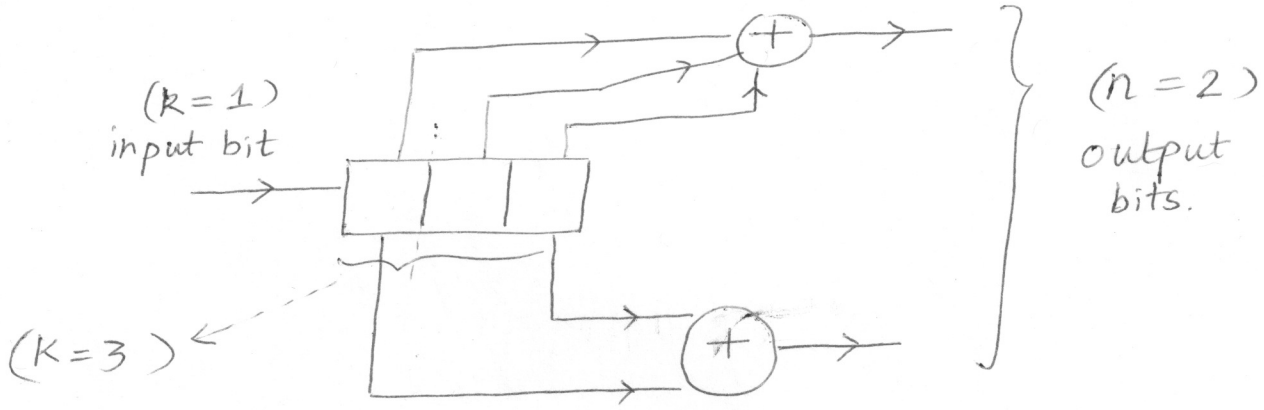
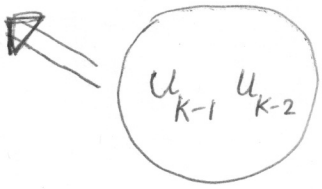
Viterbi Decoding for Convolutional Codes

- We will focus on a specific convolutional code
- $n = 2$, $k = 1$, $K = 3$



- (Possible) States of the code → 00, 01, 10, 11 → corresponding to the last 2 bits (or LSB's) of the shift register.



If $\{u_k\}_{k=0}^{\infty}$ is the input sequence...

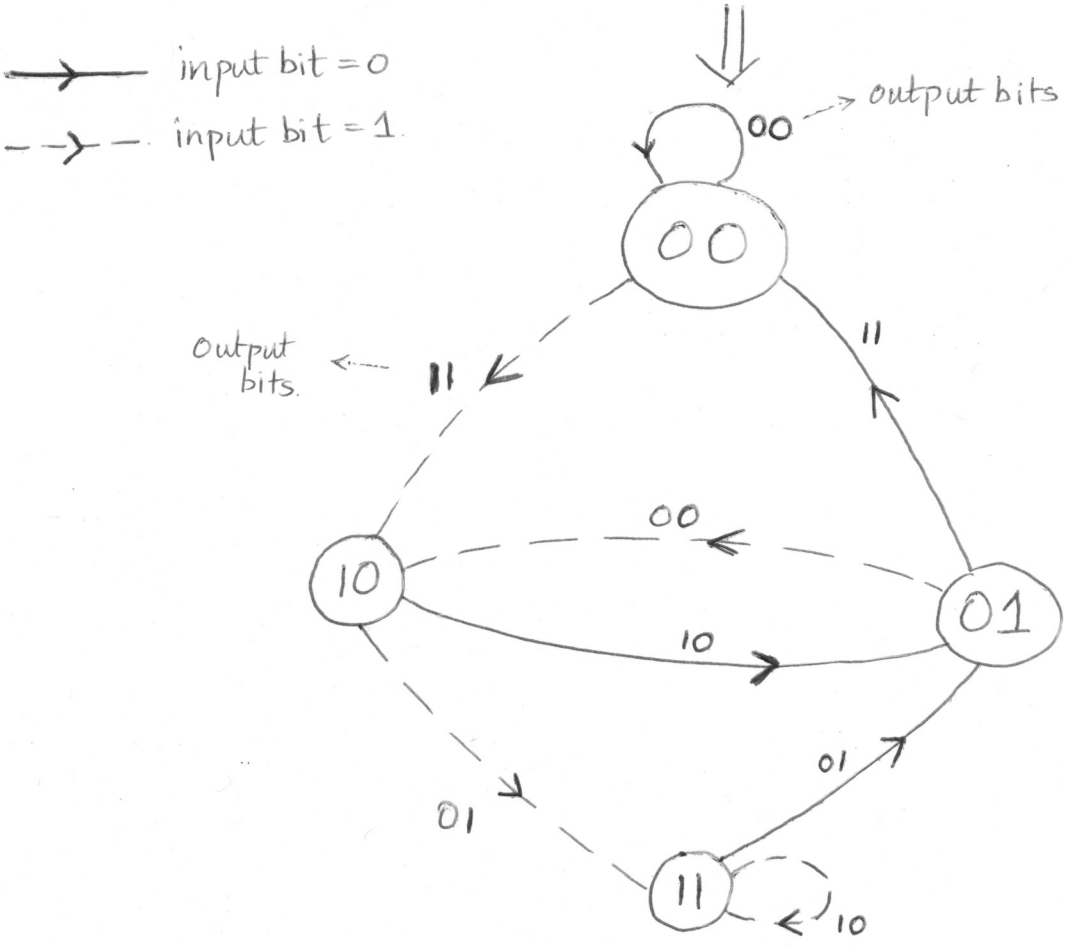
Then the output sequence (we denote it by $\{(c_k^{(1)} c_k^{(2)})\}$) is then given by:

$n = 2$ (output bits)

$$\begin{cases} c_k^{(1)} = u_k + u_{k-1} + u_{k-2} \\ c_k^{(2)} = u_k + u_{k-2}. \end{cases}$$

State diagram Representation

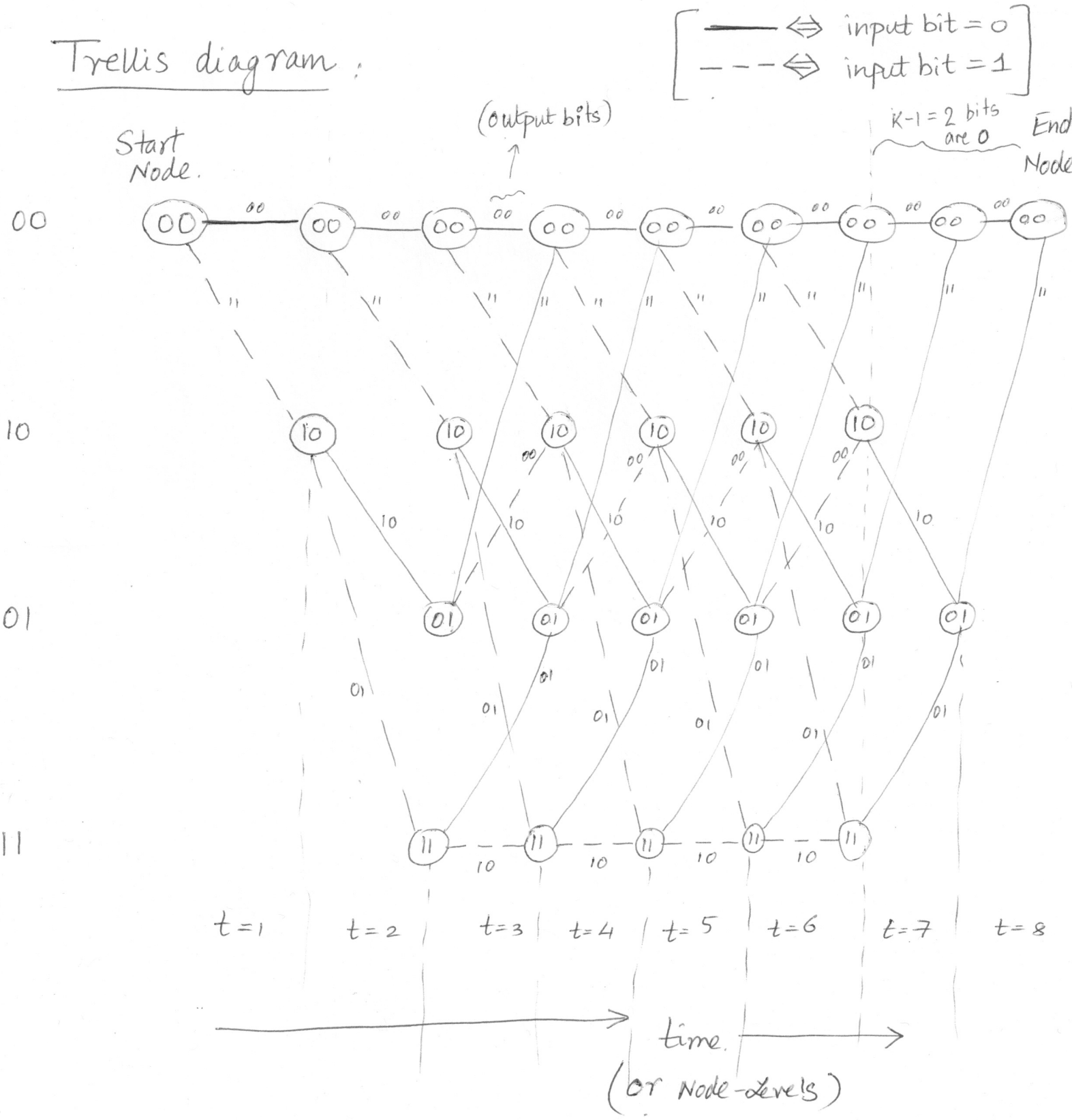
(k=1) Input bit	(Current) State	(n=2) Output bits	(Next) State
0	00	00	00
1	00	11	10
0	10	10	01
1	10	01	11
0	01	11	00
1	01	00	10
0	11	01	01
1	11	10	11



Typically, we wish to start in the "all-0" state
- and, end in the "all-0" state.

We can artificially insert (K−1) zeros at the end of the message sequence to achieve this goal.

Trellis diagram:



- Each path in the Trellis corresponds to a message / codeword
- message / codeword & the path have a one-to-one mapping.

(convolutional encoder)

Suppose we send a information sequence $\{u_k\}$ → codeword sequence $\{c_k\}$

Receiver.

Goal: Find the codeword sequence $\{c_k\}$ which is closest in Hamming distance to the received sequence $\{r_k\}$.

← received sequence $\{r_k\}$.

[If messages are equally likely, & the channel is a BSC, the above decoding rule is equivalent to the Maximum-Likelihood (ML) decoding rule].

Main Issue: → There are several sequences (or paths) on the Trellis to compare with $\{r_k\}$

→ Viterbi Algorithm gives a low complexity solution for this issue without searching over all possible paths (or codewords/sequences).

Example of Viterbi Decoding:

input sequence $\{u_k\} \Rightarrow$ 10 11 10 : 00 ($(K-1)=2$ "forced" zeros)

[also known as trailing or terminating zeros to bring back to the "all-0" state]

output sequence $\{c_k\} \Rightarrow$ ~~11 10 11 11 01~~ → Two errors

11 10 00 01 10 01 11 00

Send over the channel.

[suppose two erros occur]

received sequence $\{r_k\}$.

11 10 10 00 10 01 11 00

using this, the receiver must estimate $\{u_k\}$, i.e, the information sequence.

Viterbi-decoding