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Problem 1

Generator matrix, $G = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$

ⓐ Interchanging first and 3rd rows, we obtain the systematic form

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

ⓑ Parity check matrix, $H = [P^T : I_{n-k}]$

here $n = 7$

$k = 3$.

$\therefore H = [P^T : I_4]$

Generator matrix, $G = [I_3 : P] = \left[\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right]$

$$P = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$$P^T = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \qquad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ $H = \left[P^T : I_4 \right] = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$

(c) We have a $(7,3)$ code, there are $2^3 = 8$ valid code words, and 2^4 possible syndromes from these syndromes the all zero one corresponds to no error 7 will correspond to single errors and 8 will corresponds to double errors.

Error pattern	syndrome
0 0 0 0 0 0 0	0 0 0 0
0 0 0 0 0 0 1	0 0 0 1
0 0 0 0 0 1 0	0 0 1 0
0 0 0 0 1 0 0	0 1 0 0
0 0 0 1 0 0 0	1 0 0 0
0 0 1 0 0 0 0	1 1 0 1
0 1 0 0 0 0 0	0 1 1 1
1 0 0 0 0 0 0	1 1 1 1
1 0 0 0 0 0 1	1 1 0 0
1 0 0 0 0 1 0	1 0 1 0
1 0 0 0 1 0 0	0 1 1 0
1 0 0 1 0 0 0	0 0 1 1
1 0 1 0 0 0 0	1 0 0 1
1 1 0 0 0 0 0	0 1 0 1

0 1 0 0 0 1 0	0 1 0 1
0 0 0 1 1 0 1	1 0 1 1

(d) There is 3 linearly independent columns in H. hence there is a codeword C_m with weight $w_m = 4$ such that $C_m H^T = 0$, accordingly: $d_{min} = 4$.
This can be also obtained by generating all 8 codewords for this code and checking their minimum weight.

(e) 101 generates the codeword:

$$101 \rightarrow c = 1010011$$

then $cH^T = [0\ 0\ 0\ 0]$

Problem 2

If t is the number of correcting errors and if each codeword is viewed as the center of a sphere of radius t, then largest value that t may have without intersection of any pair of 2^k sphere is $t = \frac{1}{2}(d_{min} - 1)$

The probability of m errors in a block of n bits is $P(m,n) = \binom{n}{m} p^m (1-p)^{n-m}$

Therefore probability of a codeword error is upper bounded by $P_e \leq \sum_{m=t+1}^{n} P(m,n)$

for high SNR that is small values of p

$$P_e \simeq \binom{n}{t+1} p^{t+1} (1-p)^{n-t-1}$$

The number of codewords in a sphere of radius $t = \frac{1}{2}(d_{min} - 1)$ is

$$1 + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{t} = \sum_{i=0}^{t} \binom{n}{i}$$

Since there is $M = 2^k$ possible transmitted codewords, there are 2^k non overlapping spheres, each having radius t. The total no. of codewords enclosed in 2^k sphere can not exceed 2^n possible received codewords. Thus t error correcting code must satisfy

$$2^n \geq 2^k \sum_{i=0}^{t} \binom{n}{i}$$

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Golay code is an example of perfect code.

It is a linear block code with

block length = 23.

message length = 12.

Rate = 12/23.

minimum distance = 7.

problem 3

parity check matrix, $H = \left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] = [P^T : I_{n-k}]$

(1) generator matrix, $G = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & P \\ 0 & 0 & 1 & 0 & \\ 0 & 0 & 0 & 1 & \end{array}\right]$

$P^T = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$, $P = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

$\therefore G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$

(2) $M = 2^k = 2^4 = 16$ code words and any three columns of H are linearly independent. but we can find four columns that are linearly dependent. Hence $d_{min} = 3$.

3. The coding gain is $R_c d_{min} = \frac{1}{2} \times 3 = 1.5$

roughly equal to 1.8 dB.

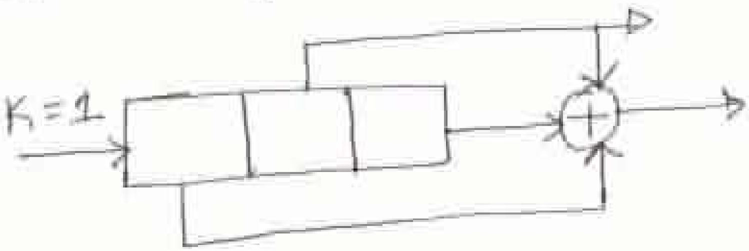
4. The ~~coding gain~~ With hard decision coding, number of errors that this code can correct

$$e_c = (d_{min} - 1)/2 = 1$$

5. It is easy to verify that each row of G is orthogonal to itself and all others rows. Hence any linear combination of the rows is also orthogonal to any linear combination of them.

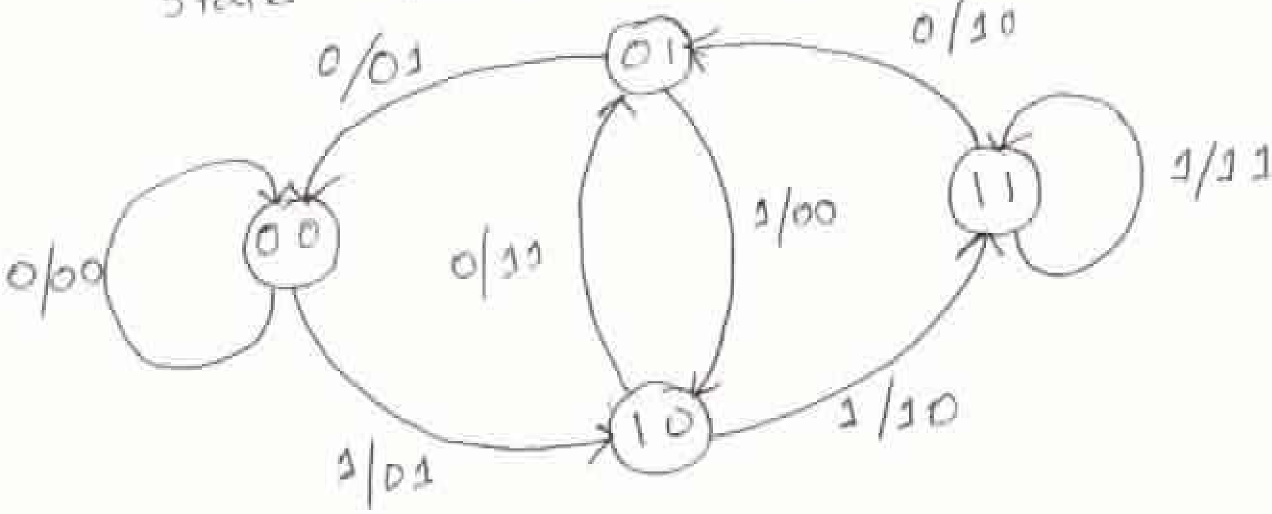
Problem 4

Block diagram of convolutional code



ⓐ Here $n = 2$

State transition diagram is shown below.



ⓑ The received sequence is

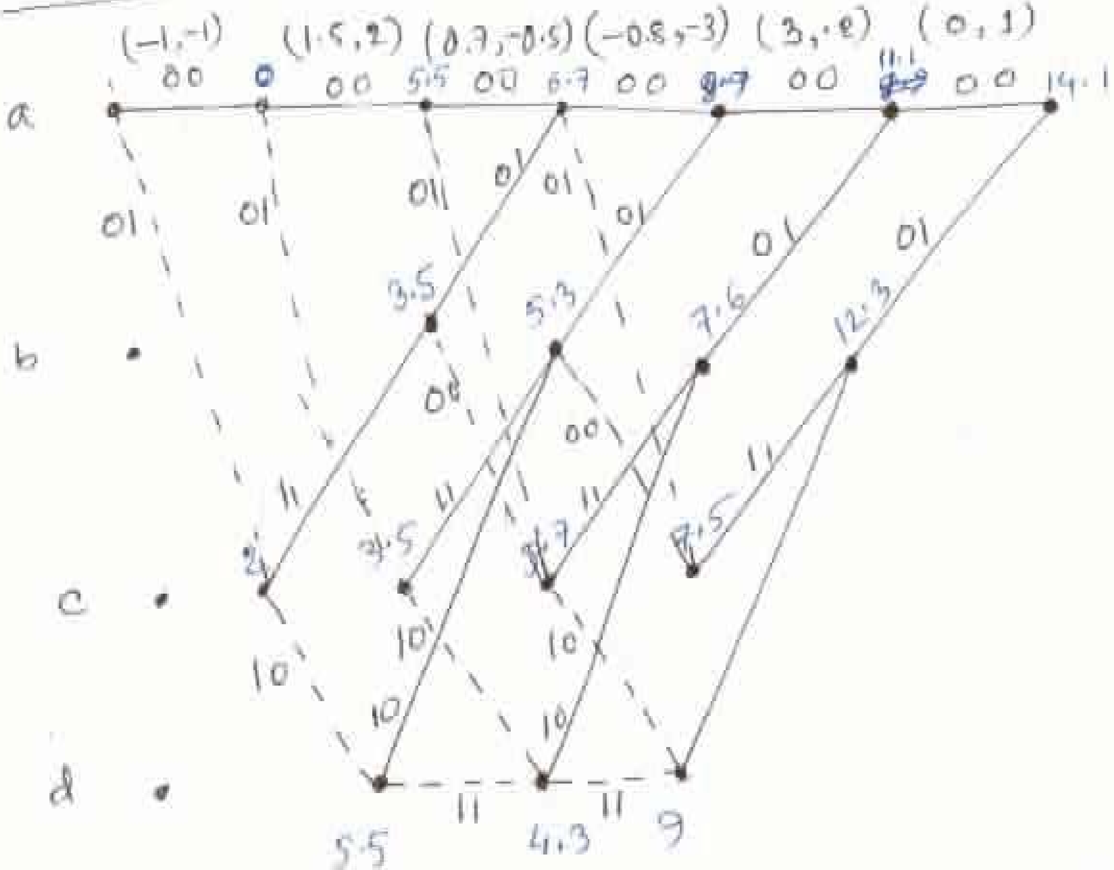
$r = (-1, -1, 1.5, 2, 0.7, -1.5, -0.8, -3, 3, 0.2, 0, 1)$

So the received sequence is 12.

$n = 2$.

So we need a trellis of depth 6 which is shown below

7 Soft decision decoding :-



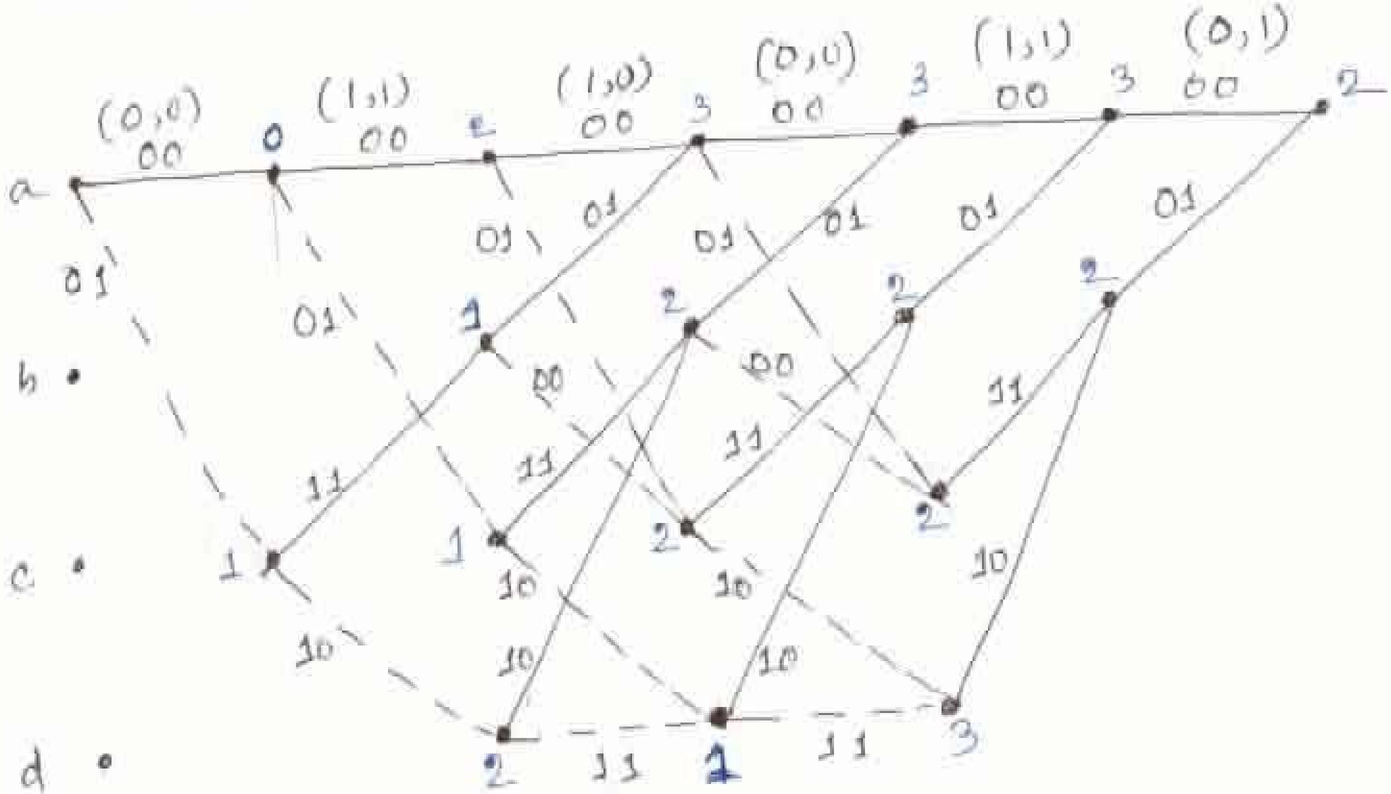
Here each ~~1~~ 0 corresponds to −1 and each one corresponds to +1. The accumulated matrics are shown in blue node. It is clear that the optimum path corresponds to the information sequence 100000. Since the last two zeros indicate the two zeros padded to the original information sequence we conclude that the transmitted sequence is 1000.

Hard decision decoding :-

After hard decision decoding

$$y = (0,0,1,1,1,0,0,0,1,0,1)$$

The trellis and metrics for hard decision decoding are shown below



The decoded sequence is either 110100 or 010100. Thus the information sequence can be either 1101 or 0101.