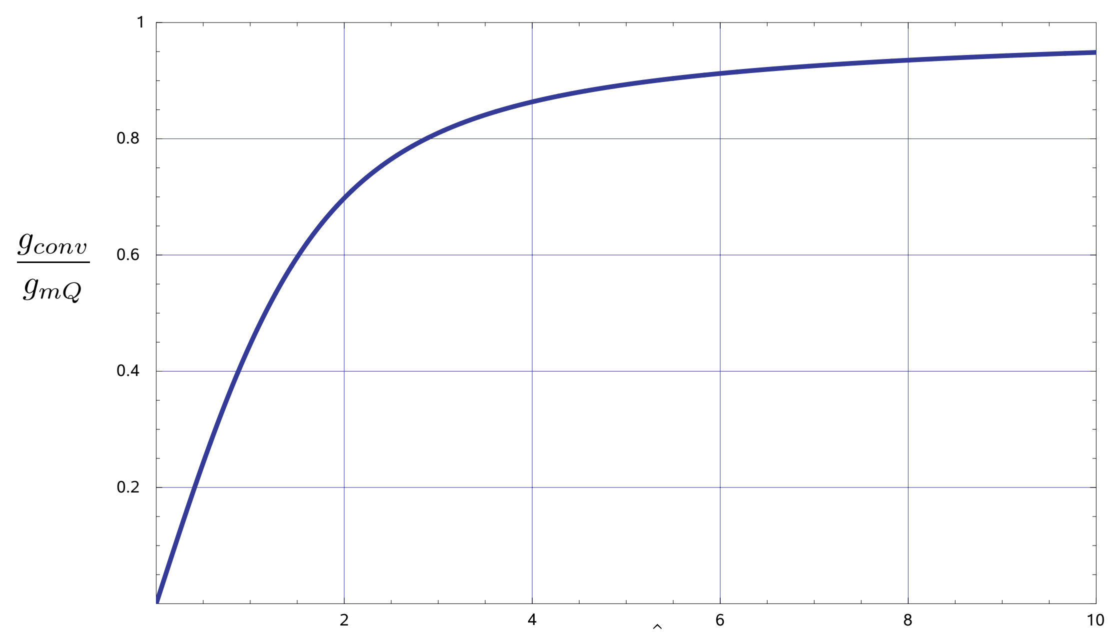
BJT Mixer Analysis (cont)

- The output at the IF is therefore given by

$$i_C\big|_{\omega_{IF}} = \hat{V}_s \underbrace{\frac{qI_Q}{kT}}_{g_{mQ}} \frac{I_1(b)}{I_0(b)} \cos(\underbrace{\omega_0 - \omega_s}_{\omega_{IF}})t$$

- The conversion gain is given by

$$g_{conv} = g_{mQ} \frac{I_1(b)}{I_0(b)}$$



LO Signal Drive

- For now, let's ignore the small-signal input and determine the impedance seen by the LO drive. If we examine the collector current

$$I_C = I_Q \left(1 + \frac{2I_1(b)}{I_0(b)} \cos \omega t + \frac{2I_2(b)}{I_0(b)} \cos 2\omega t + \cdots \right)$$

- The base current is simply I_C/β, and so the input impedance seen by the LO is given by

$$Z_i|_{\omega_0} = \frac{\hat{V}_o}{i_{B,\omega_0}} = \frac{\beta \hat{V}_o}{i_{C,\omega_0}} = \frac{\beta \hat{V}_o}{I_Q \frac{2I_1(b)}{I_0(b)}} = \frac{\beta b V_t}{I_Q \frac{2I_1(b)}{I_0(b)}}$$

$$= \frac{b}{2} \frac{\beta}{g_{mQ}} \frac{I_0(b)}{I_1(b)} = \frac{\beta}{G_m}$$

RF Signal Drive

- The impedance seen by the RF singal source is also the base current at the ω_s components. Typically, we have a high-Q circuit at the input that resonates at RF.

$$i_B(t) = \frac{i_C(t)}{\beta}$$

$$= \frac{1}{\beta}\frac{qI_Q}{kT}\left(\hat{V}_s \cos\omega_s t + \frac{2I_1(b)}{I_0(b)}\cos(\omega_0 \pm \omega_s)t + \cdots\right)$$

- The input impedance is thus the same as an amplifier

$$R_{in} = \frac{\hat{V}_s}{|\text{component in } i_B \text{ at } \omega_s|} = \beta\frac{kT}{qI_Q} = \frac{\beta}{g_{mQ}}$$

Mixer Analysis: General Approach

- If we go back to our original equations, our major assumption was that the mixer is a linear time-varying function relative to the RF input. Let's see how that comes about

$$I_C = I_S e^{v_{BE}/V_t}$$

where

$$v_{BE} = v_{in} + v_o + V_A$$

or

$$I_C = I_S e^{V_A/V_t} \times e^{b \cos \omega_0 t} \times e^{\frac{\hat{V}_s}{V_t} \cos \omega_s t}$$

- If we assume that the RF signal is weak, then we can approximate $e^x \approx 1 + x$

General Approach (cont)

- Now the output current can be expanded into

$$I_C = I_Q \left(1 + \frac{2I_1(b)}{I_0(b)} \cos \omega t + \frac{2I_2(b)}{I_0(b)} \cos 2\omega t + \cdots \right)$$

$$\times \left(1 + \frac{\hat{V}_s}{V_t} \cos \omega_s t \right)$$

- In other words, the output can be written as

$$= \text{BIAS} + \text{LO} + \text{Conversion Products}$$

- In general we would filter the output of the mixer and so the LO terms can be minimized. Likewise, the RF terms are undesired and filtered from the output.

Distortion in Mixers

- Using the same formulation, we can now insert a signal with two tones

$$v_{in} = \hat{V_{s1}} \cos \omega_{s1} t + \hat{V_{s2}} \cos \omega_{s2}$$

$$I_C = I_S e^{V_A/V_t} \times e^{b \cos \omega_0 t} \times e^{\frac{\hat{V_{s1}}}{V_t} \cos \omega_{s1} t + \frac{\hat{V_{s2}}}{V_t} \cos \omega_{s2} t}$$

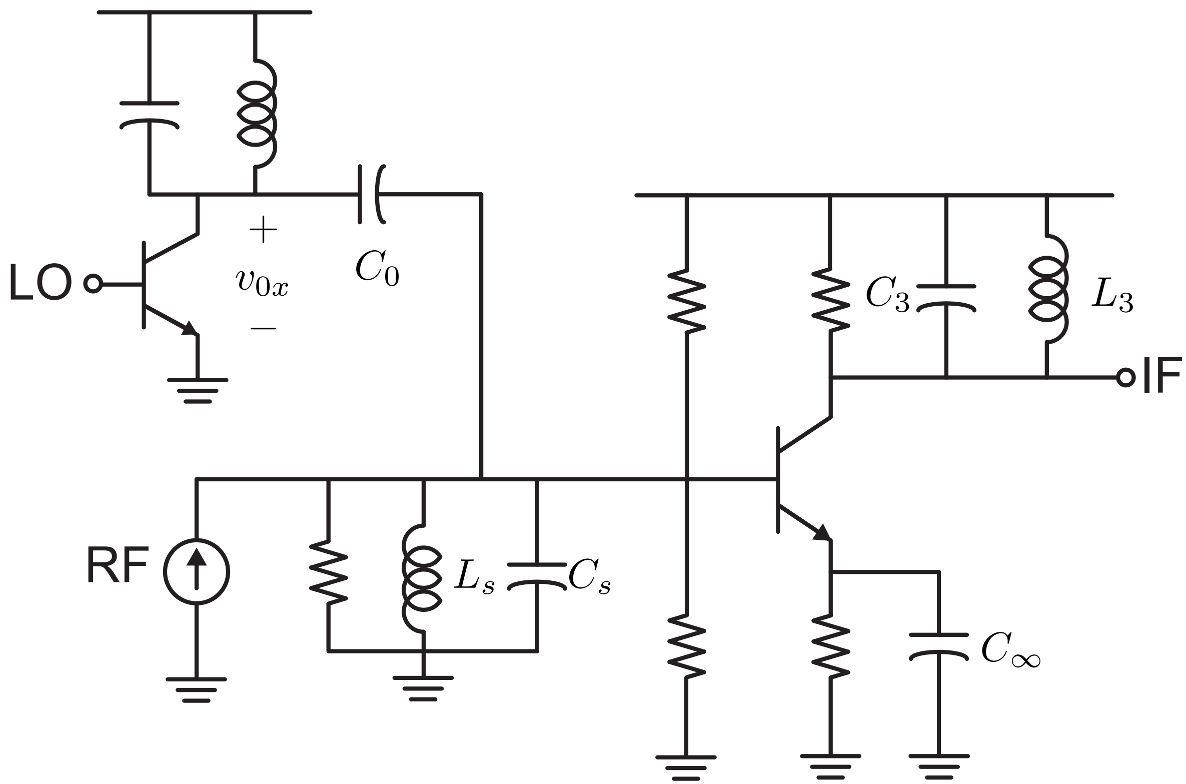
- The final term can be expanded into a Taylor series

$$I_C = I_S e^{V_A/V_t} \times e^{b \cos \omega_0 t} \times$$

$$\left(1 + V_{s1} \cos \omega_s 1 t + V_{s2} \cos \omega_s 2 t + (\ )^2 + (\ )^3 + \cdots\right)$$

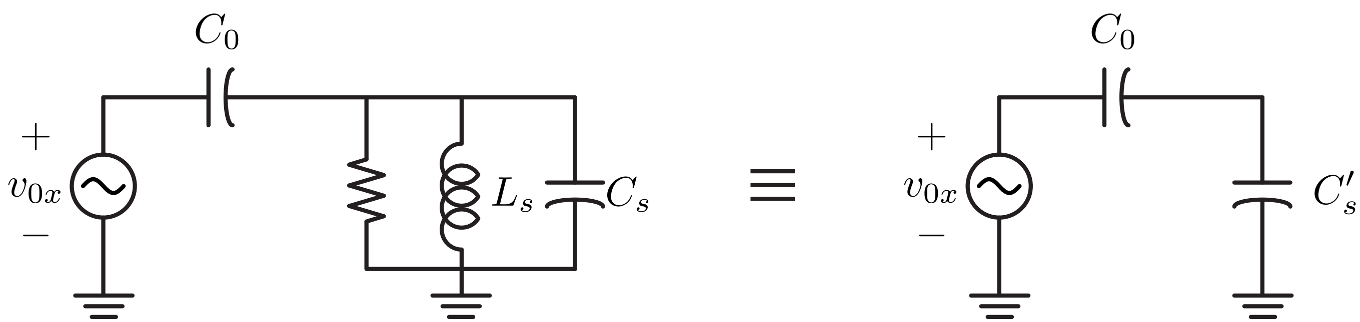
- The square and cubic terms produce IM products as before, but now these products are frequency translated to the IF frequency

Another BJT Mixer



- The signal from the LO driver is capacitively coupled to the BJT mixer

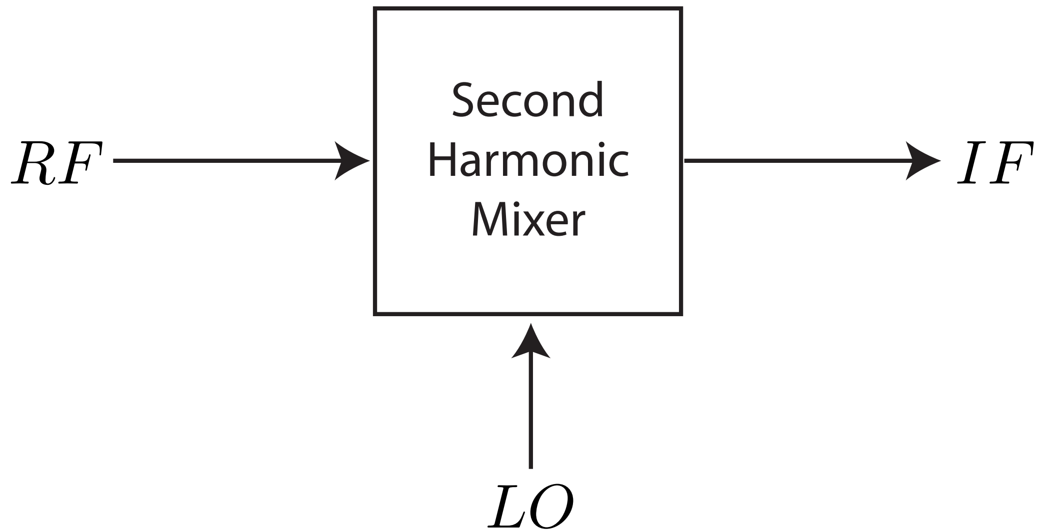
LO Capacitive Divider



- Assume that $\omega_{LO} > \omega_{RF}$, or a high side injection
- Note beyond resonance, the input impedance of the tank appears capacitive. Thus C_s' is the effective capacitance of the tank. The equivalent circuit for the LO drive is therefore a capacitive divider

$$v_o = \frac{C_o}{C_o + C_s'} v_{ox}$$

Harmonic Mixer



- We can use a harmonic of the LO to build a mixer.
- Example, let $LO = 500\mathrm{MHz}$, $RF = 900\mathrm{MHz}$, and $IF = 100\mathrm{MHz}$.
- Note that $IF = 2LO - RF = 1000 - 900 = 100$

Harmonic Mixer Analysis

- The nth harmonic conversion tranconductance is given by

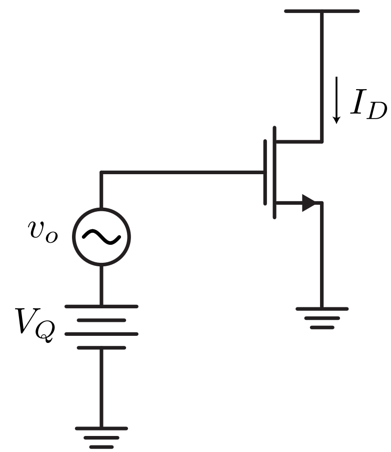
$$g_{conv,n} = \frac{|\text{IF current out}|}{|\text{input signal voltage}|} = \frac{g_n}{2}$$

- For a BJT, we have

$$g_{conv,n} = g_{mQ} \frac{I_n(b)}{I_0(b)}$$

- The advantage of a harmonic mixer is the use of a lower frequency LO and the separation between LO and RF.
- The disadvantage is the lower conversion gain and higher noise.

FET Large Signal Drive

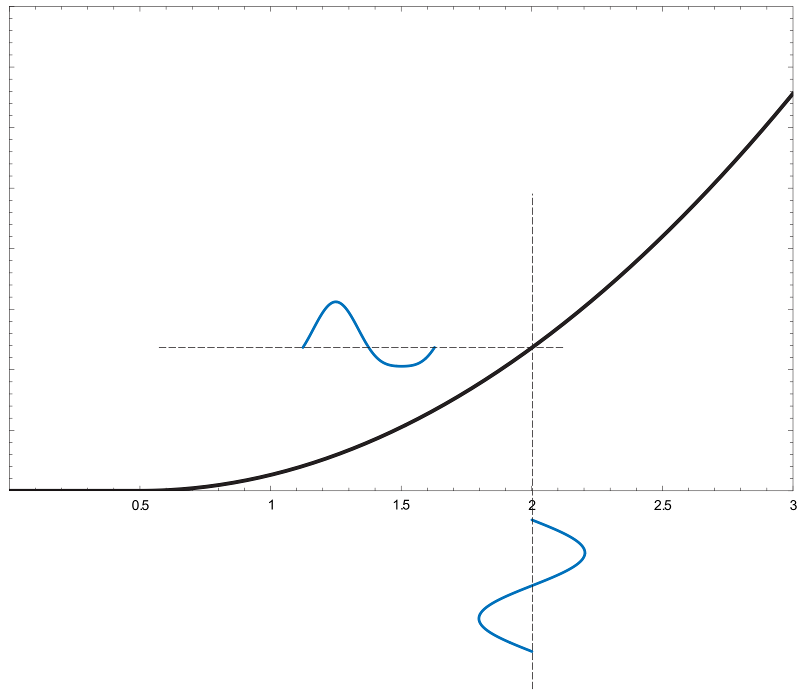


- Consider the output current of a FET driven by a large LO signal

$$I_D = \frac{\mu C_{ox}}{2}\frac{W}{L}(V_{GS} - V_T)^2(1 + \lambda V_{DS})$$

- where $V_{GS} = V_A + v_{LO} = V_A + V_o \cos\omega_0 t$. Here we implicitly assume that V_o is small enough such that it does not take the device into cutoff.

FET Large Signal Drive (cont)



- That means that $V_A + V_0 \cos\omega_0 t > V_T$, or $V_A - V_0 > V_T$, or equivalently $V_0 < V_A - V_T$. Under such a case we expand the current

$$I_D \propto \left((V_A - V_T)^2 + V_0 \cos^2\omega_0 t + 2(V_A - V_T)V_0 \cos\omega_0 t\right)$$

FET Current Components

- The $\cos^2$ term can be further expanded into a DC and second harmonic term.
- Identifying the quiescent operating point

$$I_Q = \frac{\mu C_{ox}}{2}\frac{W}{L}(V_A - V_T)^2(1+\lambda V_{DS})$$

$$I_D = I_{DQ} + \mu C_{ox}\frac{W}{L}\left(\underbrace{\frac{1}{4}V_0^2}_{\text{bias point shift}} + \underbrace{(V_A - V_T)V_o\cos\omega_0 t}_{\text{LO modulation}} + \underbrace{\frac{V_0^2}{4}\cos^2\omega_0 t}_{\text{LO 2nd harmonic}}\right)(1+\lambda V_{DS})$$

FET Time-Varying Transconductance

- The transconductance of a FET is given by (assuming strong inversion operation)

$$g(t) = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox} \frac{W}{L}(V_{GS} - V_T)(1 + \lambda V_{DS})$$

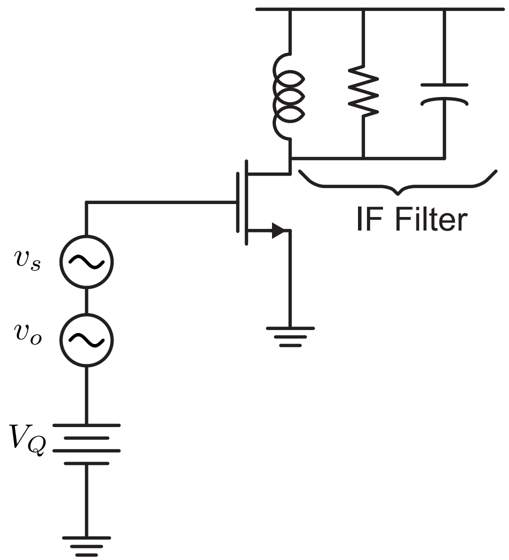
$$V_{GS}(t) = V_A + V_0 \cos \omega_0 t$$

$$g(t) = \mu C_{ox} \frac{W}{L}(V_A - V_T + V_0 \cos \omega_0 t)(1 + \lambda V_{DS})$$

$$g(t) = g_{mQ} \left(1 + \frac{V_0}{V_A - V_T} \cos \omega_0 t \right) (1 + \lambda V_{DS})$$

- This is an almost ideal mixer in that there is no harmonic components in the transconductance.

MOS Mixer



- We see that we can build a mixer by simply injecting an $LO + RF$ signal at the gate of the FET (ignore output resistance)

$$i_0 = g(t)v_s = g_{mQ}\left(1 + \frac{V_0}{V_A - V_T}\cos\omega_0 t\right)V_s\cos\omega_s t$$

$$i_0|_{IF} = \frac{g_{mQ}}{2}\frac{V_0}{V_A - V_T}\cos(\omega_0 \pm \omega_s)tV_s$$

$$g_c = \frac{i_0|_{IF}}{V_s} = \frac{g_{mQ}}{2}\frac{V_0}{V_A - V_T}$$

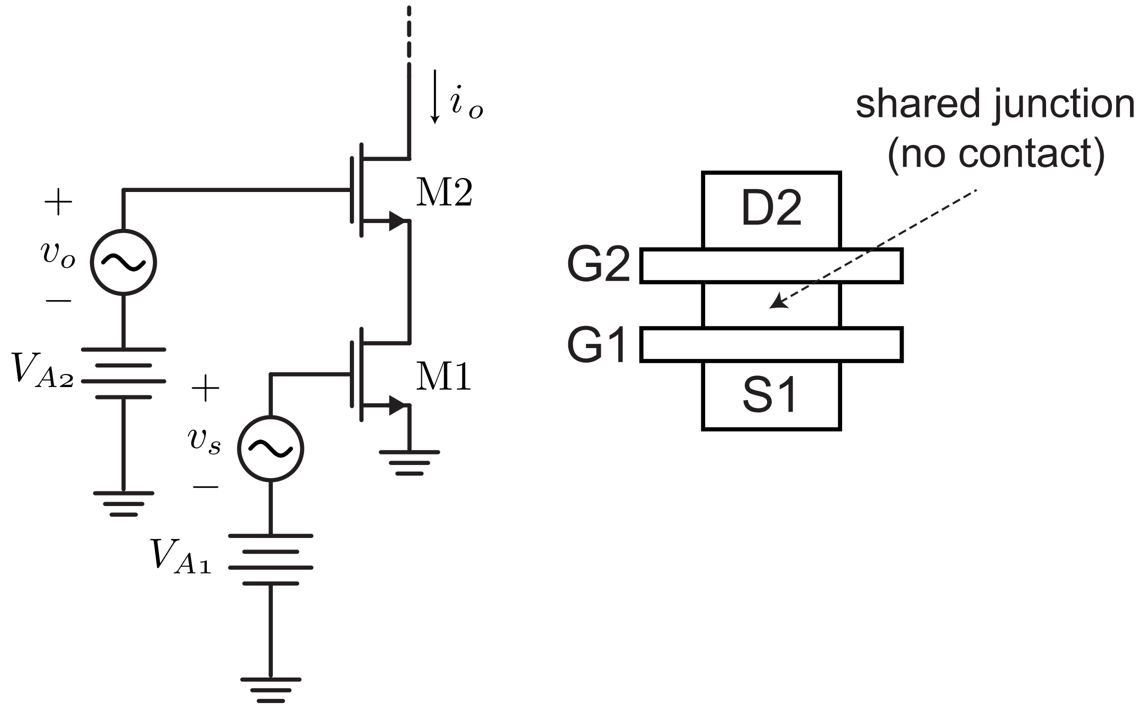
MOS Mixer Summary

- But $g_{mQ} = \mu C_{ox} \frac{W}{L}(V_A - V_T)$

$$g_c = \frac{\mu C_{ox}}{2} \frac{W}{L} V_0$$

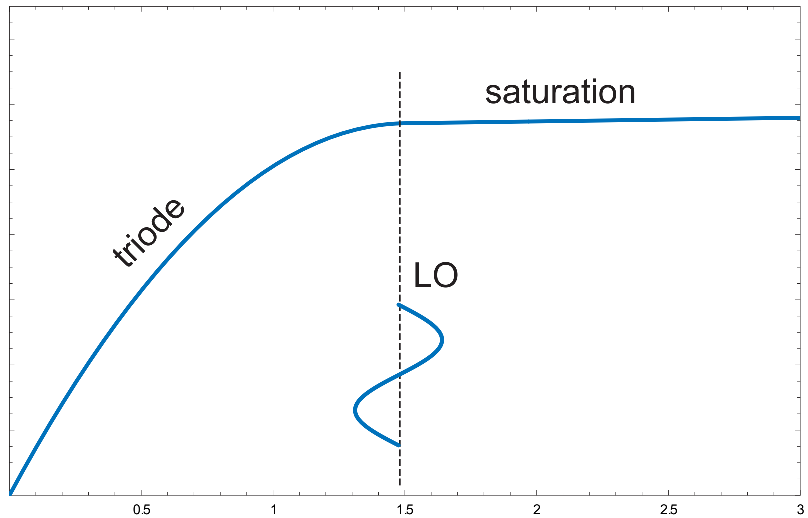
- which means that g_c is independent of bias V_A. The gain is controlled by the LO amplitude V_0 and by the device aspect ratio.
- Keep in mind, though, that the transistor must remain in forward active region in the entire cycle for the above assumptions to hold.
- In practice, a real FET is not square law and the above analysis should be verified with extensive simulation. Sub-threshold conduction and output conductance complicate the picture.

“Dual Gate” Mixer



- The “dual gate” mixer, or more commonly a cascode amplifier, can be turned into a mixer by applying the LO at the gate of $M2$ and the RF signal at the gate of $M1$. Using two transistors in place of one transistor results in area savings since the signals do not need to be combined with a transformer or capacitively .

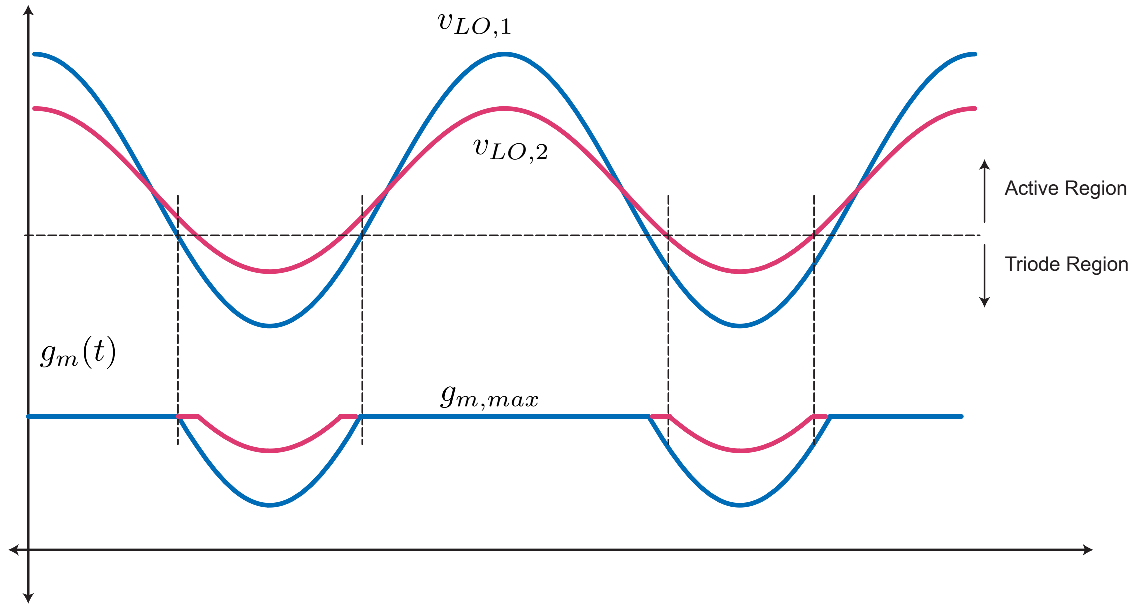
Dual Gate Mixer Operation



- Without the LO signal, this is simply a cascode amplifier. But the LO signal is large enough to push M1 into triode during part of the operating cycle.
- The transconductance of $M1$ is therefore modulated periodically

$$g_m\big|_{\text{sat}} = \mu C_{ox}\frac{W}{L}(V_{GS} - V_T) \qquad g_m\big|_{\text{triode}} = \mu C_{ox}\frac{W}{L}V_{DS}$$

Dual Gate Waveforms



- V_{GS2} is roughly constant since $M1$ acts like a current source.

$$V_{D1} = v_{LO} - V_{GS2} = V_{A2} + V_0 \cos\omega_0 t - V_{GS2}$$

$$g(t) = \begin{cases} \mu C_{ox}\frac{W}{L}(V_{GS1} - V_T) & V_{D1} > V_{GS} - V_T \\ \mu C_{ox}(V_{A2} - V_{GS2} - |V_0 \cos\omega_0 t|) & V_{D1} < V_{GS} - V_T \end{cases}$$

Realistic Waveforms

- A more sophisticated analysis would take sub-threshold operation into account and the resulting $g(t)$ curve would be smoother. A Fourier decomposition of the waveform would yield the conversion gain coefficient as the first harmonic amplitude.

Mixer Analysis: Time Domain

- A generic mixer operates with a periodic transfer function $h(t+T) = h(t)$, where $T = 1/\omega_0$, or T is the LO period. We can thus expand $h(t)$ into a Fourier series

$$y(t) = h(t)x(t) = \sum_{-\infty}^{\infty} c_n e^{j\omega_0 n t} x(t)$$

- For a sinusoidal input, $x(t) = A(t)\cos\omega_1 t$, we have

$$y(t) = \sum_{-\infty}^{\infty} \frac{c_n}{2} A(t) \left(e^{j(\omega_1 + \omega_0 n)t} + e^{j(-\omega_1 + \omega_0 n)t} \right)$$

- Since $h(t)$ is a real function, the coefficients $c_{-k} = c_k$ are even. That means that we can pair positive and negative frequency components.

Time Domain Analysis (cont)

- Take c_1 and c_{-1} as an example

$$= c_1 \frac{e^{j(\omega_1+\omega_0)t} + e^{j(-\omega_1+\omega_0)t}}{2} A(t) + c_1 \frac{e^{j(\omega_1-\omega_0)t} + e^{j(-\omega_1-\omega_0)t}}{2} A(t) +$$

$$= c_1 A(t) \cos(\omega_1 + \omega_0)t + c_1 A(t) \cos(\omega_1 - \omega_0)t + \cdots$$

- Summing together all the components, we have

$$y(t) = \sum_{-\infty}^{\infty} c_n \cos(\omega_1 + n\omega_0)t$$

- Unlike a perfect multiplier, we get an infinite number of frequency translations up and down by harmonics of ω_0.

Frequency Domain Analysis

- Since multiplication in time, $y(t) = h(t)x(t)$, is convolution in the frequency domain, we have

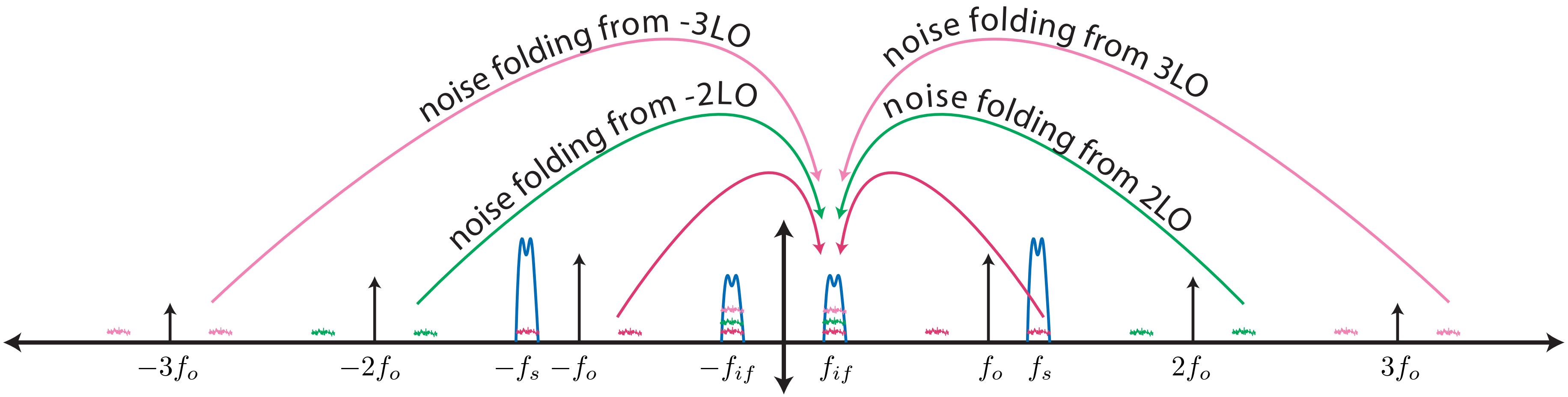
$$Y(f) = H(f) * X(f)$$

- The transfer function $H(f) = \sum_{-\infty}^{\infty} c_n \delta(f - nf_0)$ has a discrete spectrum. So the output is given by

$$Y(f) = \int_{-\infty}^{\infty} \sum_{-\infty}^{\infty} c_n \delta(\sigma - nf_0) X(f - \sigma) d\sigma$$

$$= \sum_{-\infty}^{\infty} c_n \int_{-\infty}^{\infty} \delta(\sigma - nf_0) X(f - \sigma) d\sigma$$

Frequency Domain (cont)



- By the frequency sifting property of the $\delta(f-\sigma)$ function, we have

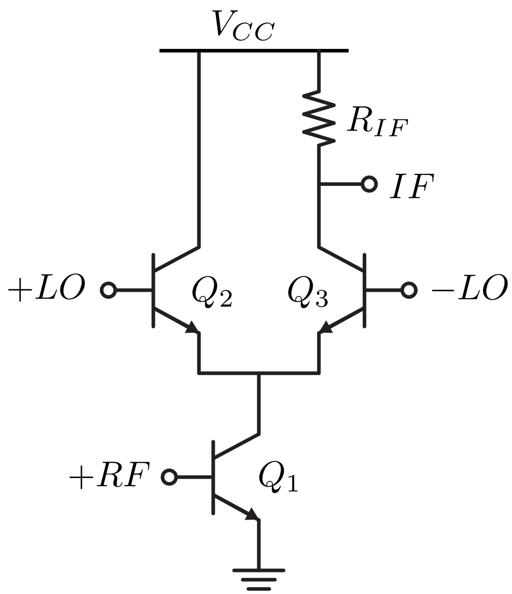
$$Y(f) = \sum_{-\infty}^{\infty} c_n X(f - nf_0)$$

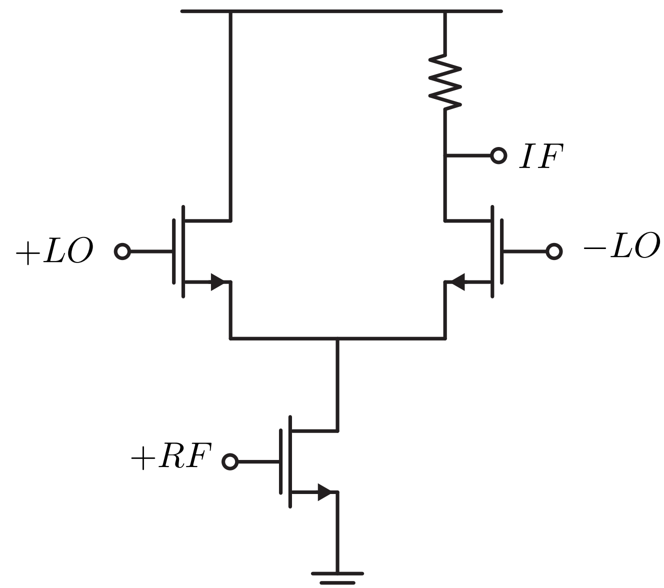
- Thus, the input spectrum is shifted by all harmonics of the LO up and down in frequency.

Noise/Image Problem

- Previously we examined the “image” problem. Any signal energy a distance of IF from the LO gets downconverted in a perfect multiplier. But now we see that for a general mixer, any signal energy with an IF of any harmonic of the LO will be downconverted !
- These other images are easy to reject because they are distant from the desired signal and a image reject filter will be able to attenuate them significantly.
- The noise power, though, in all image bands will fold onto the IF frequency. Note that the noise is generated by the mixer source resistance itself and has a white spectrum. Even though the noise of the antenna is filtered, new noise is generated by the filter itself!

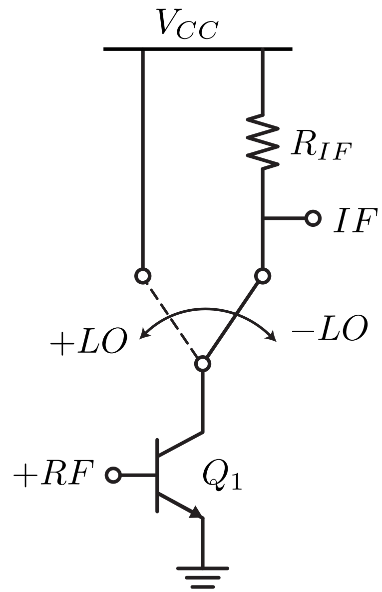
Current Commutating Mixers





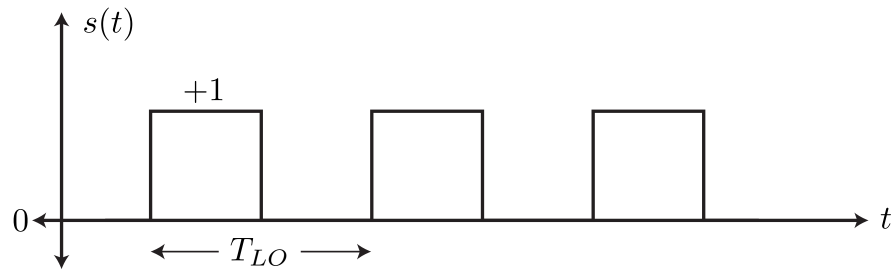
- A popular alternative mixer topology uses a differential pair LO drive and an RF current injection at the tail. In practice, the tail current source is implemented as a transconductor.
- The LO signal is large enough to completely steer the RF current either through Q1 or Q2.

Current Commutating Mixer Model



- If we model the circuit with ideal elements, we see that the current I_{C1} is either switched to the output or to supply at the rate of the LO signal.
- When the LO signal is positive, we have a cascode dumping its current into the supply. When the LO signal is negative, though, we have a cascode amplifier driving the output.

Conversion Gain



- We can now see that the output current is given by a periodic time varying transconductance

$$i_o = g_m(t) v_s = g_{mQ} s(t) v_s$$

- where $s(t)$ is a square pulse waveform (ideally) switching between 1 and 0 at the rate of the LO signal. A Fourier decomposition yields

$$i_o = g_{mQ} v_s \left(0.5 + \frac{2}{\pi} \cos \omega_0 t - \frac{2}{\pi} \frac{1}{3} \cos 3\omega_0 t + \cdots \right)$$

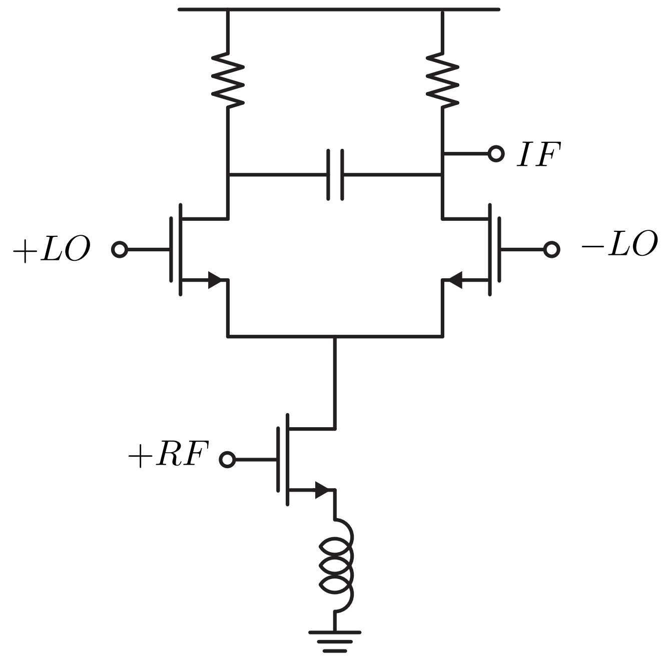
Conversion Gain (cont)

- So the RF signal v_s is amplified (feed-thru) by the DC term and mixed by all the harmonics

$$\frac{i_o}{V_s} = \frac{g_{mQ}}{2}\left(\frac{1}{2}\cos\omega_s t + \frac{2}{\pi}\cos(\omega_0 \pm \omega_s)t - \frac{2}{3\pi}\cos(3\omega_0 \pm \omega_s)t + \cdots\right)$$

- The primary conversion gain is $g_c = \frac{1}{\pi} g_{mQ}$.
- Since the role of $Q1$ (or $M1$) is to simply create an RF current, it can be degenerated to improve the linearity of the mixer. Inductance degeneration can be employed to also achieve an impedance match.
- MOS version acts in a similar way but the conversion gain is lower (lower g_m) and it requires a larger LO drive.

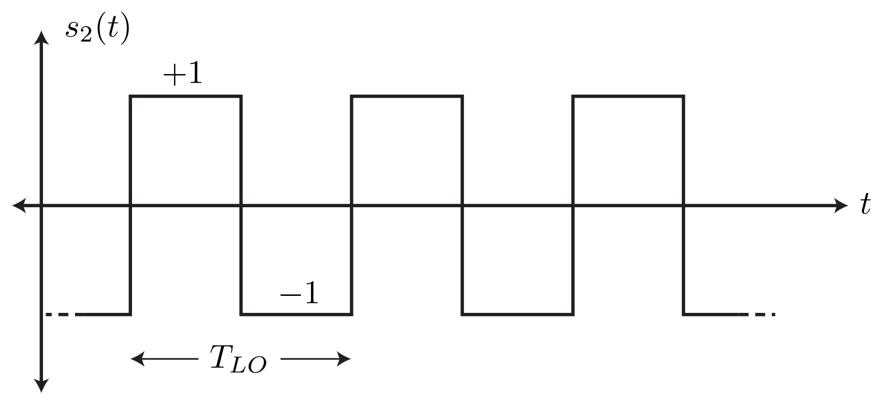
Differential Output



- This block is commonly known as the Gilbert Cell
- If we take the output signal differentially, then the output current is given by

$$i_o = g_m(t)v_s = g_{mQ}s_2(t)v_s$$

Differential Output Gain



- The pulse waveform $s_2(t)$ now switches between ± 1, and thus has a zero DC value

$$s_2(t) = \frac{4}{\pi} \cos \omega_0 t - \frac{1}{3} \frac{4}{\pi} \cos 3\omega_0 t + \cdots$$

- The lack of the DC term means that there is ideally no RF feedthrough to the IF port. The conversion gain is doubled since we take a differential output $g_c / g_{mQ} = \frac{2}{\pi}$