ECE 5654: Spring 2014 Homework 4

[20 points] (Due: 5:00 pm, Thursday, April 10)

Problem 1 [5 points] The generator matrix for a linear binary code is

$$\boldsymbol{G} = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

a. Express $\boldsymbol{G}$ in systematic $[\boldsymbol{I}|\boldsymbol{P}]$ form.
b. Determine the parity check matrix $\boldsymbol{H}$ for the code.
c. Construct the table of syndromes for the code.
d. Determine the minimum distance of the code.
e. Demonstrate that the codeword $\boldsymbol{c}$ corresponding to the information sequence 101 satisfies $\boldsymbol{c}\boldsymbol{H}^t = \boldsymbol{0}$.

Problem 2 [4 points]

Show that for any (n, k) code which can correct t errors must satisfy

$$2^n \geq 2^k \sum_{i=0}^{t} \binom{n}{i}$$

Codes which satisfy this bound with equality are also known as perfect codes.

Give an example of a perfect code (we have discussed a few in class)

Problem 3 [5 points]

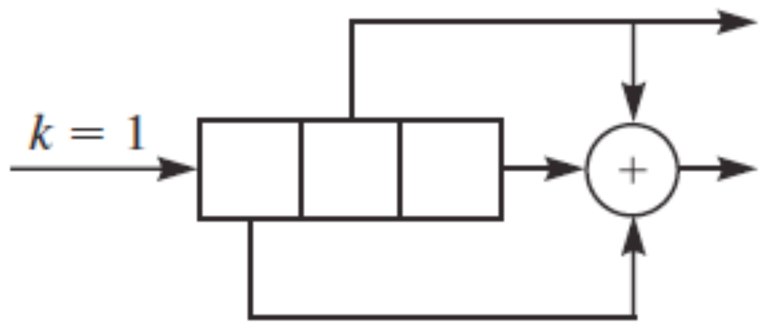
The parity check matrix of a linear block code is given below:

$$\boldsymbol{H} = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

1. Determine the generator matrix for this code in the systematic form.
2. How many codewords are in this code? What is the $d_{\min}$ for this code?
3. What is the coding gain for this code (soft decision decoding and BPSK modulation over an AWGN channel are assumed)?
4. Using hard decision decoding, how many errors can this code correct?
5. Show that any two codewords of this code are orthogonal, and in particular any codeword is orthogonal to itself.

Problem 4 [6 points]

The block diagram for a convolutional code is shown below



a) Draw the state transition diagram for this code

b) This convolutional code is used with a binary antipodal signaling scheme for transmission over a noisy channel, with the input-output relationship

$$r_i = c_i + n_i$$

where $c_i \in \{\pm 1\}$ and the noise components are i.i.d. with PDF $p(n) = \frac{1}{2}e^{-|n|}$
At the output of the matched filter, the received sequence is

$$\mathbf{r} = (-1, -1, 1.5, 2, 0.7, -0.5, -0.8, -3, 3, 0.2, 0, 1)$$

Find the most likely information sequence if the receiver employs

- soft decision decoding

- hard decision decoding

(you can assume that the code is terminated at the zero state)