ECE 5205

Spring 2014 Semester

pn junction - revisited

Revisted material
from ECE 3214, ECE 4234, ECE 5200

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Website with the lectures recorded:

http://media1.vbs.vt.edu/content/classes/z3851 ???

Grading:

Midterm Exam	25%	(3/18/14)
Final Exam	40%	(5/9/14)
Homework	35%	

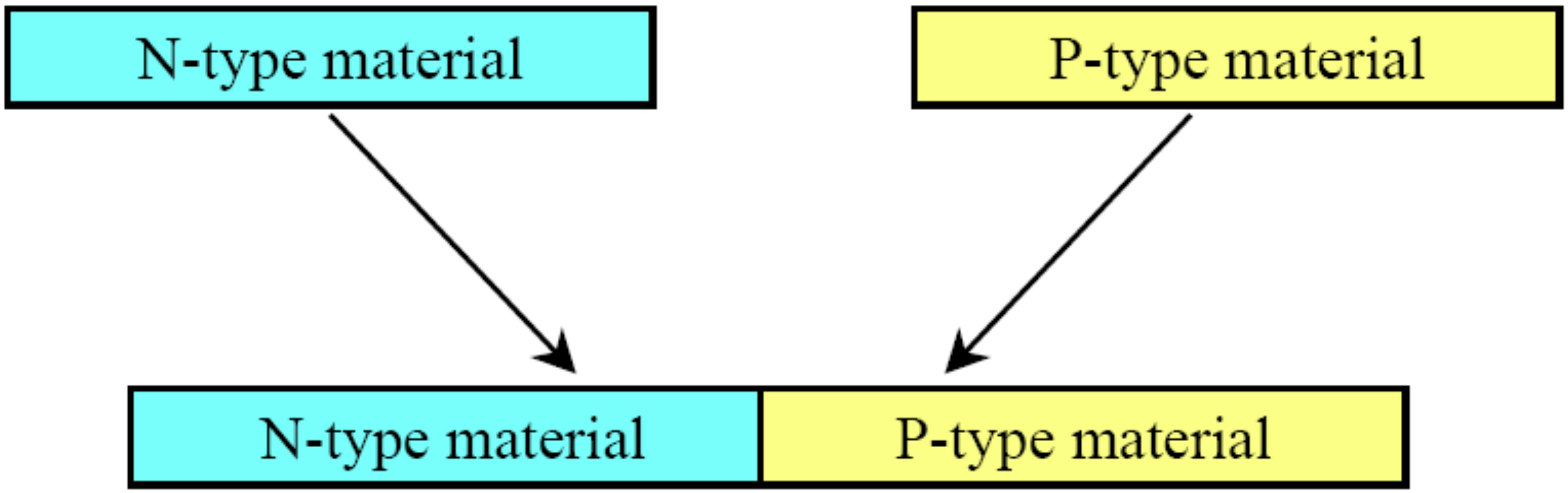
Midterm Exam: March 19, 2013 during lecture time

Final exam: May 9, 2014 DURH 261 10:05 am – 12:05 pm

Content

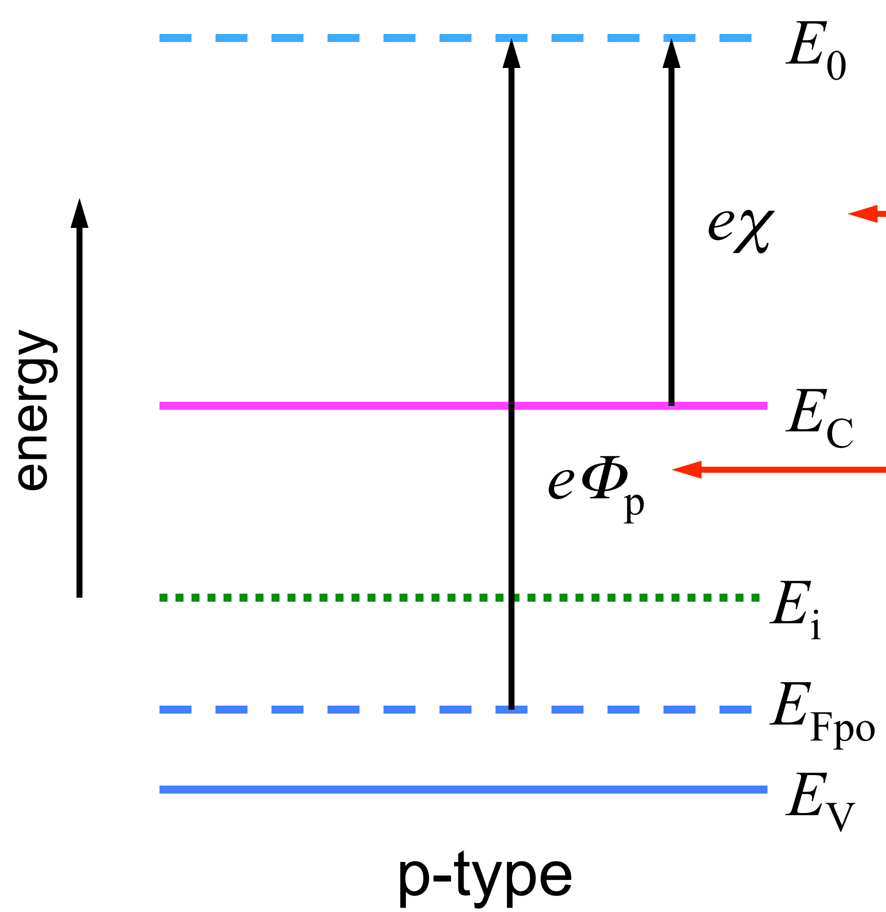
1. The Basics of the pn Junction

1.1 Basic Structure of the pn junction



A p-n junction diode is made by forming a p-type region of material directly next to a n-type region.

Electron Affinity and Work-function for Semiconductors



vacuum level

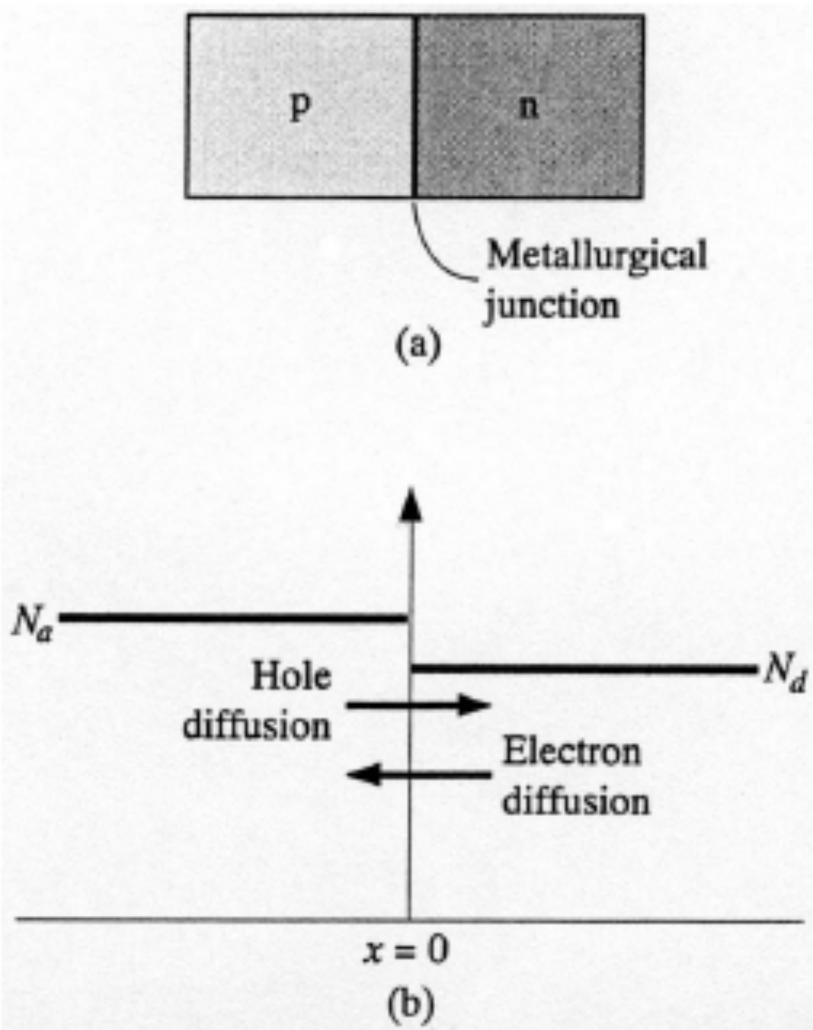
electron affinity

$$\boxed{e\chi_{Si} = e\chi_{GaAs} = 4.05\ \text{eV}}$$

work function

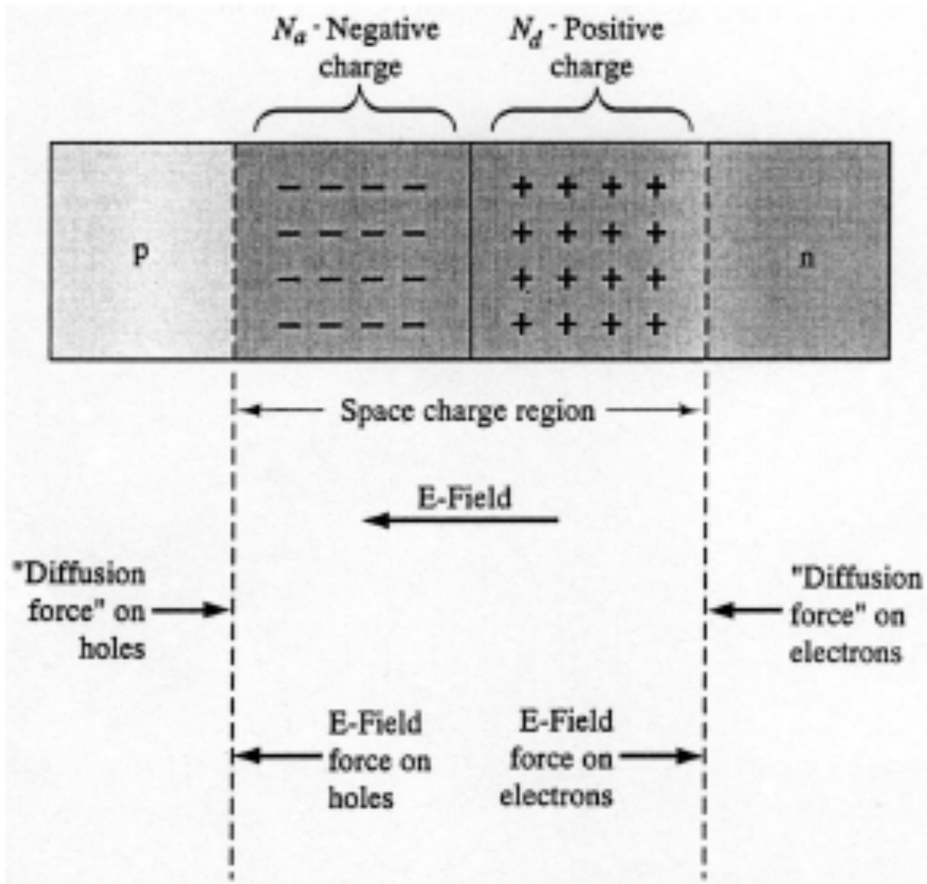
The workfunction for a semiconductor is given by E_o-E_F and not E_o-E_c (affinity), **despite the fact that there are no electrons at the energy E_F (in contrast to metals for example).** Thermal equilibrium requires that only a fraction of electrons are in conduction band. When an electron is removed from the conduction band, thermal equilibrium can be only maintained if an electron is excited from VB to CB, which involves absorbing heat (energy) from the environment; thus it takes more energy than $e\chi$ to remove an electron.

Basic Structure of the pn Junction



- The entire semiconductor is a single crystal.
- The interface is called the ***metallurgical junction***.
- A step junction with uniform doping in each region and an abrupt change in doping at the interface.
- Electrons diffuse from the n-region to p-region and holes diffuse in the reverse direction.
- An electric field is established in the direction from the n-region to the p-region.

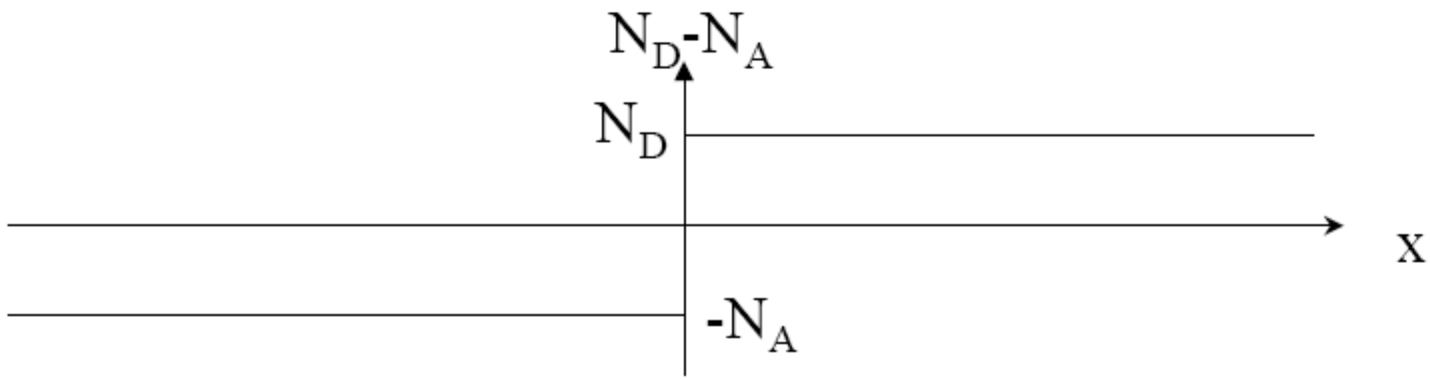
Basic Structure of the pn Junction



The negatively and positively charged regions are called the ***space charge region***, or the ***depletion region***.

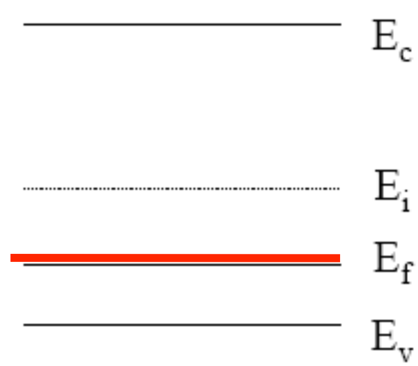
1.2 Space Charge Width and Electric Field

How to Draw Energy Bands for a pn-junction?

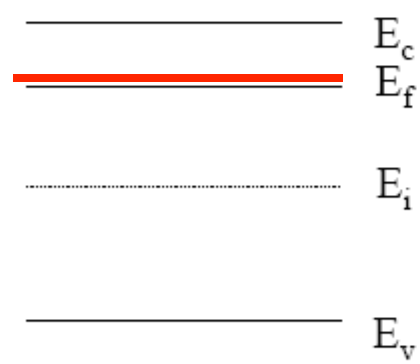


Junction

In regions far away from the “junction” the band diagram looks like:



p-type semiconductor

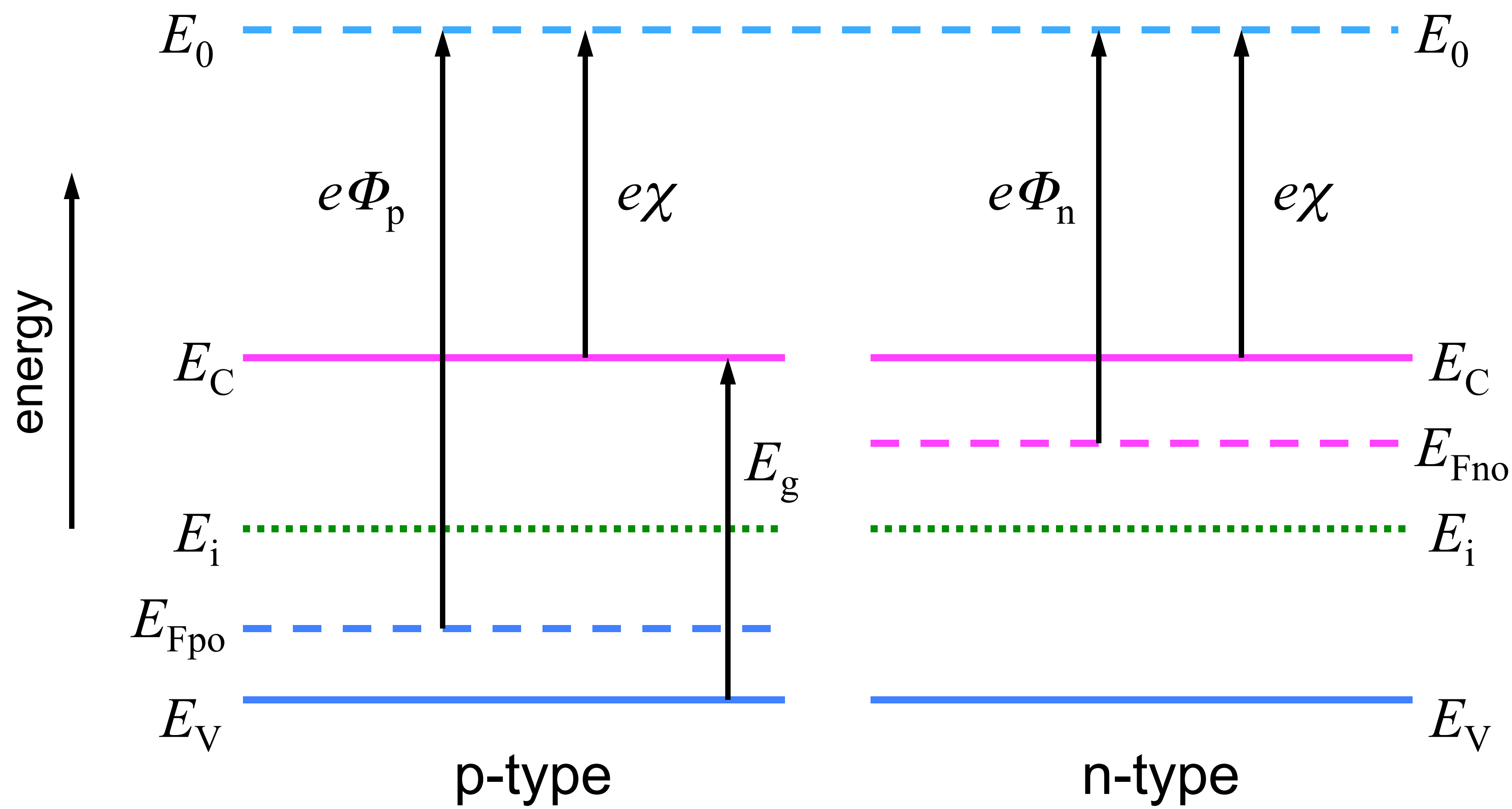


n-type semiconductor

Before Connection of p and n regions

$$\boxed{e\Phi_p = e\chi + E_g - (E_{Fpo} - E_V)}$$

$$\boxed{e\Phi_n = e\chi + (E_C - E_{Fno})}$$



FERMI-DIRAC DISTRIBUTION

The Fermi-Dirac distribution function for electrons is defined as:

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

And describes probability to find an electron at energy E, provided that there are energy states available at that energy E.

The parameter (energy level) E_F is called the **FERMI LEVEL**.

This level is a reference energy where the probability of carrier occupancy is defined to be **1/2** for any temperature.

FERMI-DIRAC DISTRIBUTION

The probability of **hole occupancy** (missing electron) is denoted by:

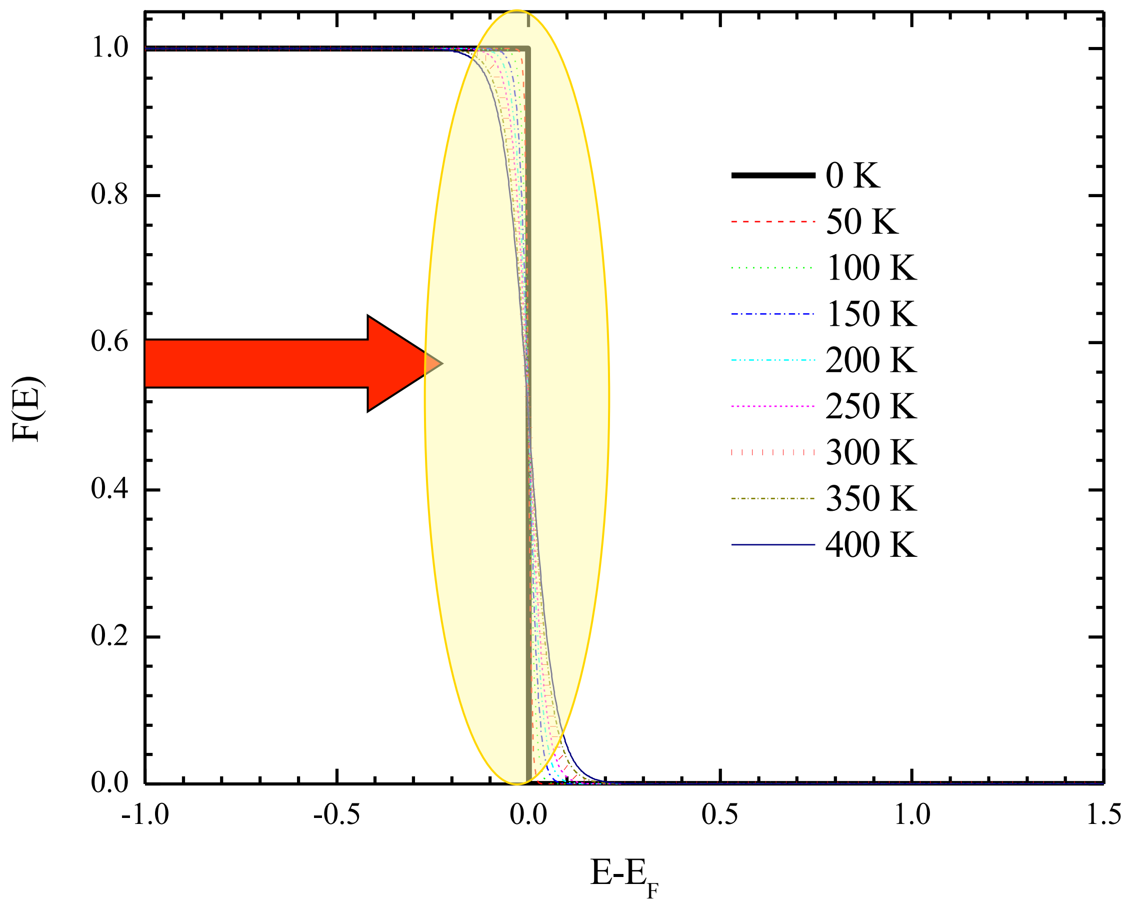
$$1 - f(E) = 1 - \frac{1}{1 + e^{(E - E_F)/kT}}$$

We recognize that this is the **probability that a state is vacated by an electron**.

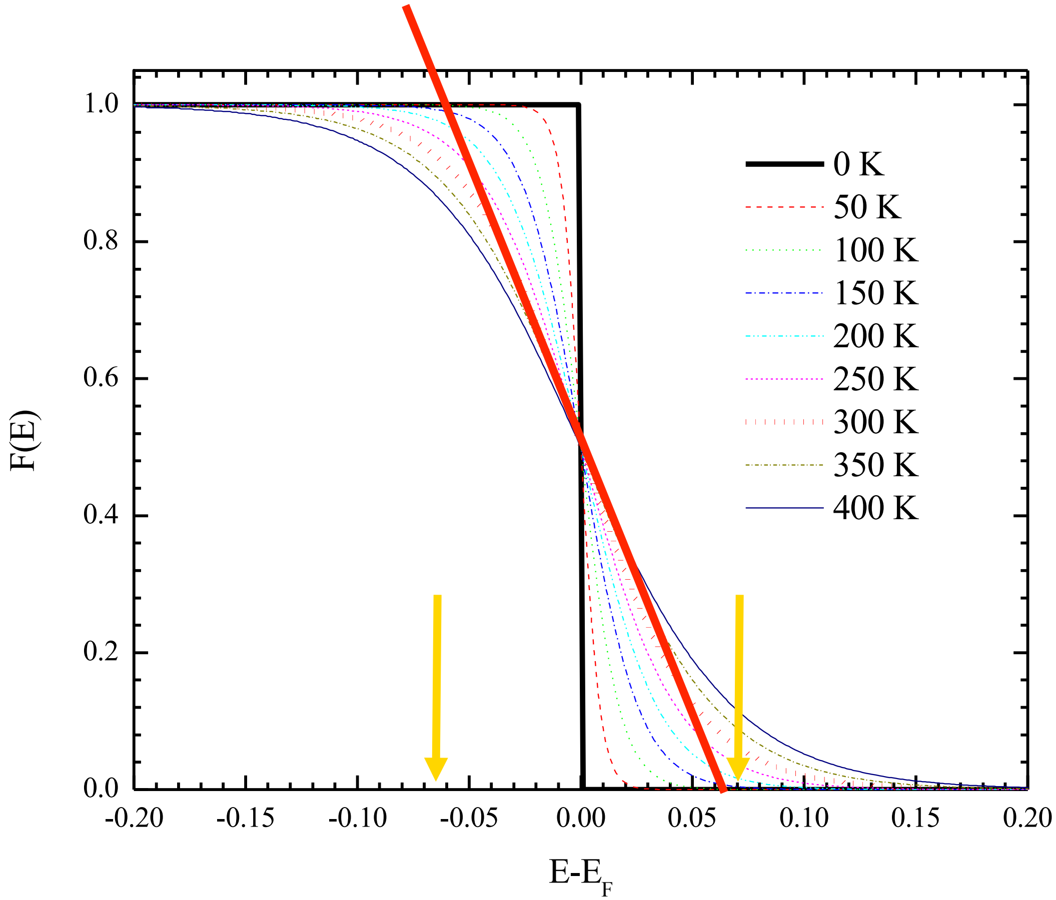
This level is a reference energy where the probability of carrier occupancy is defined to be **1/2** for any temperature.

FERMI-DIRAC DISTRIBUTION

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$



FERMI-DIRAC DISTRIBUTION



At 0 K, F(E) is a step function.

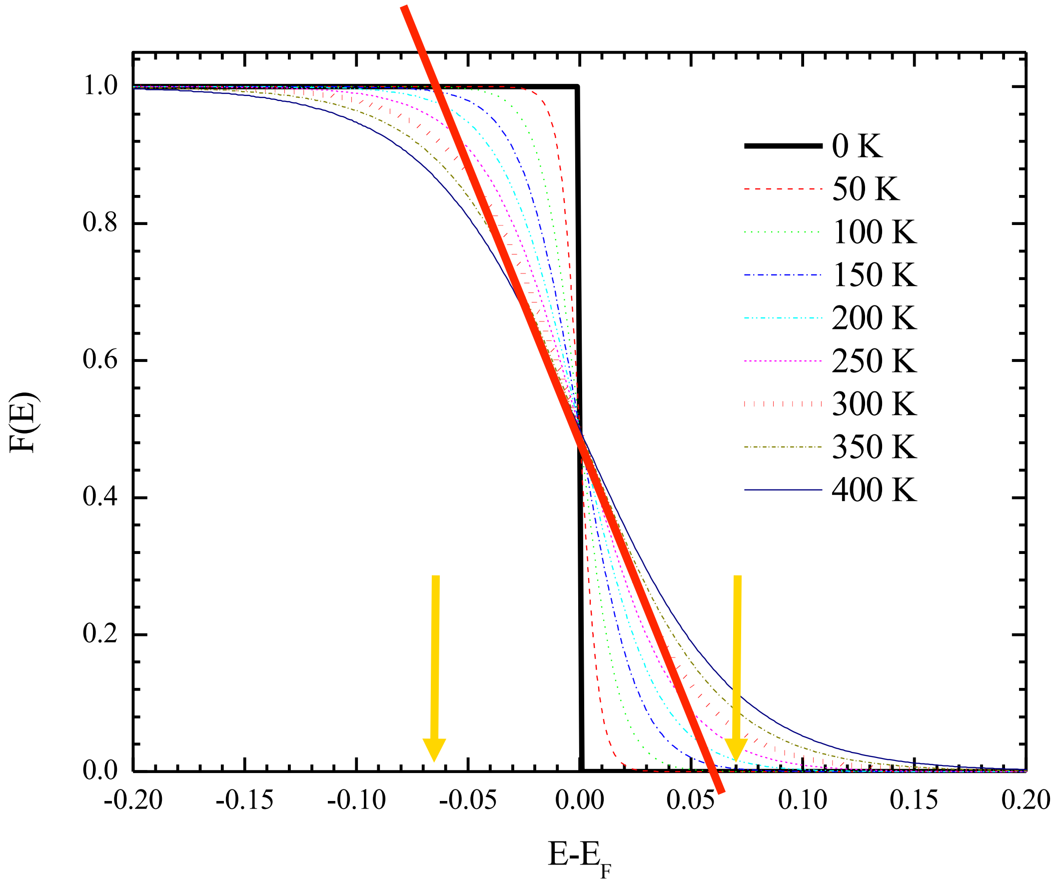
The majority of change in F (E) happens for energies within 3kT of E_f.

The function is almost linear over this region (-3kT,+3kT).

kT at 300k = (8.62×10^{-5} eV/K)(300 K)=0.0259 eV

3kT at 300k = 0.0777 eV

FERMI-DIRAC DISTRIBUTION



For energies less than 3kT, the exponential becomes smaller than 0.05

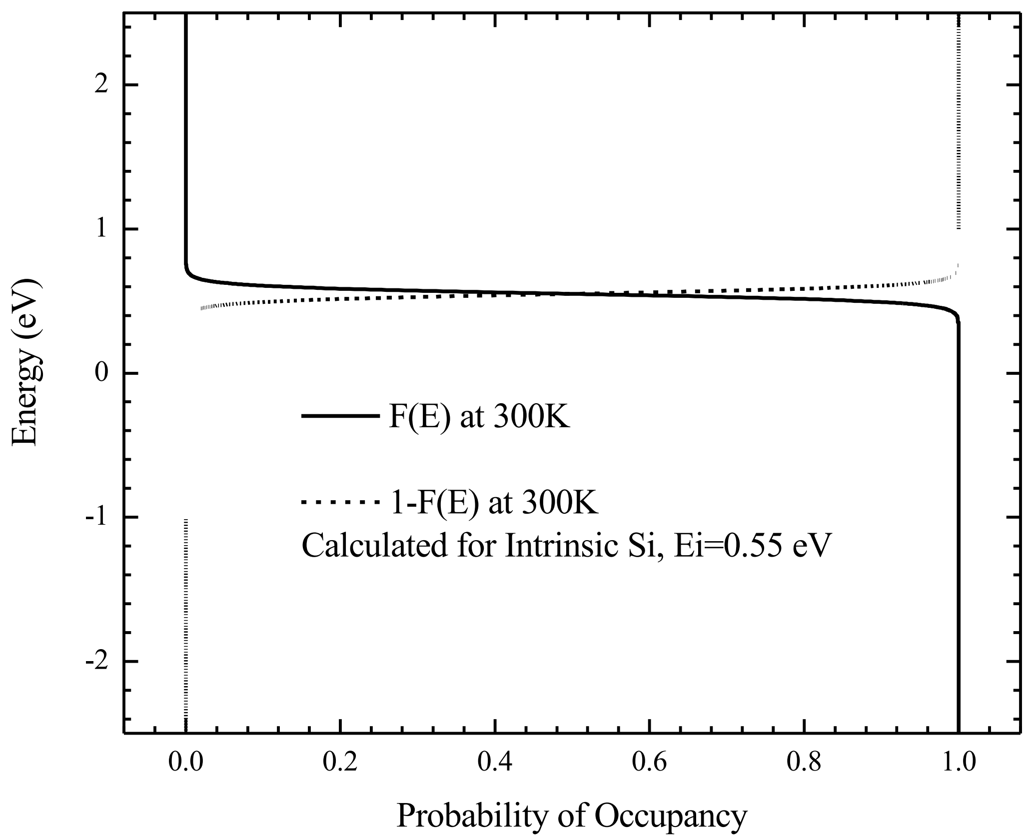
In this regime, BOLTZMAN statistics may accurately describe F(E)

$$F(E) \cong e^{-(E-E_F)/kT} \quad \text{for} \quad (E-E_F) > 3kT$$

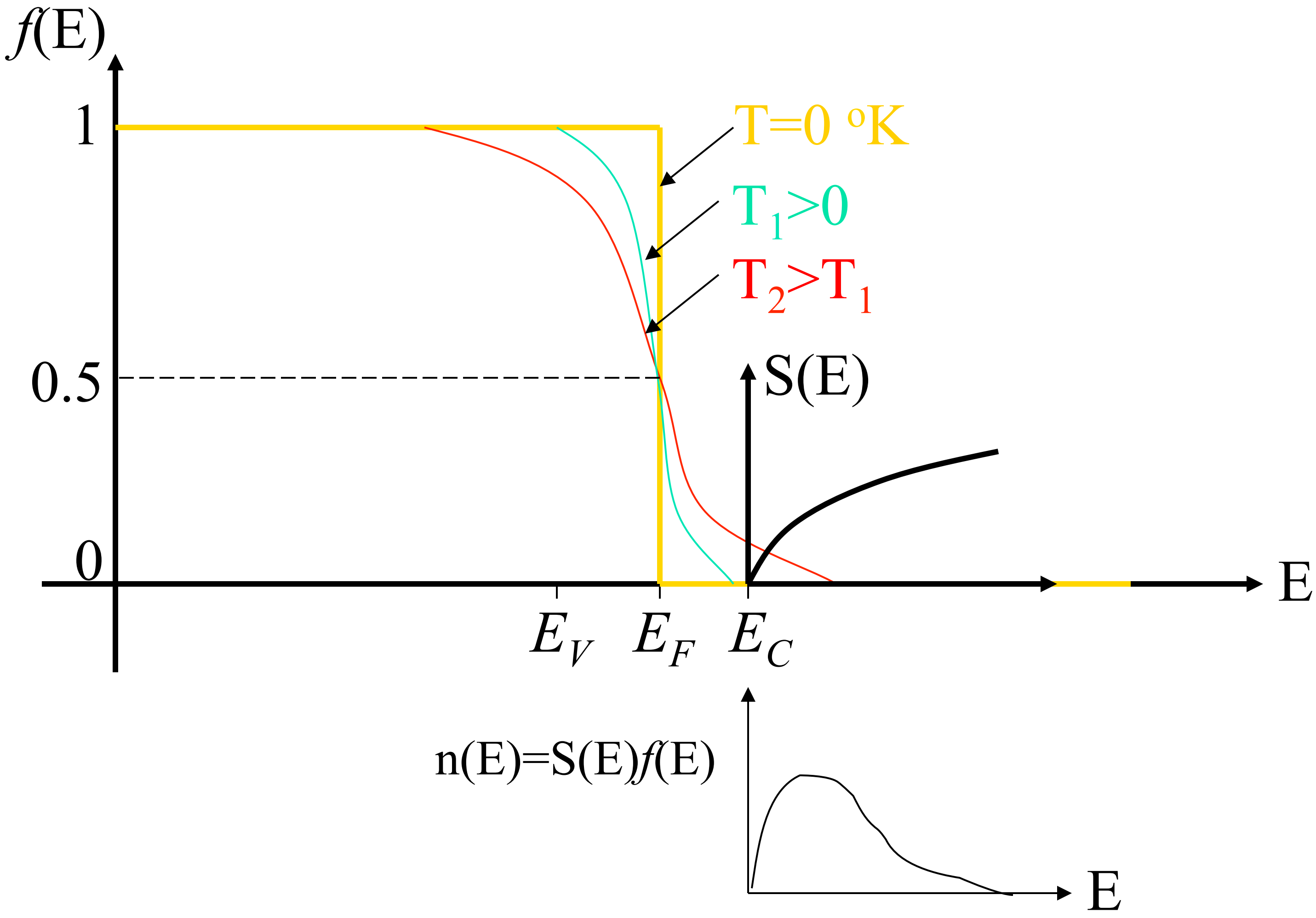
$$F(E) \cong 1 - e^{-(E-E_F)/kT} \quad \text{for} \quad (E-E_F) < 3kT$$

FERMI-DIRAC DISTRIBUTION

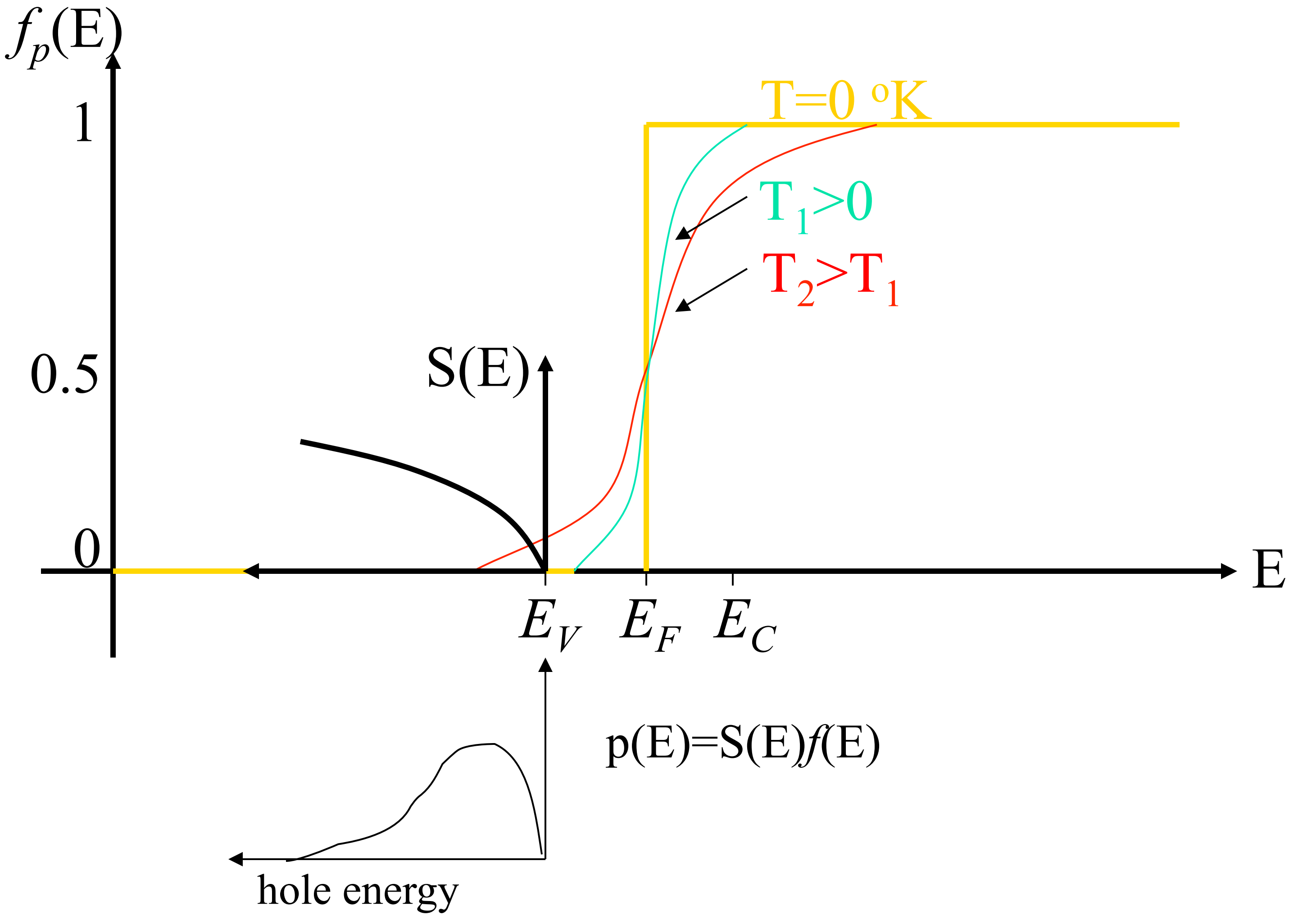
We may re-plot the hole and electron occupancy functions on the same graph (note: the probability is shown on the x-axis):



Electron distribution, n(E)



Hole distribution, p(E)



Finding n_o and p_o

$$n_0 = \int_{Ec}^{Ec(\max)} S(E) f(E) dE \approx \frac{1}{2\pi^2}\left(\frac{2m_{dse}^*}{\hbar^2}\right)^{3/2} \int_{Ec}^{\infty} \sqrt{E - E_C}\; e^{-[(E-E_F)/kT]} dE$$

$$= N_C \exp[-(E_C - E_F)/kT] \qquad \ldots where\ N_C = 2\left(\frac{m_{dse}^* kT}{2\pi\hbar^2}\right)^{3/2}$$

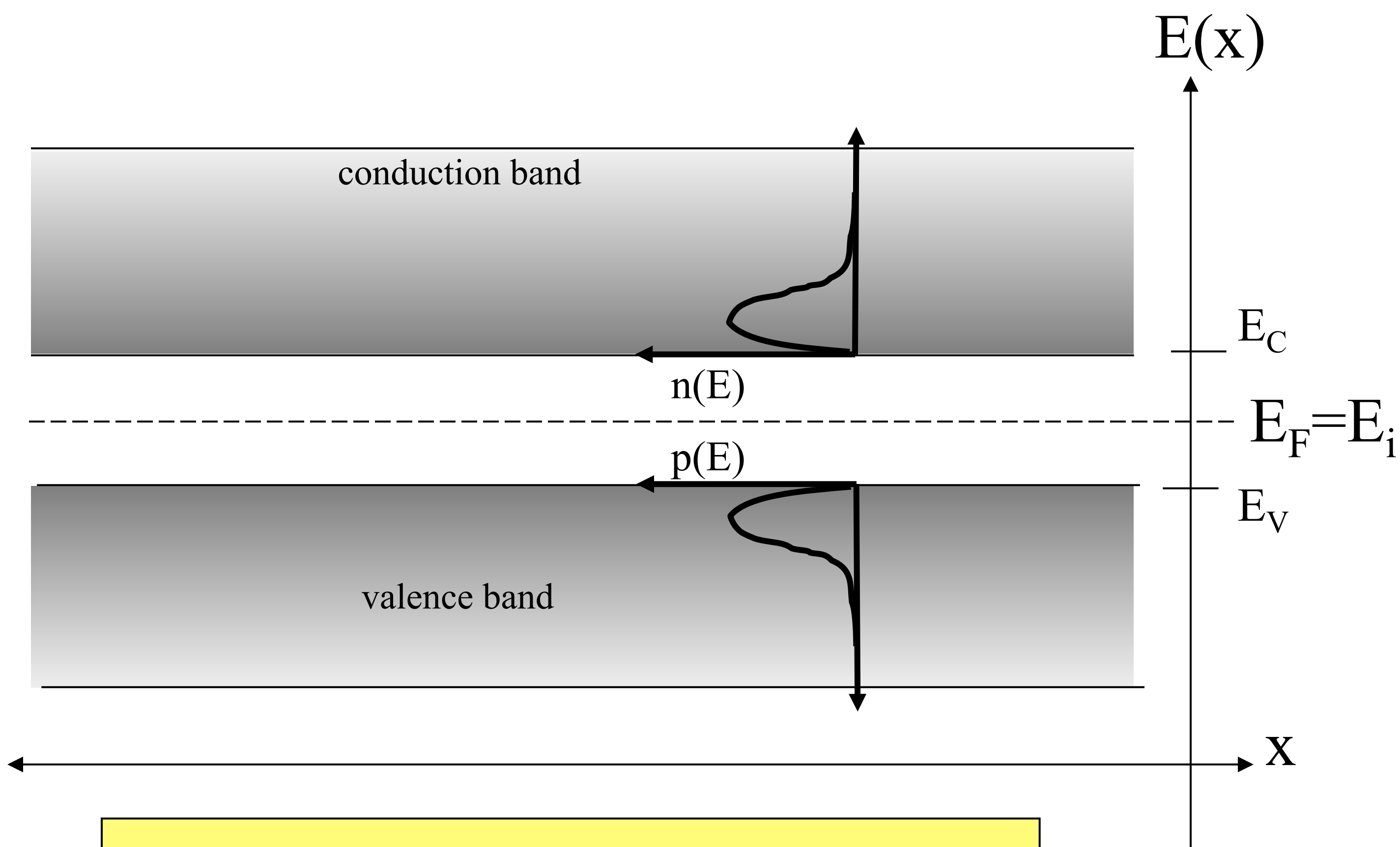
the effective density of states in the conduction band

$$p_0 = \int_{Ev(\min)}^{Ev} S(E) f_p(E) dE \approx \frac{1}{2\pi^2}\left(\frac{2m_{dsh}^*}{\hbar^2}\right)^{3/2} \int_{-\infty}^{Ev} \sqrt{E_V - E}\; e^{-[(E_F-E)/kT]} dE$$

$$= N_V \exp[-(E_F - E_V)/kT] \qquad \ldots where\ N_V = 2\left(\frac{m_{dsh}^* kT}{2\pi\hbar^2}\right)^{3/2}$$

Energy Band Diagram

intrinisic semiconductor: $n_o = p_o = n_i$

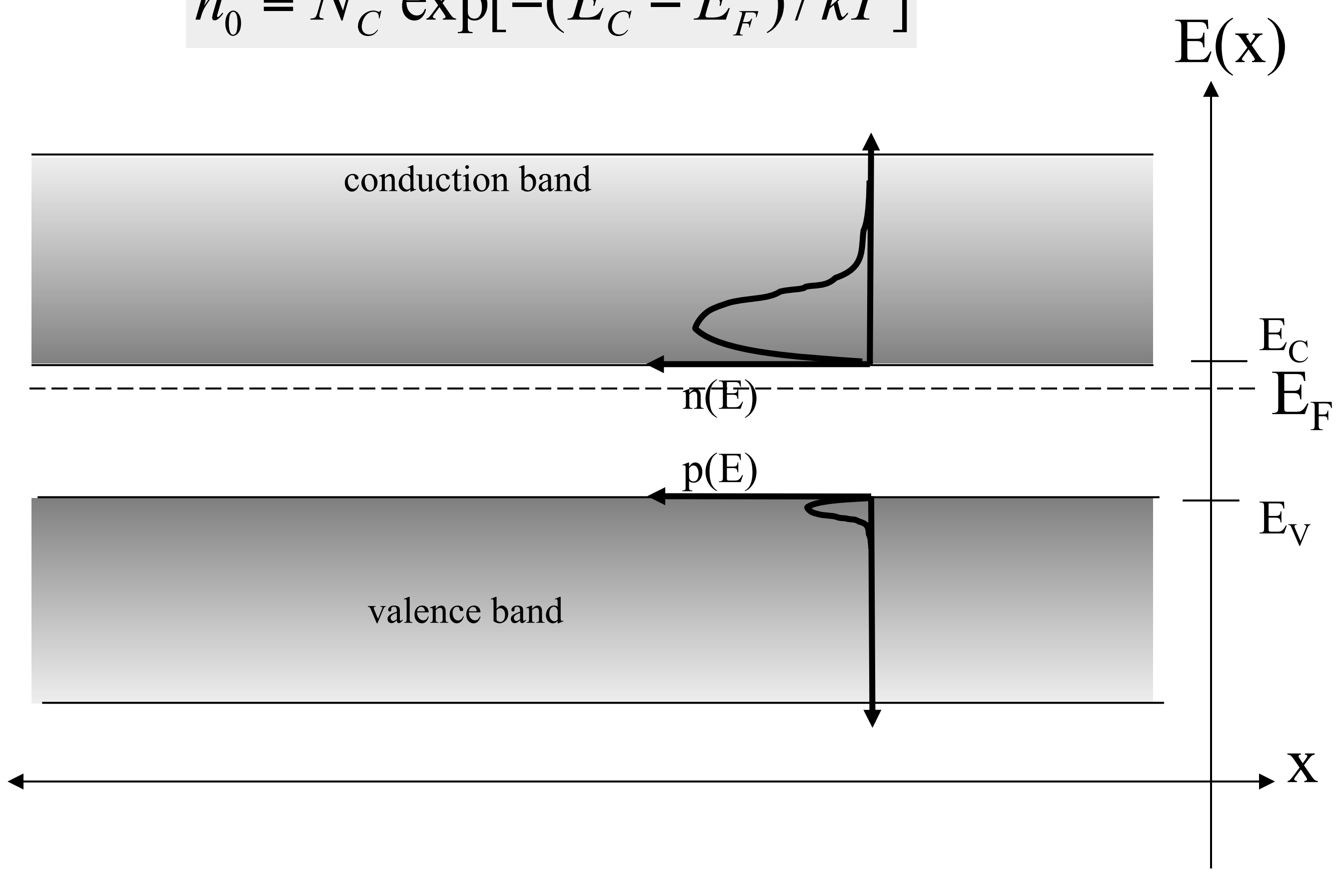


where E_i is the *intrinsic Fermi level*

Energy Band Diagram

n-type semiconductor: $n_o > p_o$

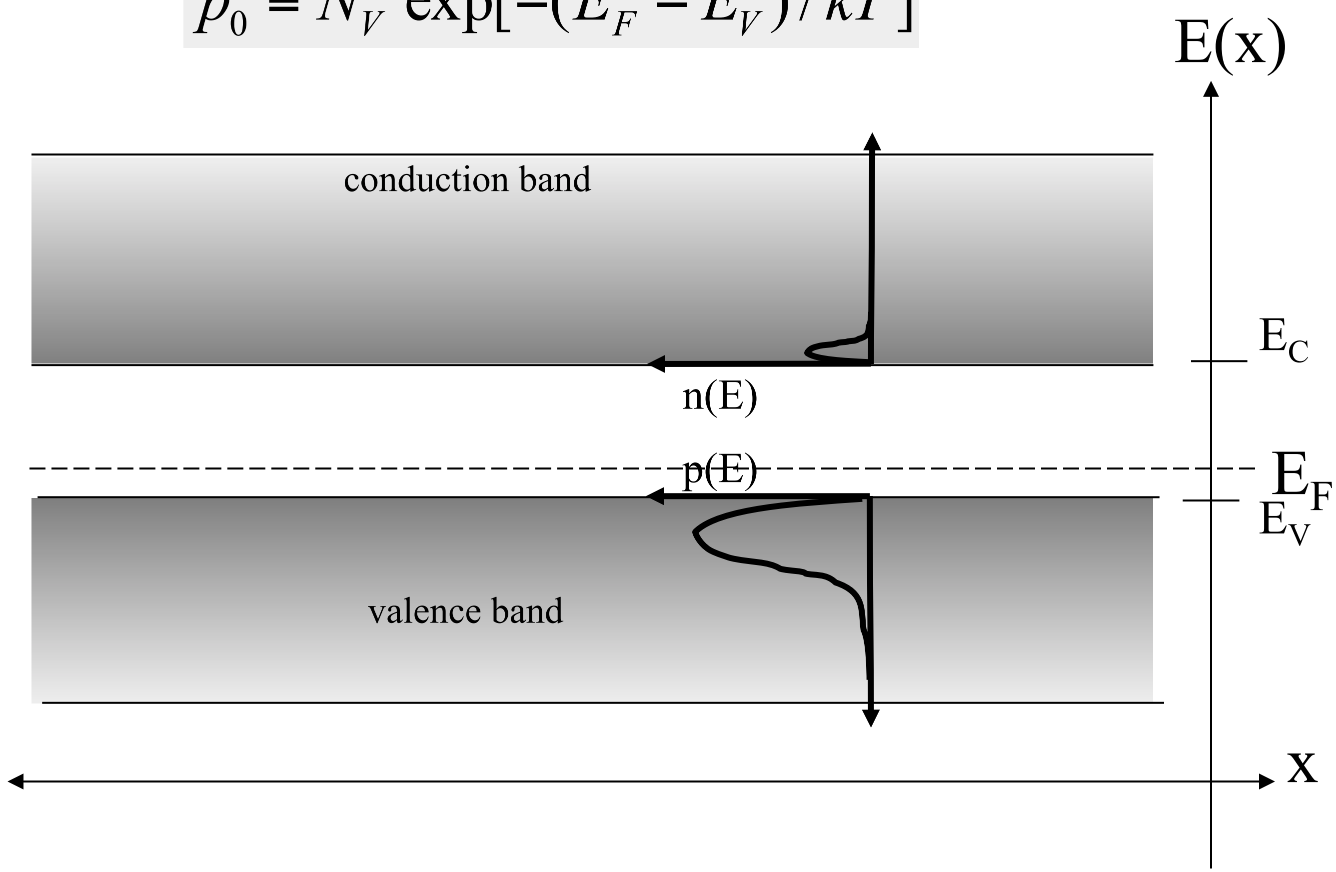
$$n_0 = N_C \exp[-(E_C - E_F)/kT]$$



Energy Band Diagram

p-type semiconductor: $p_o > n_o$

$$p_0 = N_V \exp[-(E_F - E_V)/kT]$$



A very useful relationship

$$n_0 p_0 = N_C \exp[-(E_C - E_F)/kT] \times N_V \exp[-(E_F - E_V)/kT]$$
$$= N_C N_V \, e^{-(Ec-Ev)/kT} = N_C N_V \, e^{-E_g/kT}$$

…which is independent of the Fermi Energy

Recall that $n_i = n_o = p_o$ for an intrinsic semiconductor, so

$$n_o p_o = n_i^2$$

for all non-degenerate semiconductors.

(that is as long as E_F is not within a few kT of the band edge)

$$n_0 p_0 = N_C N_V \, e^{-E_g/kT} = n_i^2$$
$$n_i = \sqrt{N_C N_V} e^{-E_g/2kT}$$

The intrinsic carrier density

$$n_0 p_0 = N_C N_V \, e^{-E_g / kT} = n_i^2$$

$$n_i = \sqrt{N_C N_V} e^{-E_g / 2kT}$$

is sensitive to the *energy bandgap*, *temperature,* and m^*

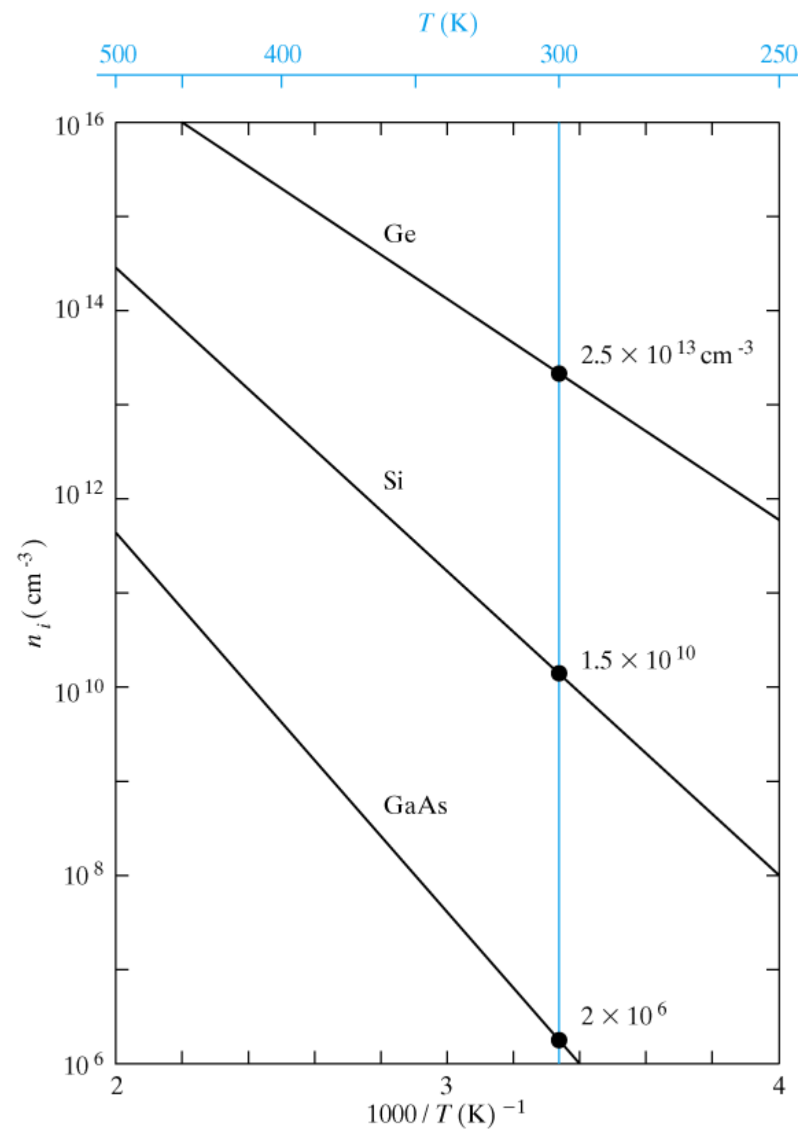
$$N_C = 2\left(\frac{m^*_{dse} kT}{2\pi\hbar^2}\right)^{3/2}$$

The intrinsic carrier density

$$\boxed{n \cdot p = N_c N_V e^{\left[-E_g\right]/kT} = n_i^2}$$

n_i represents the intrinsic carrier concentration

$$\boxed{n_i = \sqrt{N_c N_V}\, e^{\left[-E_g\right]/2kT}}$$



The intrinsic Fermi Energy (E_i)

For an intrinsic semiconductor, $n_o = p_o$ and $E_F = E_i$

$$N_C \exp[-(E_C - E_i)/kT] = N_V \exp[-(E_i - E_V)/kT]$$

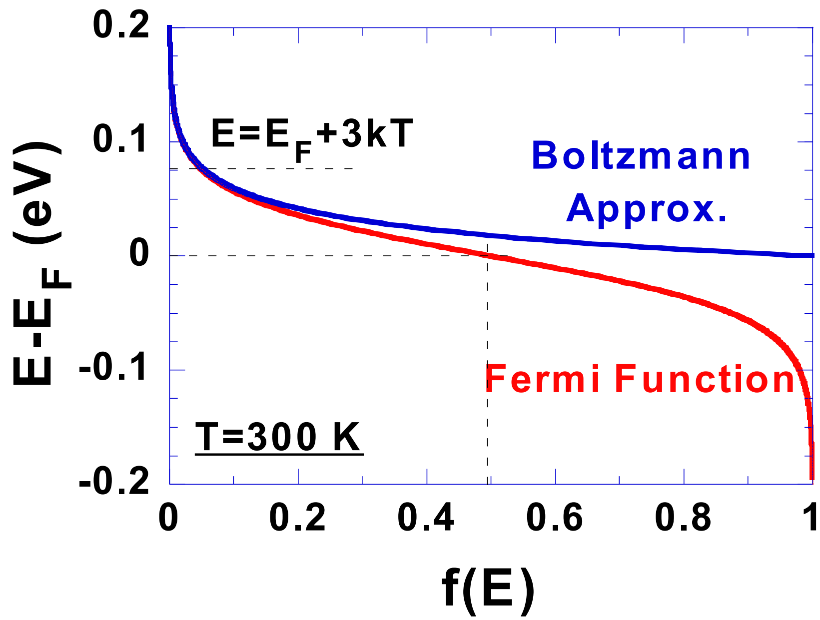
which gives

$$E_i = (E_C + E_V)/2 + (^{kT}/_2)\ln(N_V/N_C)$$

so the intrinsic Fermi level is approximately in the middle of the bandgap.

Recall: Fermi Level

$$f_D(E) = \frac{1}{1 + \exp[(E - E_F)/kT]}$$



$$f_D(E = E_F) = 1/2$$

$$f_D(E << E_F) \approx 1$$

$$f_D(E >> E_F) \approx 0$$

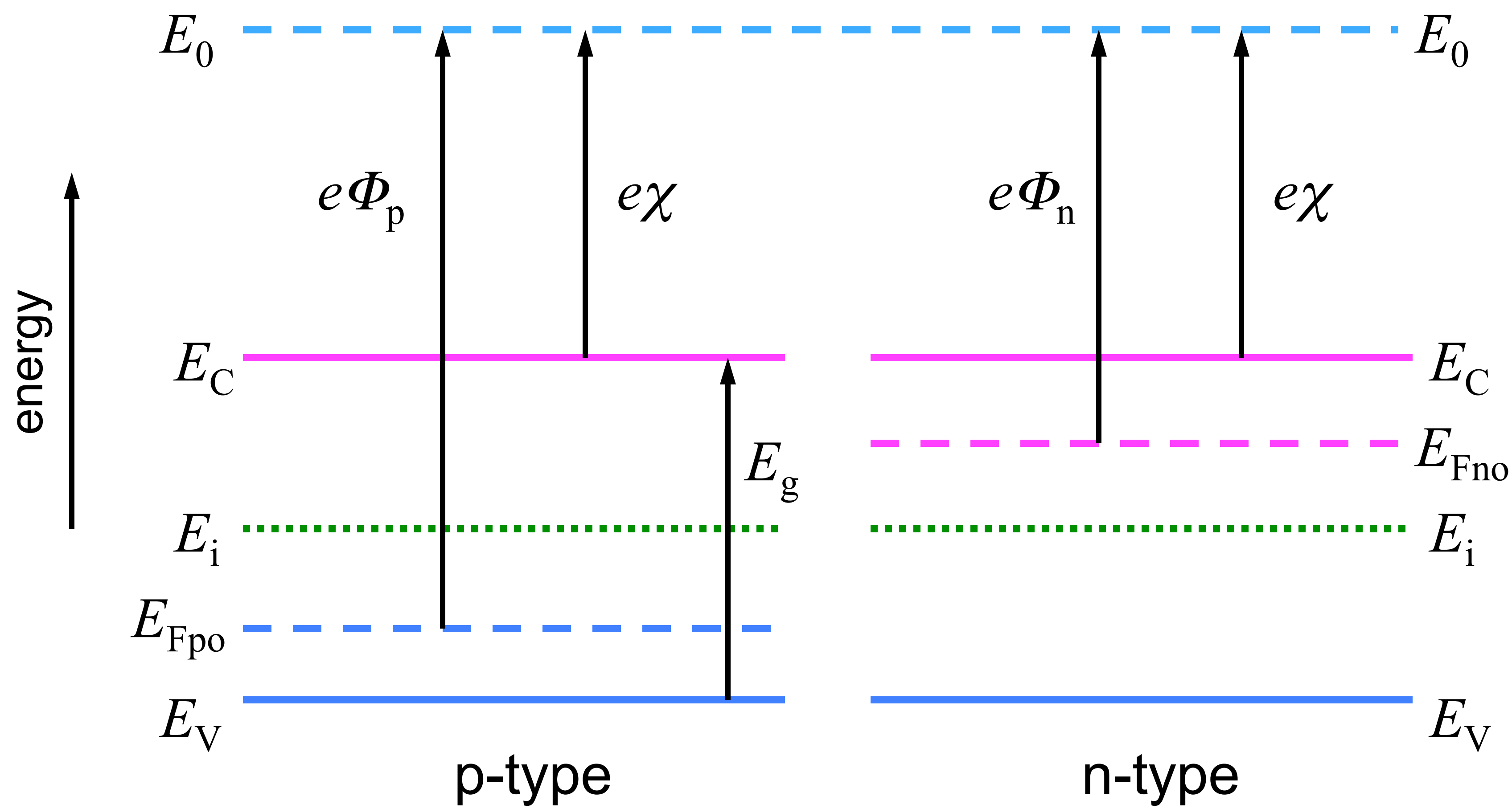
Fermi level indicates only a probability of occupancy. It does not contain any information about available states for occupancy. The information about the availibility of states is given by g(E) – density of available states.

The actual distribution of electrons is given by the product $g(E)f_D(E)$.

Before Connection of p and n regions

$$\boxed{e\Phi_p = e\chi + E_g - \left(E_{Fpo} - E_V\right)}$$

$$\boxed{e\Phi_n = e\chi + \left(E_C - E_{Fno}\right)}$$

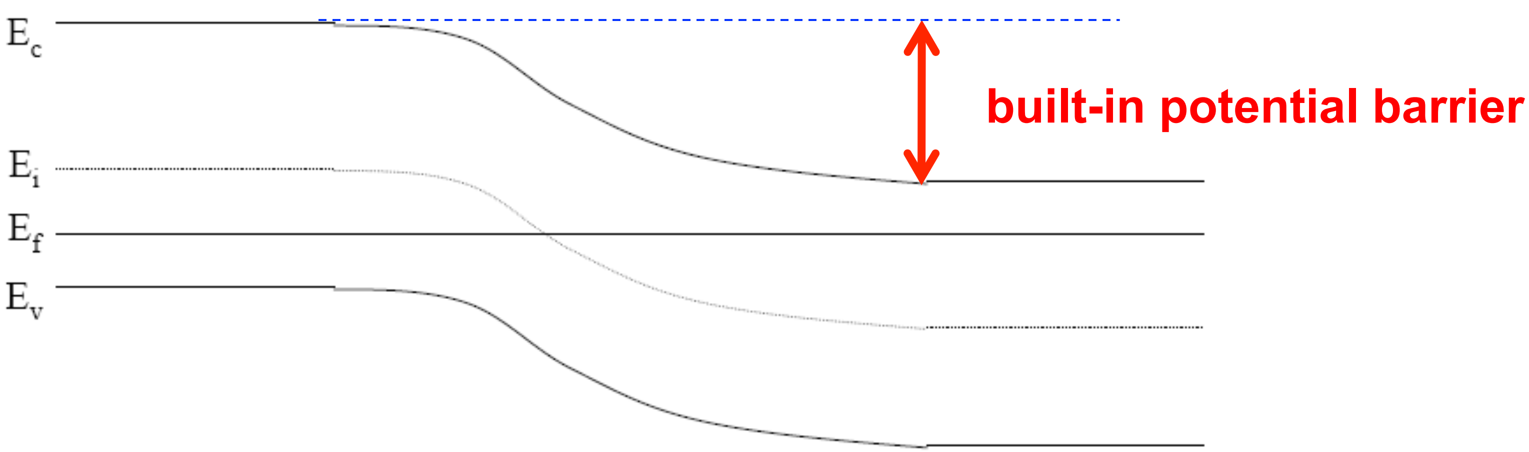


How to Draw Energy Bands for a pn-junction?

when the device has no external applied forces, no current can flow. Thus, the fermi-level must be flat!

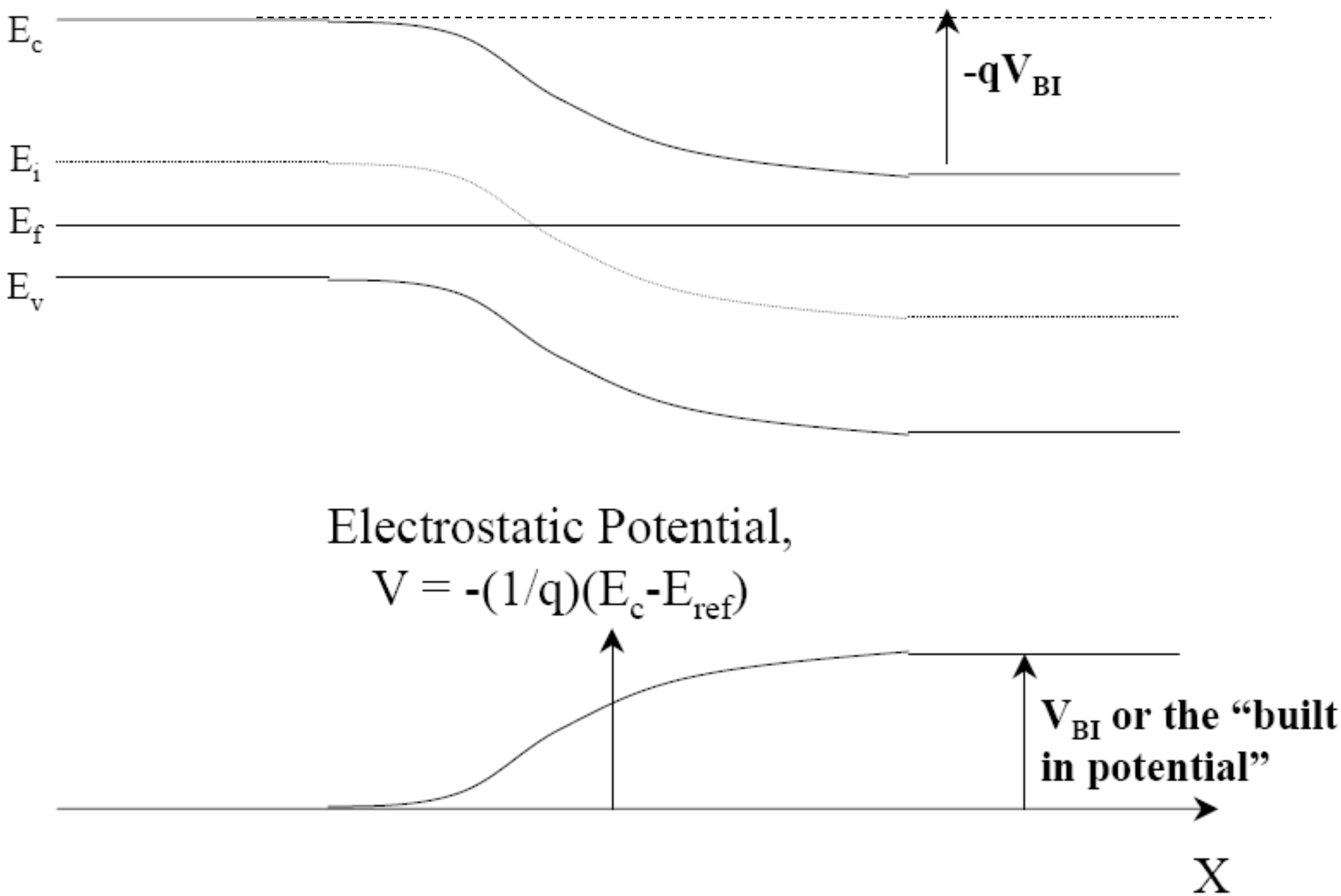
We can then fill in the junction region of the band diagram as:

How to Draw Energy Bands for a pn-junction?

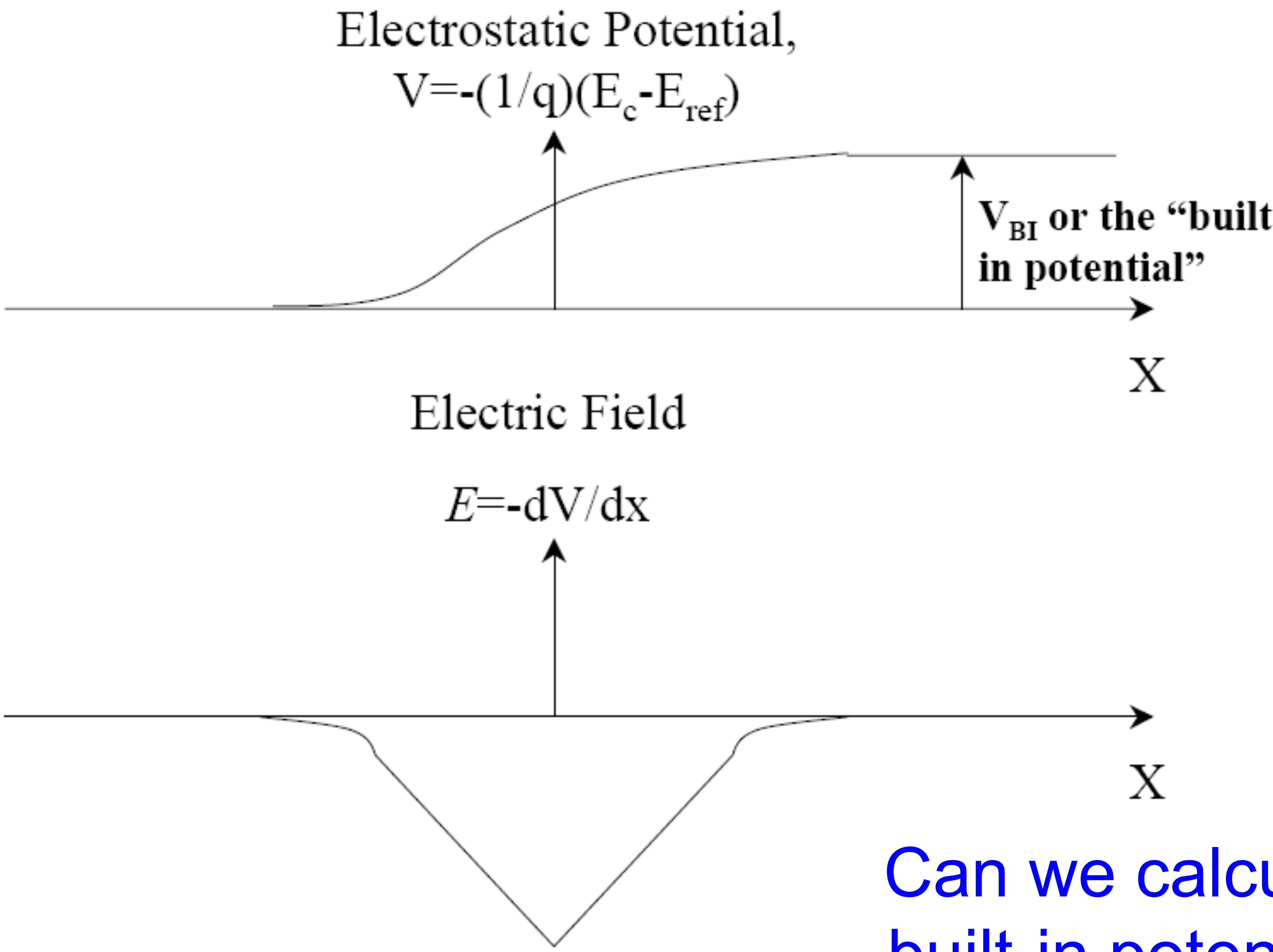


The conduction and valence band must bend since the relative position of the Fermi level to the conduction and valence band is different for the p-type and n-type semiconductor. The band bending produces a potential barrier, which is referred to as the ***built-in potential barrier***.

Electrostatic Potential in pn-junction



Electric Field in pn-junction

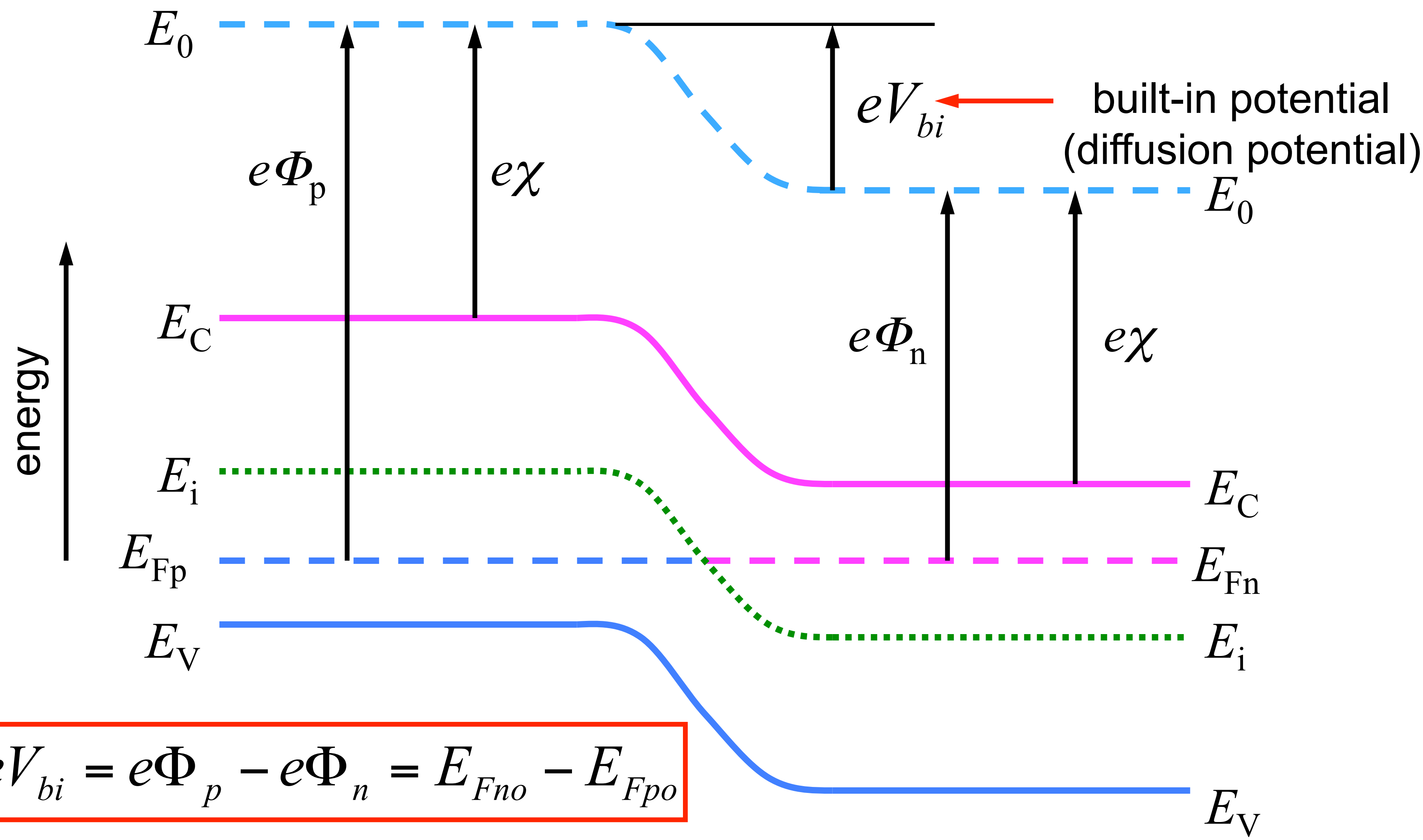


Can we calculate built-in potential?

After Connection

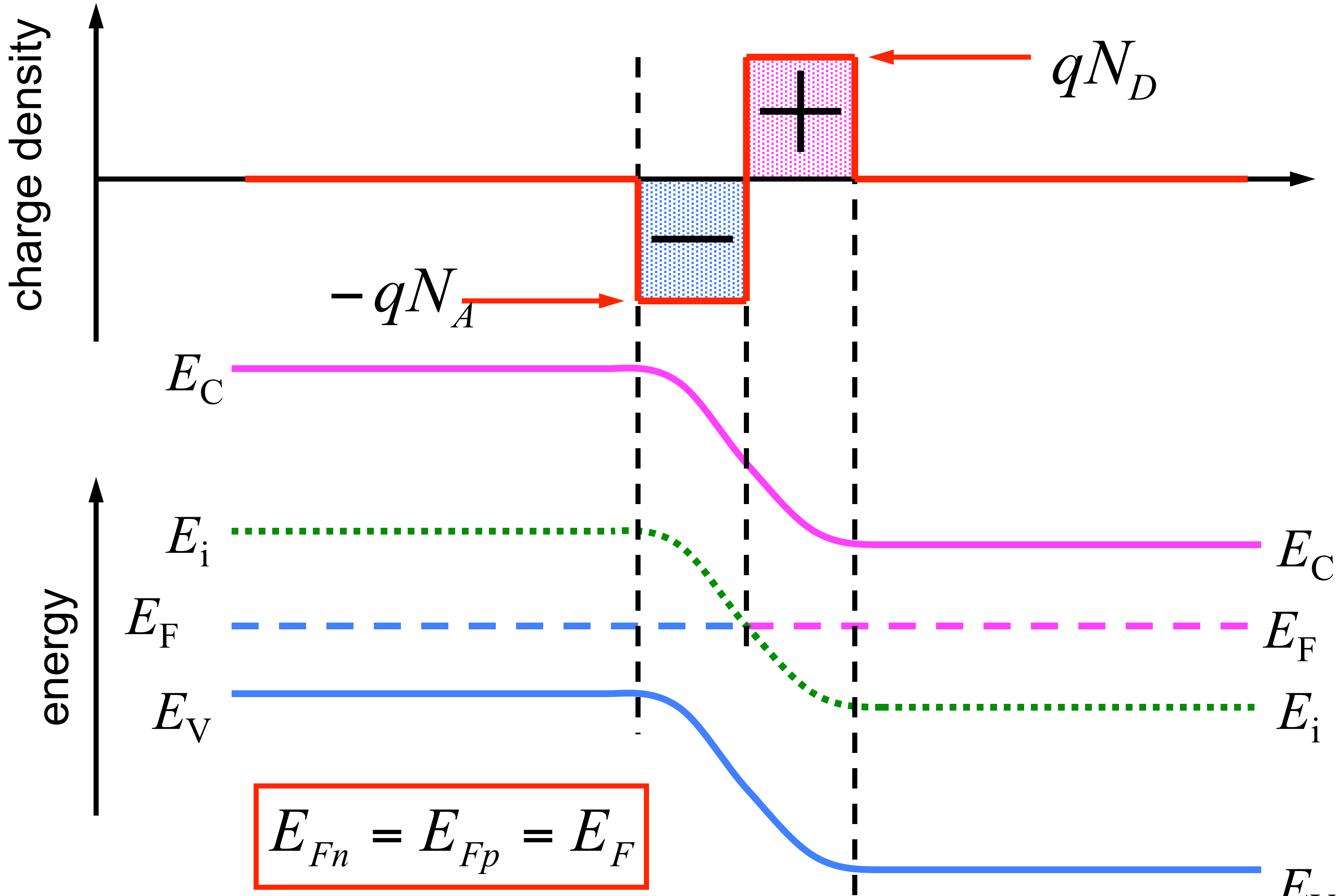
$$e\Phi_p = e\chi + E_g - \left(E_{Fpo} - E_V\right)$$

$$e\Phi_n = e\chi + \left(E_C - E_{Fno}\right)$$



$$eV_{bi} = e\Phi_p - e\Phi_n = E_{Fno} - E_{Fpo}$$

Depletion Region



For the n-region:

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] = N_d$$

$$\boxed{e\phi_{Fn} = E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)}$$

For the p-region:

$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right] = n_i \exp\left[\frac{E_{Fi} - E_F}{kT}\right] = N_a$$

$$\boxed{e\phi_{Fp} = E_F - E_{Fi} = -kT \ln\left(\frac{N_a}{n_i}\right)}$$

An important point: previously N_a and N_d denoted the concentrations in the same region. From now on, they will denote the **NET** concentrations in the individual p- and n-regions, respectively.

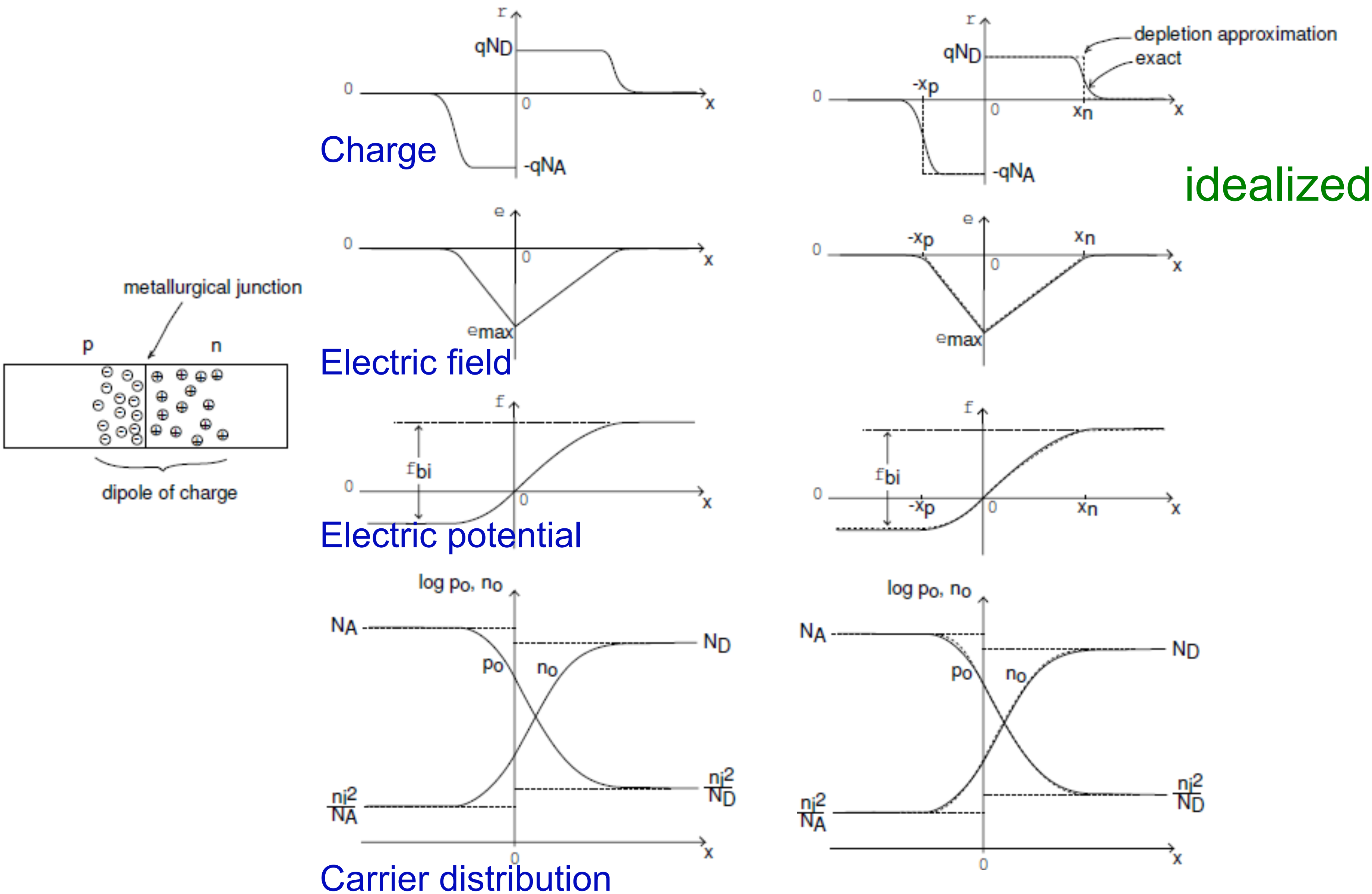
$$V_{bi} = |\phi_{Fn}| + |\phi_{Fp}| = \frac{kT}{e} \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$V_T = \frac{kT}{q}$$

V_t is the thermal voltage.

Analytical Analysis of the pn junction



1D Poisson's equation

$$div\nabla f = \nabla \cdot \nabla f = \Delta f = (Sources - Sinks)$$

$$\frac{d^2\psi(x)}{dx^2} = -\frac{d\mathbf{E}(x)}{dx} = -\frac{\rho_s(x)}{\varepsilon} =$$

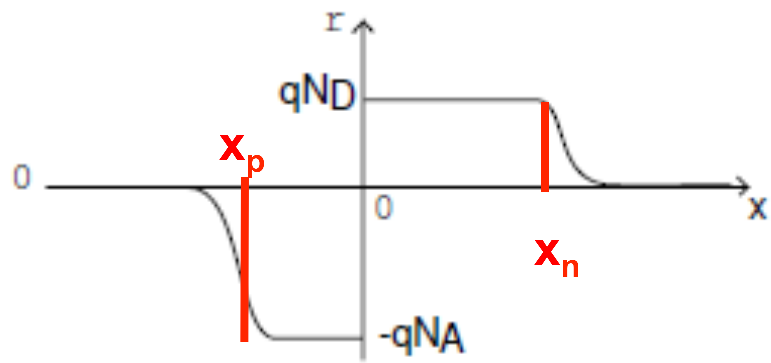
$$= -\frac{e}{\varepsilon}\left[N_D(x) - N_A(x) + p(x) - n(x)\right]$$

ψ - electrostatical potential

ρ - space charge density

ε - semiconductor permittivity

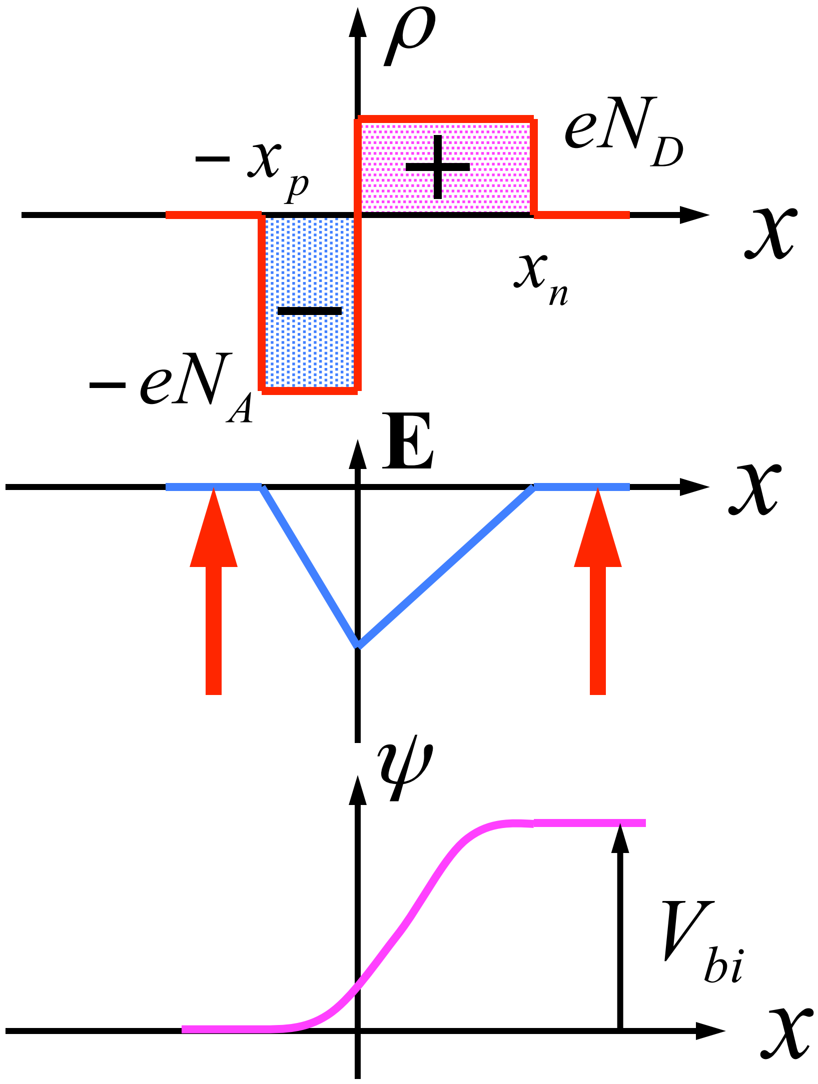
Poisson's Equation for **Abrupt** Junction And Depletion Approximations



$$\frac{d^2\psi(x)}{dx^2} = -\frac{d\mathbf{E}(x)}{dx} = -\frac{eN_A}{\varepsilon} \quad \text{for} \quad -x_p \le x < 0$$

$$\frac{d^2\psi(x)}{dx^2} = -\frac{d\mathbf{E}(x)}{dx} = \frac{eN_D}{\varepsilon} \quad \text{for} \quad 0 < x \le x_n$$

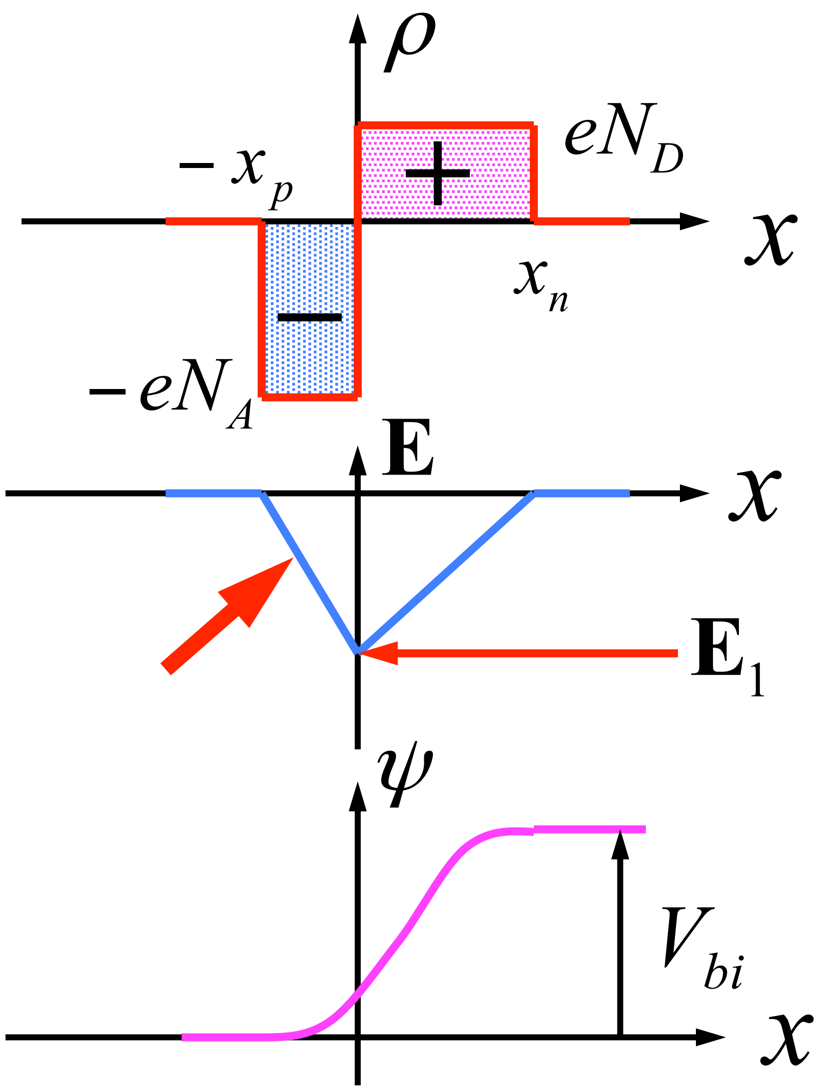
Junction Potential and Electric Field- Overview



$$\frac{d\mathbf{E}(x)}{dx} = 0$$

outside the depletion zone

Electric Field Distribution



$$\frac{d\mathbf{E}(x)}{dx} = \frac{-eN_A}{\varepsilon}$$

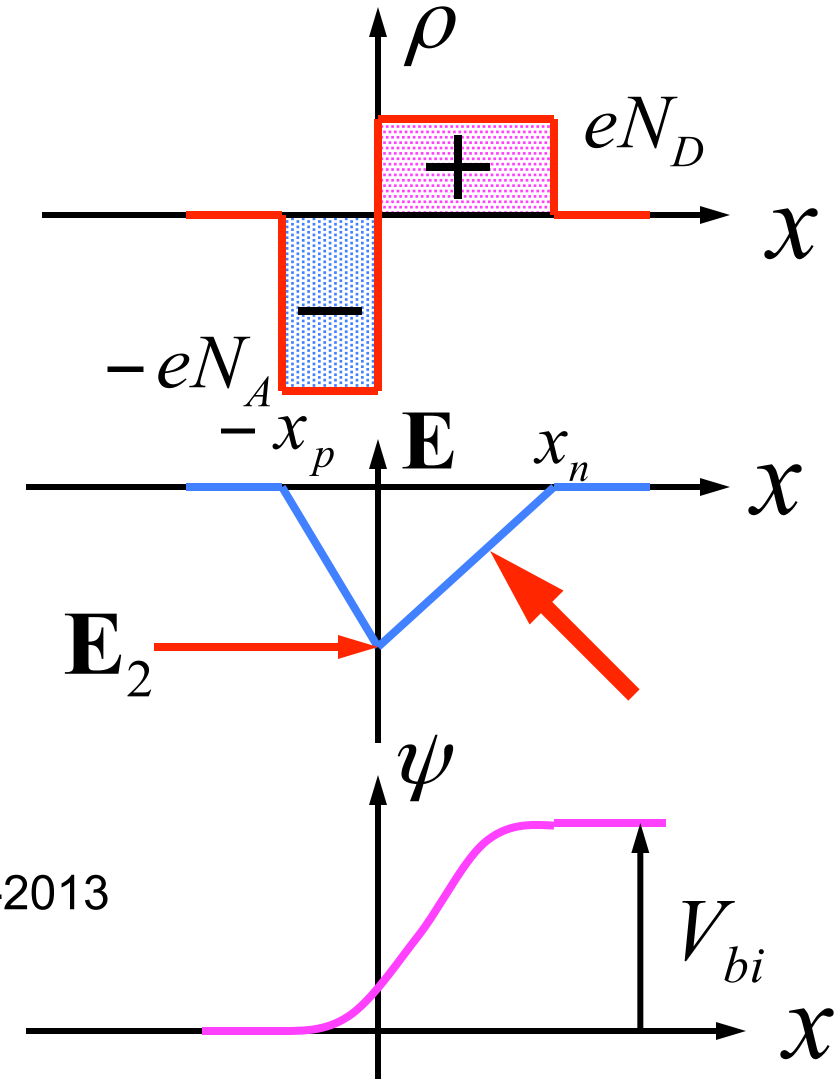
$$\mathbf{E}(x) = -\frac{eN_A}{\varepsilon}x + \mathbf{E}_1$$

$$\mathbf{E}(-x_p) = 0$$

$$\mathbf{E}_1 = -\frac{eN_A}{\varepsilon}x_p$$

$$\mathbf{E}(x) = -\frac{eN_A}{\varepsilon}\left(x + x_p\right)$$

Electric Field Distribution



$$\frac{d\mathbf{E}(x)}{dx} = \frac{eN_D}{\varepsilon}$$

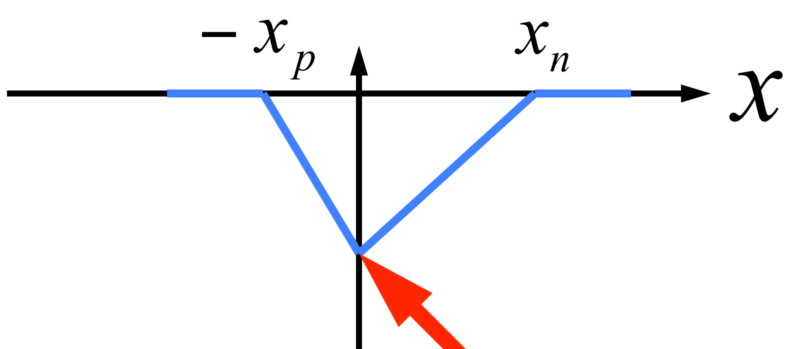
$$\mathbf{E}(x) = \frac{eN_D}{\varepsilon} x + \mathbf{E}_2$$

$$\mathbf{E}(x_n) = 0$$

$$\mathbf{E}_2 = -\frac{eN_D}{\varepsilon} x_n$$

$$\mathbf{E}(x) = \frac{eN_D}{\varepsilon}\left(x - x_n\right)$$

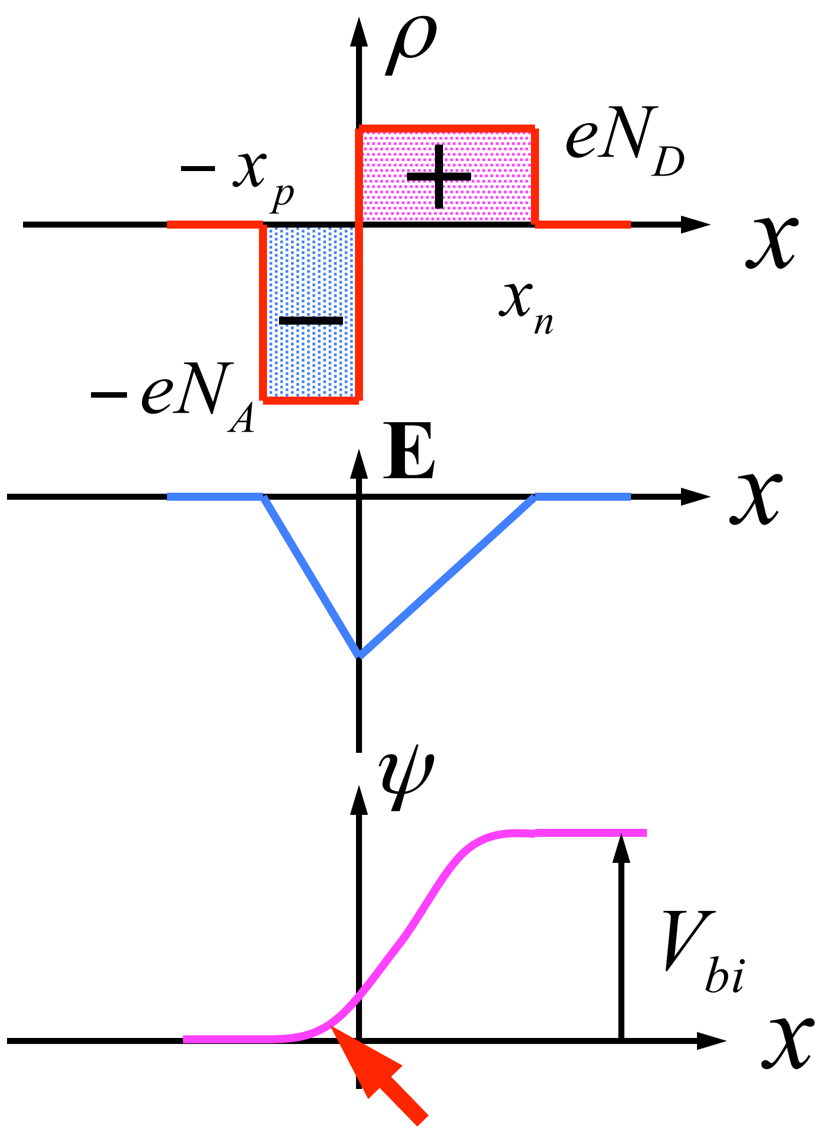
Maximum Electric Field



$$\mathbf{E}_{\max} = \mathbf{E}(0) = -\frac{eN_A}{\varepsilon}x_p = -\frac{eN_D}{\varepsilon}x_n$$

consequence: $$\boxed{N_A x_p = N_D x_n}$$

Potential Distribution



$$\mathbf{E}(x) = \frac{eN_D}{\varepsilon}x + \mathbf{E}_2$$

$$\psi(x) = \int \frac{eN_A}{\varepsilon}\left(x + x_p\right)dx$$

$$= \frac{eN_A}{\varepsilon}\left(\frac{x^2}{2} + x_p x\right) + \psi_1$$

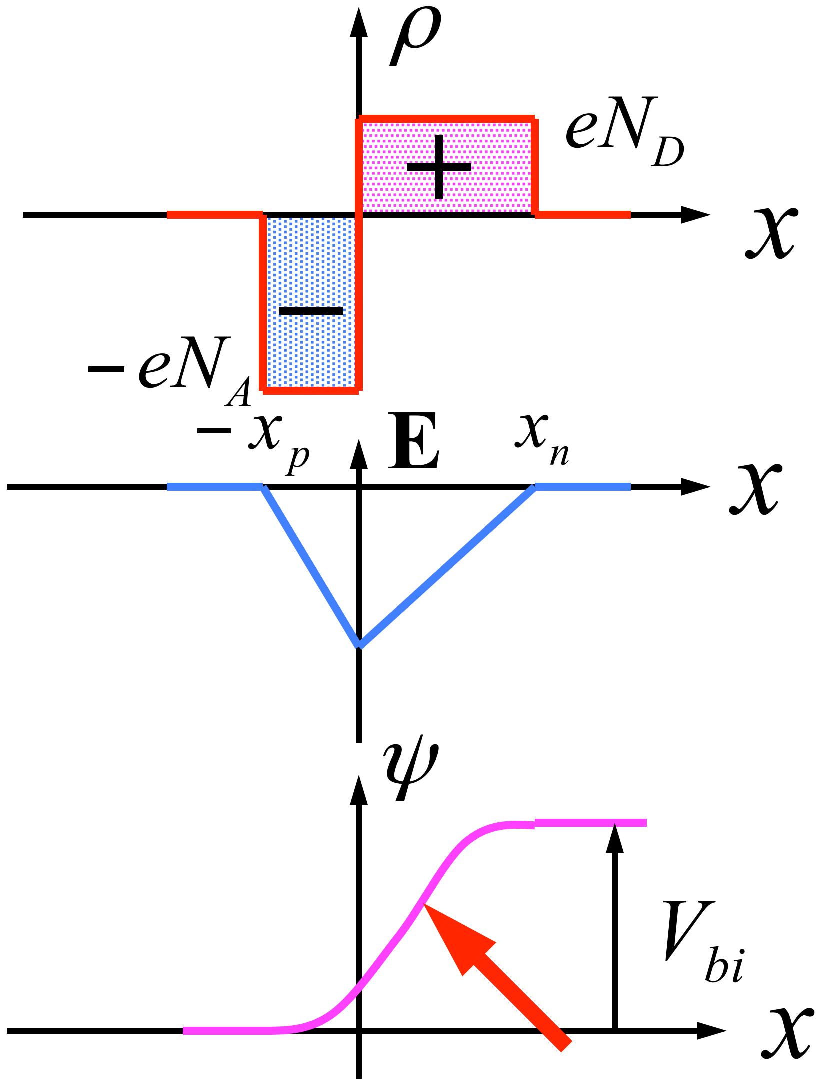
with $\psi(-x_p) = 0$

$$\psi_1 = \frac{eN_A}{\varepsilon}\frac{x_p{}^2}{2}$$

$$\boxed{\psi(x) = \frac{eN_A}{2\varepsilon}\left(x_p + x\right)^2}$$

quadratic dependence on distance !

Potential Distribution



$$\mathbf{E}(x) = \frac{eN_D}{\varepsilon}(x - x_n)$$

$$\psi(x) = \int \frac{eN_D}{\varepsilon}(x_n + x)dx$$

$$= \frac{eN_D}{\varepsilon}\left(x_n x - \frac{x^2}{2}\right) + \psi_2$$

with $\psi(x_n) = V_{bi}$

$$\psi_2 = V_{bi} - \frac{eN_D}{\varepsilon}\frac{x_n^2}{2}$$

$$\boxed{\psi(x) = V_{bi} - \frac{eN_D}{2\varepsilon}(x_n - x)^2}$$

Built-in Potential (Second Equation)

for $x = 0$ both expressions

$$\psi(x) = V_{bi} - \frac{eN_D}{2\varepsilon}(x_n - x)^2 \qquad \psi(x) = \frac{eN_A}{2\varepsilon}(x_p + x)^2$$

must give the same value:

$$\psi(0) = V_{bi} - \frac{eN_D}{2\varepsilon}x_n^{\ 2} = \frac{eN_A}{2\varepsilon}x_p^{\ 2}$$

recall

$$\boxed{V_{bi} = \frac{e}{2\varepsilon}\left(N_D x_n^{\ 2} + N_A x_p^{\ 2}\right)} \qquad \boxed{V_{bi} = \frac{kT}{e}\ln\left(\frac{N_D N_A}{n_i^{\ 2}}\right)}$$

Depletion Width Expressions

$$\boxed{W \equiv x_n + x_p}$$

$$V_{bi} = \frac{e}{2\varepsilon}\left(N_D x_n^{\,2} + N_A x_p^{\,2}\right)$$

$$N_A x_p = N_D x_n$$

$$V_{bi} = \frac{e}{2\varepsilon}\left[N_D x_n^{\,2} + N_A\left(\frac{N_D x_n}{N_A}\right)^2\right] = x_n^{\,2}\,\frac{e}{2\varepsilon}\left(N_D + \frac{N_D^{\,2}}{N_A}\right)$$

$$V_{bi} = \frac{e}{2\varepsilon}\left[N_D\left(\frac{N_A x_p}{N_D}\right)^2 + N_A x_n^{\,2}\right] = x_p^{\,2}\,\frac{e}{2\varepsilon}\left(\frac{N_A^{\,2}}{N_D} + N_A\right)$$

$$x_n = \sqrt{\frac{2\varepsilon}{e}\,\frac{N_A}{N_D N_A + N_D^{\,2}}\,V_{bi}} \qquad x_p = \sqrt{\frac{2\varepsilon}{e}\,\frac{N_D}{N_D N_A + N_A^{\,2}}\,V_{bi}}$$

Depletion Width Expressions

$$x_n = \sqrt{\frac{2\varepsilon}{e}\frac{N_A}{N_D N_A + N_D^{\,2}} V_{bi}} \qquad x_p = \sqrt{\frac{2\varepsilon}{e}\frac{N_D}{N_D N_A + N_A^{\,2}} V_{bi}}$$

$$W^2 = \left(x_n + x_p\right)^2 = \frac{2\varepsilon}{e}\frac{N_A}{N_D N_A + N_D^{\,2}} V_{bi} + \frac{2\varepsilon}{e}\frac{N_D}{N_A N_D + N_A^{\,2}} V_{bi}$$

$$+ 2 \cdot \sqrt{\frac{2\varepsilon}{e}\frac{N_A}{N_D N_A + N_D^{\,2}} V_{bi}} \cdot \sqrt{\frac{2\varepsilon}{e}\frac{N_D}{N_A N_D + N_A^{\,2}} V_{bi}} =$$

$$\frac{2\varepsilon}{e} V_{bi} \frac{N_A^{\,2} + 2N_D N_A + N_D^{\,2}}{\left(N_A + N_D\right) N_D N_A} = \frac{2\varepsilon}{e}\frac{\left(N_A + N_D\right)}{N_D N_A} V_{bi}$$

$$\boxed{W = \sqrt{\frac{2\varepsilon}{e}\frac{\left(N_A + N_D\right)}{N_D N_A} V_{bi}}}$$

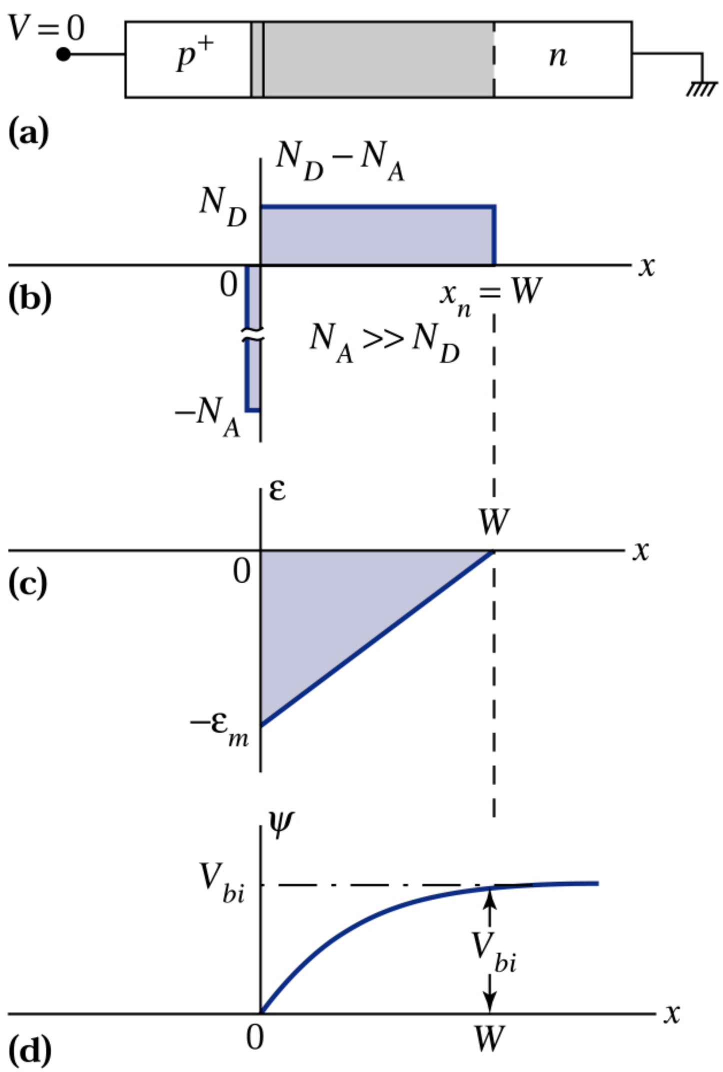
Recall: Built-in Potential (Diffusion Potential)

$$\boxed{V_{bi} = \psi_n - \psi_p = \frac{kT}{e}\ln\left(\frac{N_D N_A}{n_i^2}\right)}$$

$$\boxed{V_{bi} = \frac{e}{2\varepsilon}\left(N_D {x_n}^2 + N_A {x_p}^2\right)}$$

One-side Abrupt Junction: $N_a >> N_d$

Important for s/d regions of a MOSFET and emitter of BJT



$$W = \sqrt{\frac{2\varepsilon}{e}\frac{(N_A + N_D)}{N_D N_A} V_{bi}}$$

if $\quad x_p << x_n$

$$W \cong x_n = \sqrt{\frac{2\varepsilon V_{bi}}{e N_D}}$$

Example: consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 10^{16}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}. Calculate the space charge width and maximum electric field in the junction.

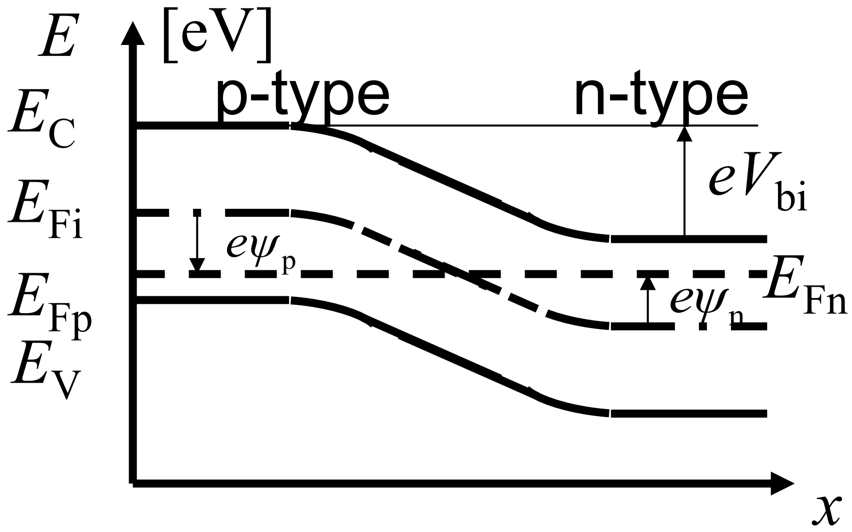
Solution: $$V_{bi} = \left(\frac{kT}{e}\right)\ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259)\ln\left[\frac{(10^{16})(10^{15})}{(1.5\times10^{10})^2}\right] = 0.635\text{V}$$

$$W = \left\{\frac{2\varepsilon_s V_{bi}}{e}\left[\frac{N_a + N_d}{N_a N_d}\right]\right\}^{1/2}$$

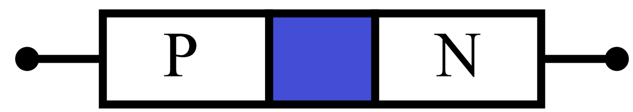
$$= \left\{\frac{2(11.7)(8.854\times10^{-14})(0.635)}{1.602\times10^{-19}}\left[\frac{10^{16}+10^{15}}{(10^{16})(10^{15})}\right]\right\}^{1/2} = 9.5\times10^{-5}\,\text{cm} = 0.95\mu\text{m}$$

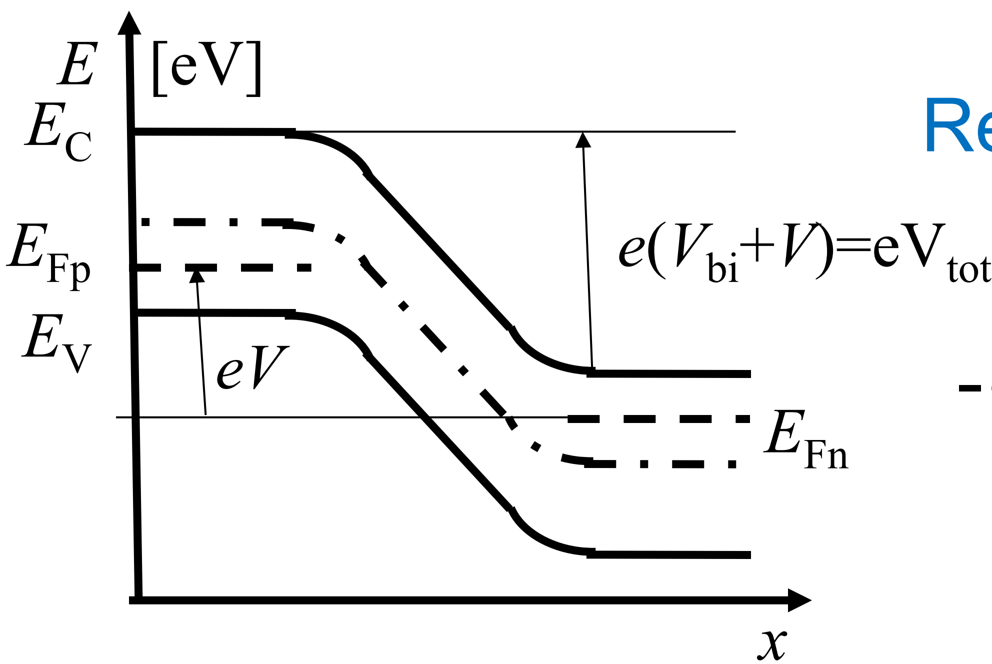
$$x_n = \frac{N_a}{N_a + N_d}W = 0.86\mu\text{m} \qquad x_p = \frac{N_d}{N_a + N_d}W = 0.086\mu\text{m}$$

$$E_{\max} = \frac{-eN_d x_n}{\varepsilon_s} = \frac{-(1.6\times10^{-19})(10^{15})(0.86\times10^{-4})}{(11.7)(8.854\times10^{-14})} = -1.33\times10^{4}\,\text{V/cm}$$



Unbiased Junction

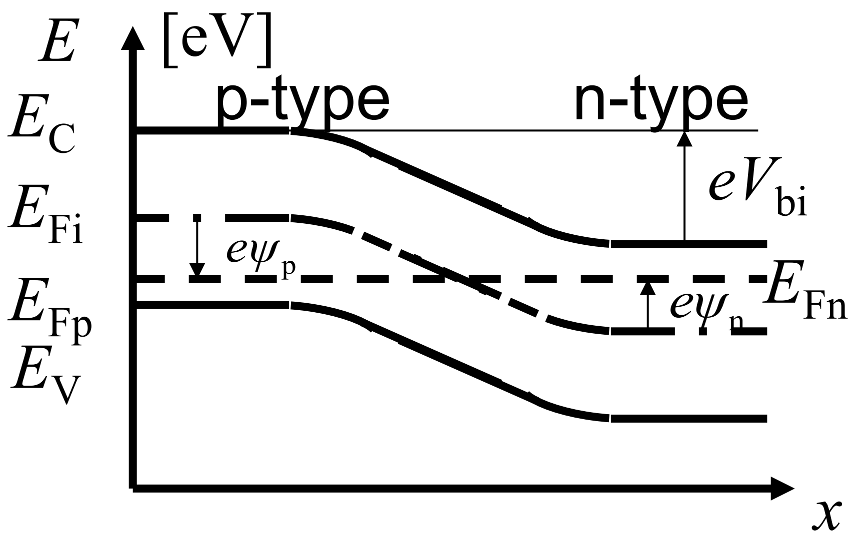


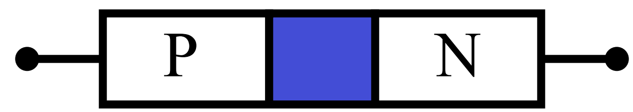


Reverse-biased Junction

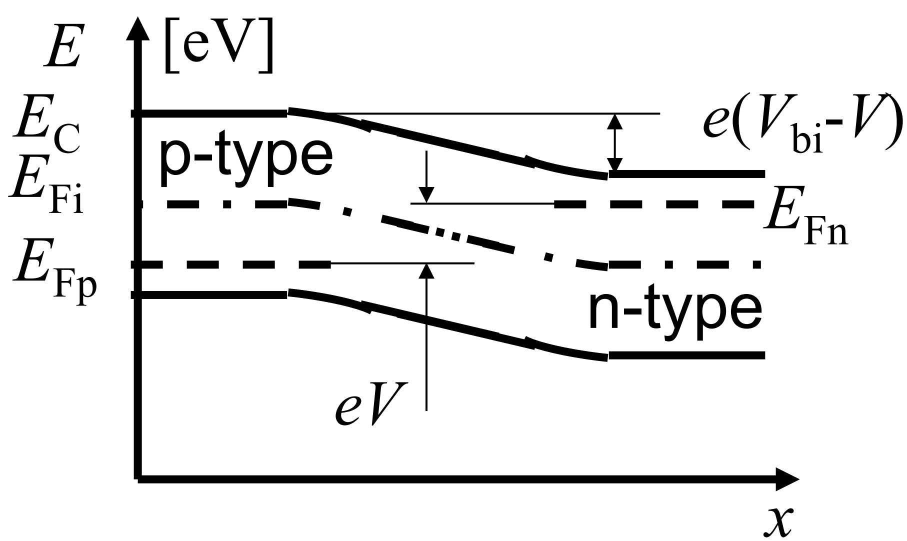
$\boxed{V < 0}$

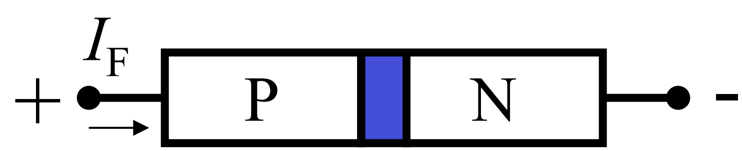
Unbiased Junction





Forward-biased Junction





$\boxed{V > 0}$

Space Charge Width and Electric Field under Reverse Applied Bias

$$W = \left\{ \frac{2\varepsilon_s \left(V_{bi} + V_R \right)}{e} \left[\frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}$$

Example: consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 10^{16}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}. Calculate the space charge width under a reverse bias of 5 V. $n_i = 1.5 \times 10^{10}$ cm^{-3}.

Space Charge Width and Electric Field under Reverse Applied Bias

The electric field is still a linear function of distance. The maximum electric field is still at the metallurgical junction.

$$x_n = \left\{ \frac{2\varepsilon_s (V_{bi} + V_R)}{e} \left[\frac{N_a}{N_d} \right] \left[\frac{1}{N_a + N_d} \right] \right\}^{1/2}$$

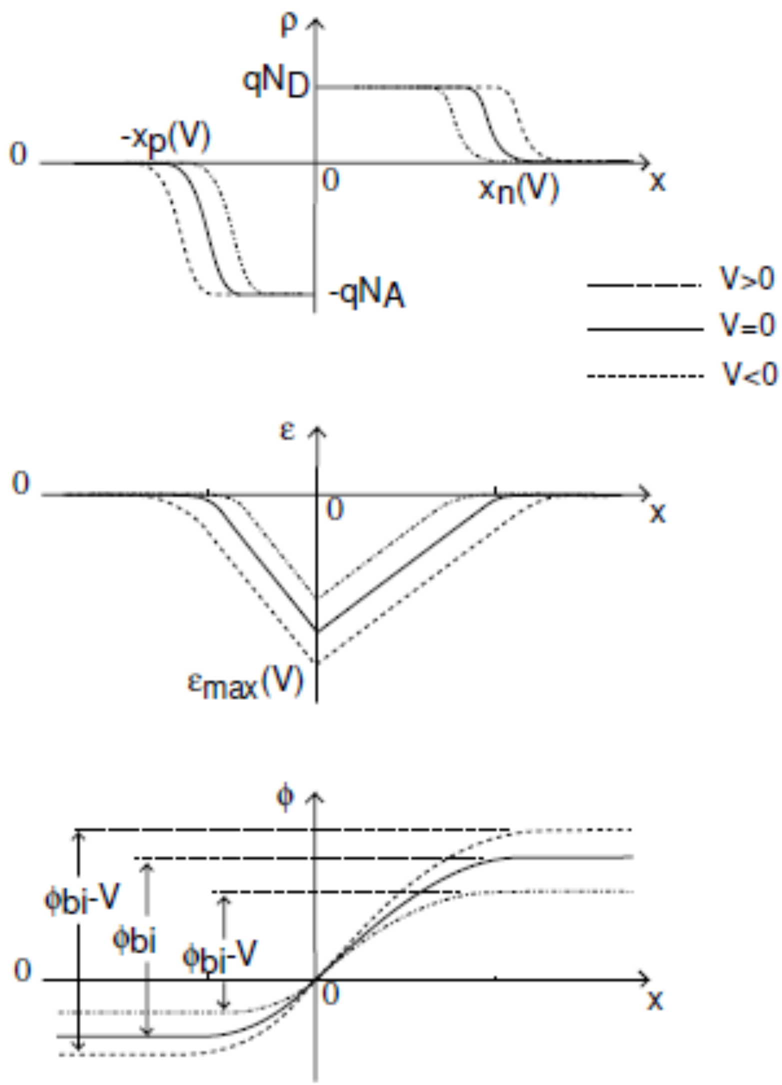
$$x_p = \left\{ \frac{2\varepsilon_s (V_{bi} + V_R)}{e} \left[\frac{N_d}{N_a} \right] \left[\frac{1}{N_a + N_d} \right] \right\}^{1/2}$$

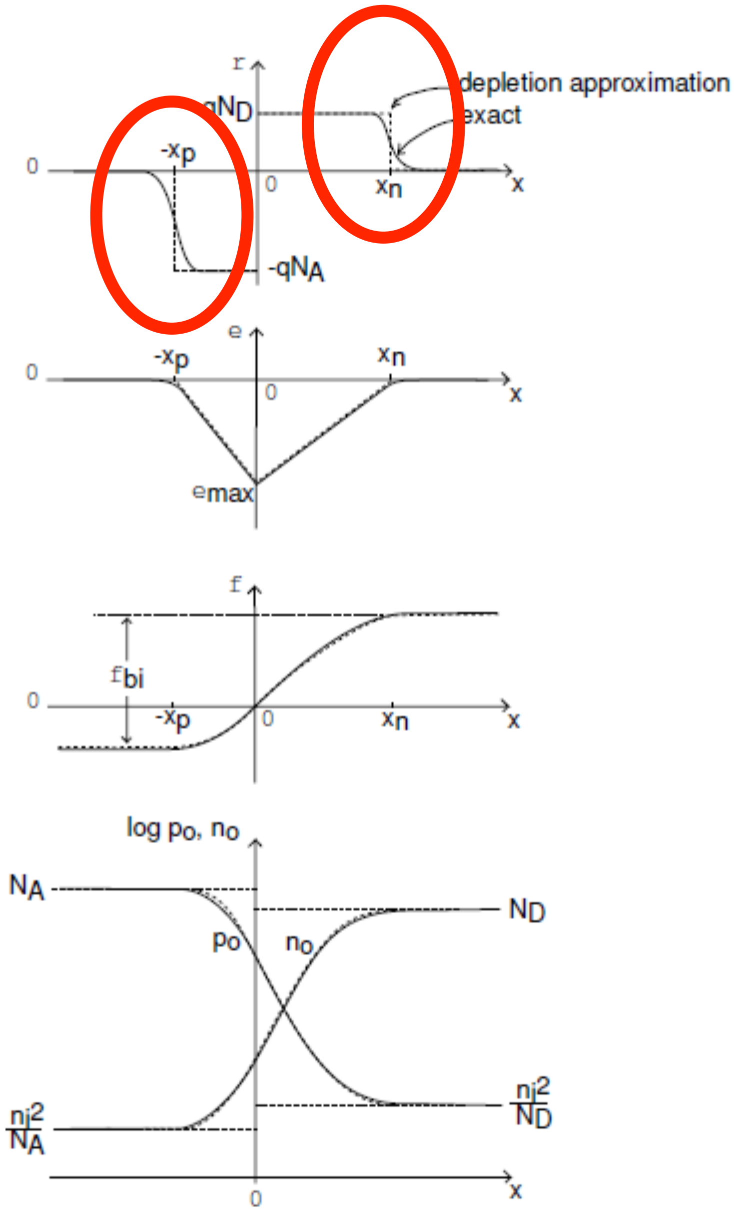
$$E_{\max} = \frac{-eN_d x_n}{\varepsilon_s} = \frac{-eN_a x_p}{\varepsilon_s}$$

$$E_{\max} = -\left\{ \frac{2e(V_{bi} + V_R)}{\varepsilon_s} \left[\frac{N_a N_d}{N_a + N_d} \right] \right\}^{1/2} = -\frac{2(V_{bi} + V_R)}{W}$$

$= -4 \times 10^4$ V/cm

Forward- and reversed biased Junction





Sharp boundary of the space charge region.

This is an idealization; in reality the transition is not sharp but gradual characterized by the Debye length L_D.

Depletion approximation

How precise is this approximation?

Debye length

$$L_D = \left[\frac{\varepsilon_s kT}{q^2(n+p)}\right]^{1/2}$$

For an extrinsic n-type semiconductor, the minority-carrier density is negligible and we can write $n+p=N_d$

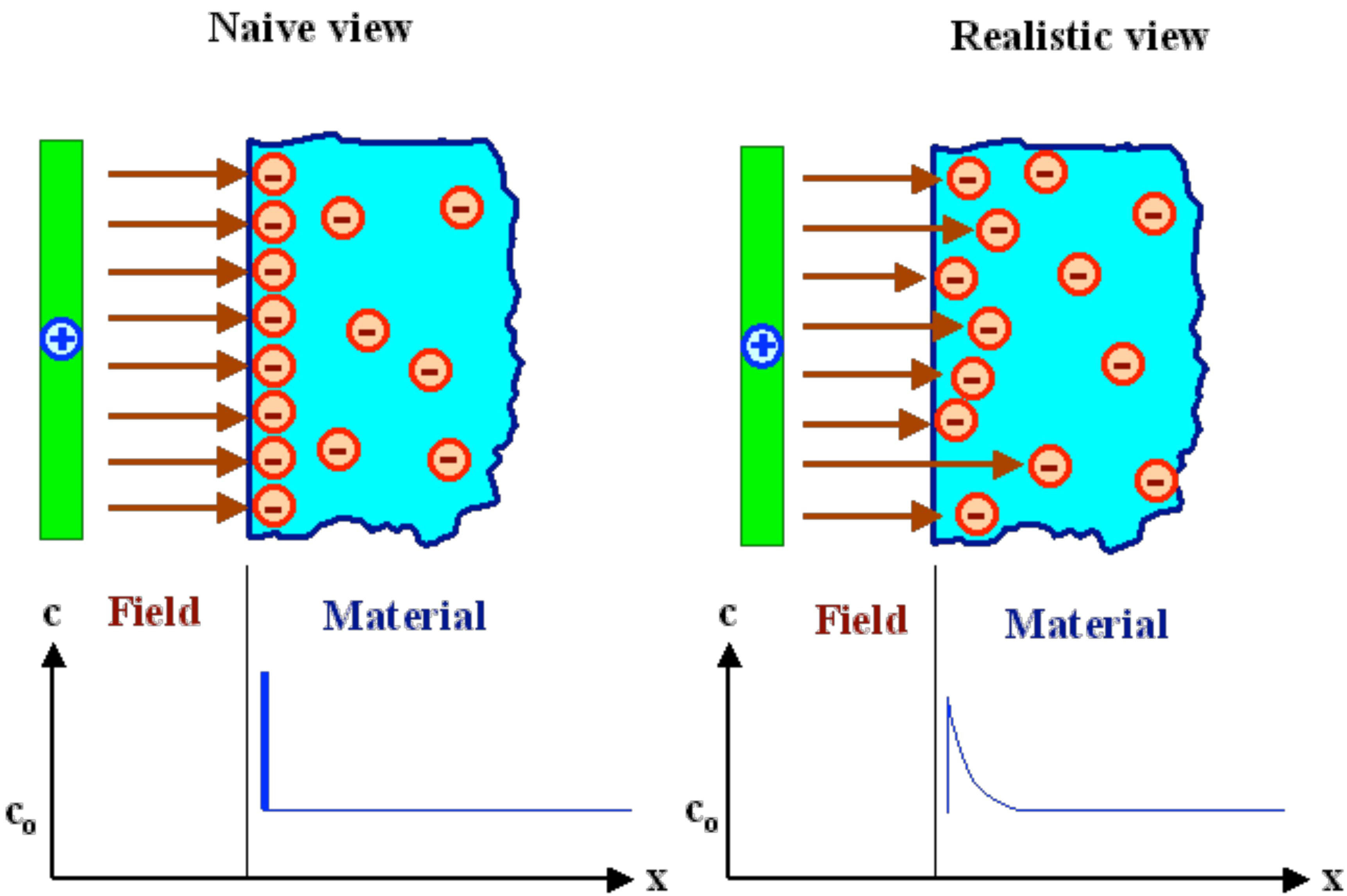
$$L_D = \left[\frac{\varepsilon_s kT}{q^2 N_d}\right]^{1/2}$$

For $N_d=10^{16}$ cm^{-3} we get $L_D=40$ nm,

for $N_d=10^{18}$ cm^{-3} we get $L_D=4$nm.

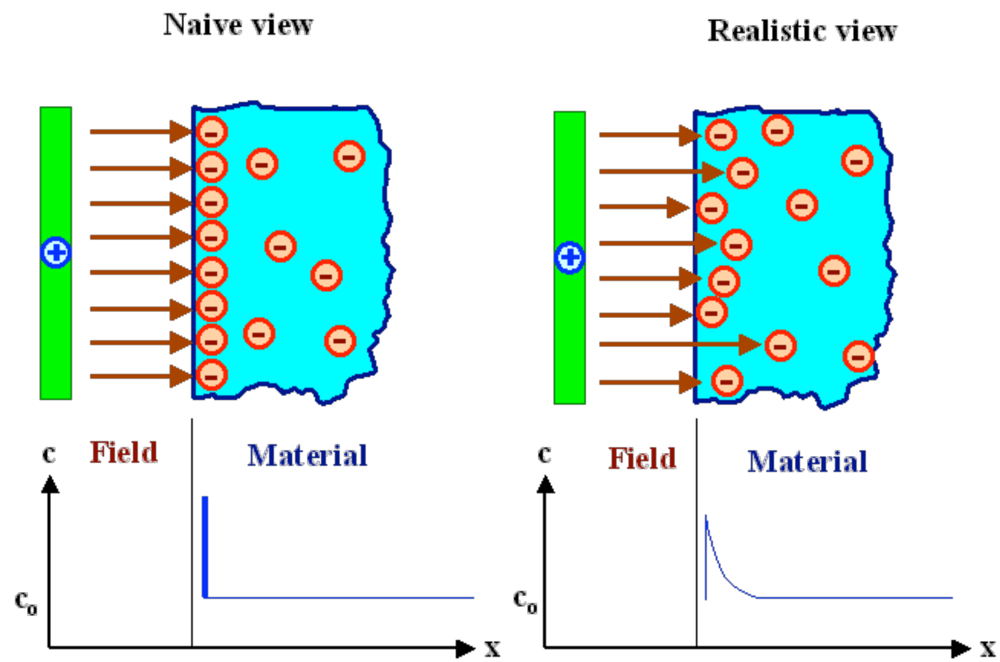
Debye length is a characteristic length associated with the spatial variation of the potential. The potential varies exponentially with distance near the edges of the space charge region equal to the Debye length. Because the carrier concentration itself depends exponentially on the potential, the carrier concentration changes rapidly from the dopant concentration to essentially zero within a few Debye lengths.

Debye length



Debye length

Imagine that we hold a positively charged plate at some (small) distance to the surface of a material having mobile negative charges (a metal, a suitable ionic conductor, a **n**-doped semiconductor, ...). In other words, the positively charged plate and the material are *insulated*, and no currents of any kind can flow between the two. However, there will be an electrical field, with field lines starting at the positive charges on the plate and ending on the negative charges inside the material. We have the following situation as depicted:



In a *naive* (and *wrong*) view, enough negatively charged carriers in the material would move to the surface to screen the field completely, i.e. prevent its penetration into the material. "Enough", to be more precise, means just the right number so that every field line originating from some charge in the positively charged plate ends on a negatively charged carrier inside the material.

But that would mean that the concentration of carriers at the surface would be pretty much a δ- function, or at least a function with a very steep slope. That does not seem to be physically sensible. We certainly would expect that the concentration varies smoothly within a certain distance, and this distance we call **Debye length**.

1.4 Junction Capacitance under Reverse Bias

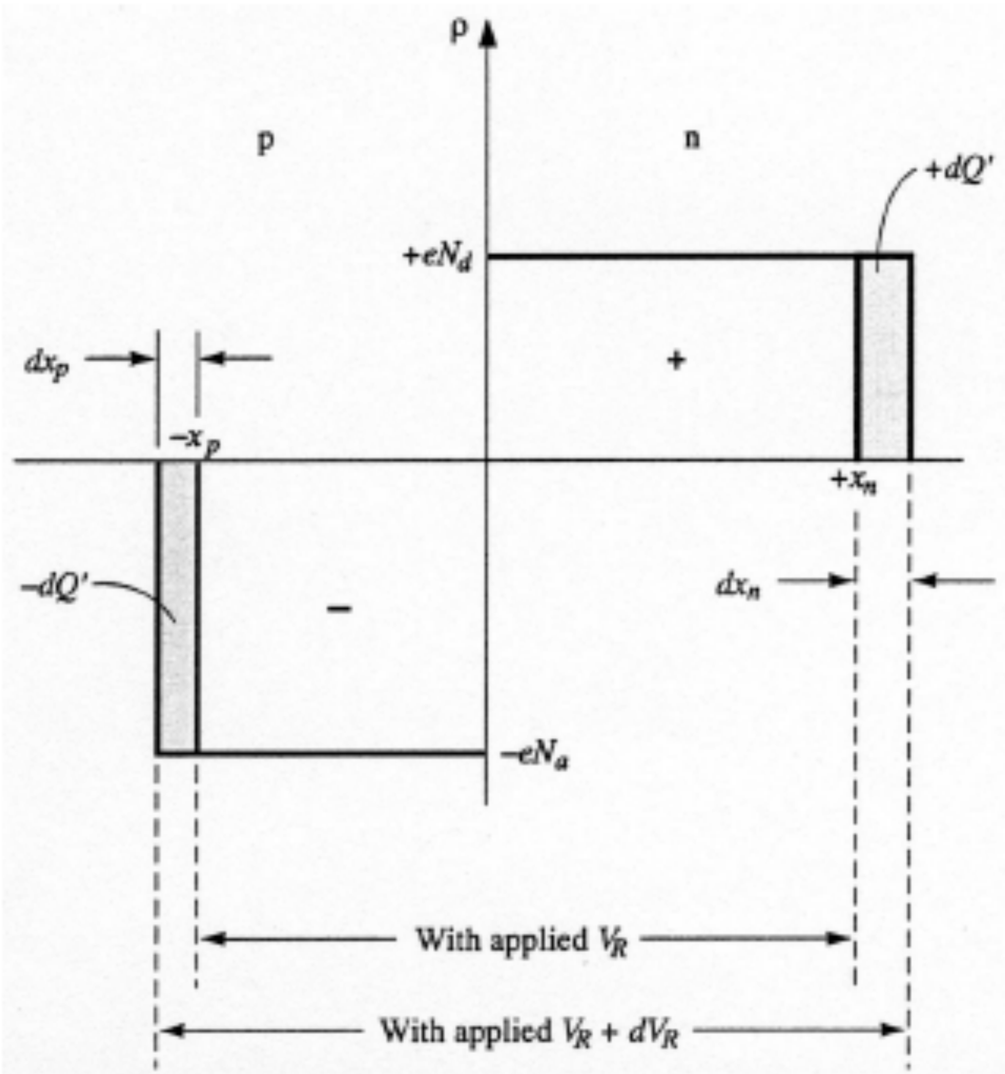
pn Junction: Depletion Capacitance

The junction depletion layer capacitance per unit area is defined as:

$$\boxed{C_j = \frac{dQ}{dV}}$$

Where dQ is the incremental charge in depletion layer charge per unit area for an incremental change in the applied voltage dV.

Junction Capacitance under Reverse Applied Bias



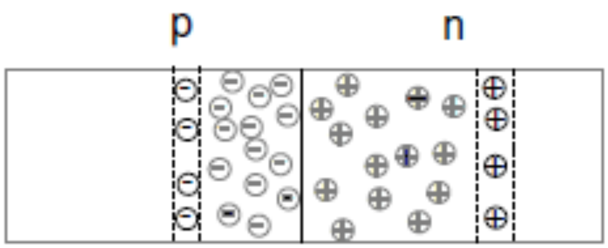
An increase in the applied reverse bias will lead to an increase in the positive and negative charges in the n- and p-regions, respectively. The junction capacitance per unit area is obtained from:

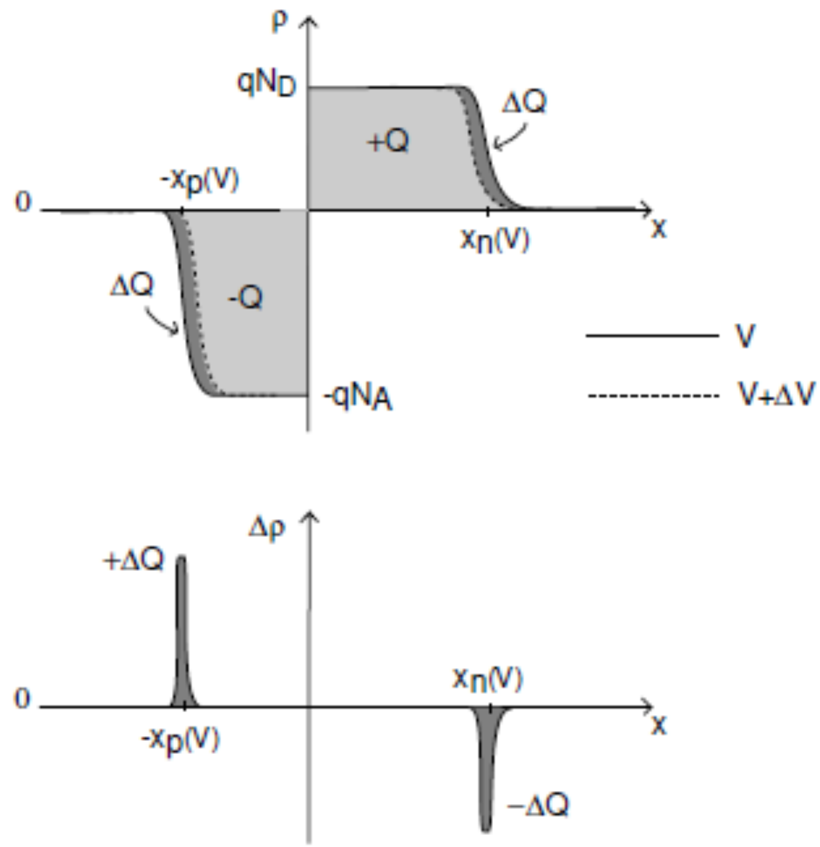
$$dQ' = eN_d dx_n = eN_a dx_p$$

$$C' = \frac{dQ'}{dV_R}$$

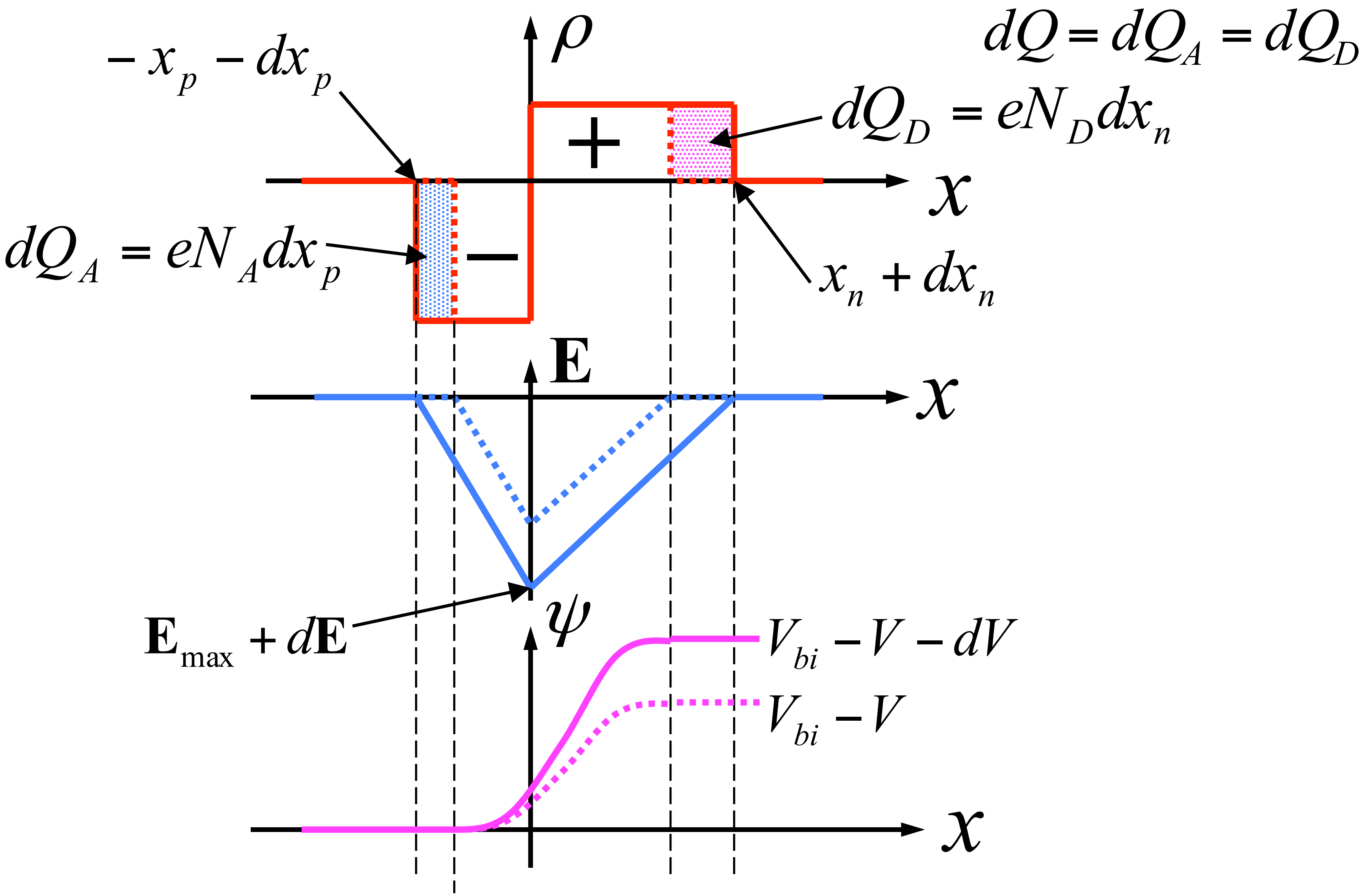
pn Junction Depletion Capacity

Think differentially:





pn Junction Depletion Capacity



Capacitance of an Abrupt Junction

$$C_j = \frac{dQ}{dV_R}$$

$$dQ = eN_D dx_n = eN_A dx_p$$

$$x_n = \sqrt{\frac{2\varepsilon}{q}\frac{N_A}{N_D N_A + N_D^2}\left(V_{bi} + V_R\right)}$$

Junction Capacitance under Reverse Applied Bias

$$x_n = \left\{ \frac{2\varepsilon_s (V_{bi} + V_R)}{e} \left[\frac{N_a}{N_d} \right] \left[\frac{1}{N_a + N_d} \right] \right\}^{1/2}$$

$$\frac{dx_n}{dV_R} = \left\{ \frac{\varepsilon_s}{2e(V_{bi} + V_R)} \left[\frac{N_a}{N_d} \right] \left[\frac{1}{N_a + N_d} \right] \right\}^{1/2}$$

$$C = eN_d \frac{dx_n}{dV_R}$$

$$\boxed{C = \left\{ \frac{e\varepsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right\}^{1/2}}$$

Junction Capacitance under Reverse Applied Bias

Example: consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 10^{16}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}. The junction is under a reverse bias of 5 V. Calculate the junction capacitance.

$$C' = \left\{ \frac{(1.6\times10^{-19})(11.7)(8.85\times10^{-14})(10^{16})(10^{15})}{2(0.635+5)(10^{16}+10^{15})} \right\}^{1/2} = 3.66\times10^{-9}\,\mathrm{F/cm^2}$$

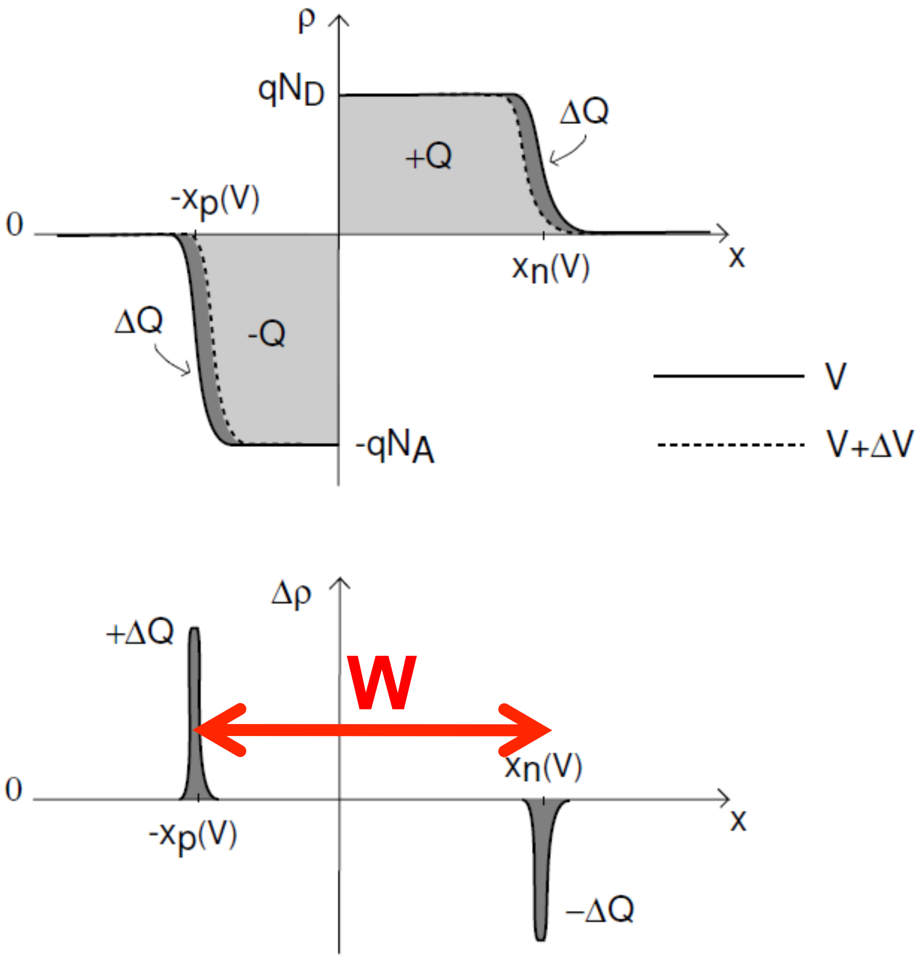
C' is in the range of nF to pF.

$$C' = \left\{ \frac{e\varepsilon_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)} \right\}^{1/2}$$

$$W = \left\{ \frac{2\varepsilon_s (V_{bi}+V_R)}{e} \left[\frac{N_a+N_d}{N_a N_d} \right] \right\}^{1/2}$$

Source/drain capacitances are important for the switching speed of a MOSFET

Junction Capacitance – Parallel-plate Capacitance



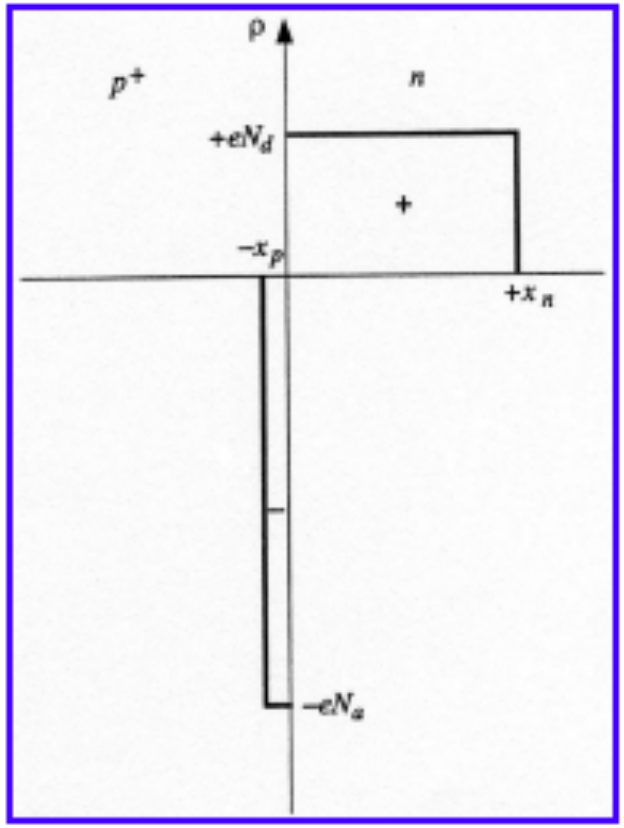
$$C = \frac{\varepsilon_s}{W}$$

1.5 One-sided Abrupt Junction under Reverse Bias

For example, $N_a >> N_d$, a p^+n junction.

$$W = \left\{ \frac{2\varepsilon_s (V_{bi} + V_R)}{e} \left[\frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}$$

$$\approx \left\{ \frac{2\varepsilon_s (V_{bi} + V_R)}{e N_d} \right\}^{1/2}$$



$$x_n = \left\{ \frac{2\varepsilon_s (V_{bi} + V_R)}{e} \left[\frac{N_a}{N_d} \right] \left[\frac{1}{N_a + N_d} \right] \right\}^{1/2}$$

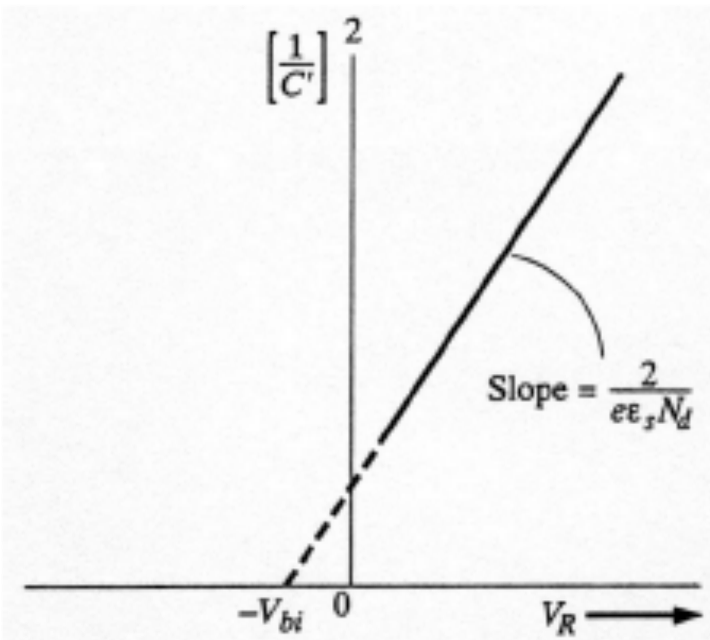
$$x_p = \left\{ \frac{2\varepsilon_s (V_{bi} + V_R)}{e} \left[\frac{N_d}{N_a} \right] \left[\frac{1}{N_a + N_d} \right] \right\}^{1/2}$$

$$\Rightarrow \begin{cases} x_p << x_n \\ W \approx x_n \end{cases}$$

One-sided Abrupt Junction under Reverse Bias

For example, $N_a >> N_d$, a p$^+$n junction.

$$C = \left\{ \frac{e\varepsilon_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)} \right\}^{1/2} \approx \left\{ \frac{e\varepsilon_s N_d}{2(V_{bi}+V_R)} \right\}^{1/2}$$



$$\left(\frac{1}{C}\right)^2 = \frac{2(V_{bi}+V_R)}{e\varepsilon_s N_d}$$

The left curve can be used to determine the doping concentration and the built-in potential.

→ Information on V_{bi} and N_d

Example: assume a silicon p^+n junction at $T = 300$ K with $n_i = 1.5 \times 10^{10}$ cm^{-3}. Assume that the intercept of the $(1/C')^2$ versus V_R curve gives $V_{bi} = 0.855$ volt and the slope is 1.32×10^{15} [(F/cm^2)$^{-2}$(volt)$^{-1}$]. Determine the doping concentrations.

One-sided Abrupt Junction under Reverse Bias

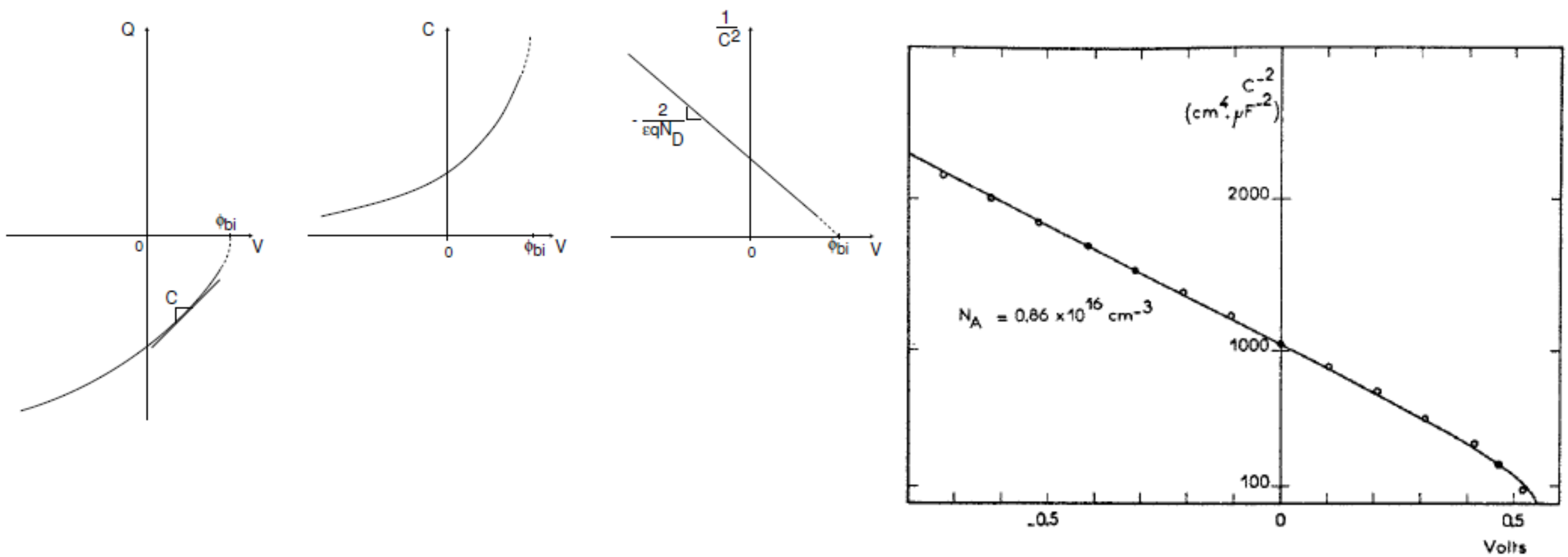
For p$^+$-n junction:

$$C(V) = \sqrt{\frac{\epsilon q N_D}{2(\phi_{bi} - V)}}$$

Capacitance dominated by lowly-doped side

Technique to extract ϕ_{bi} and N_{low}:

$$\frac{1}{C^2} = \frac{2(\phi_{bi} - V)}{\epsilon q N_D}$$

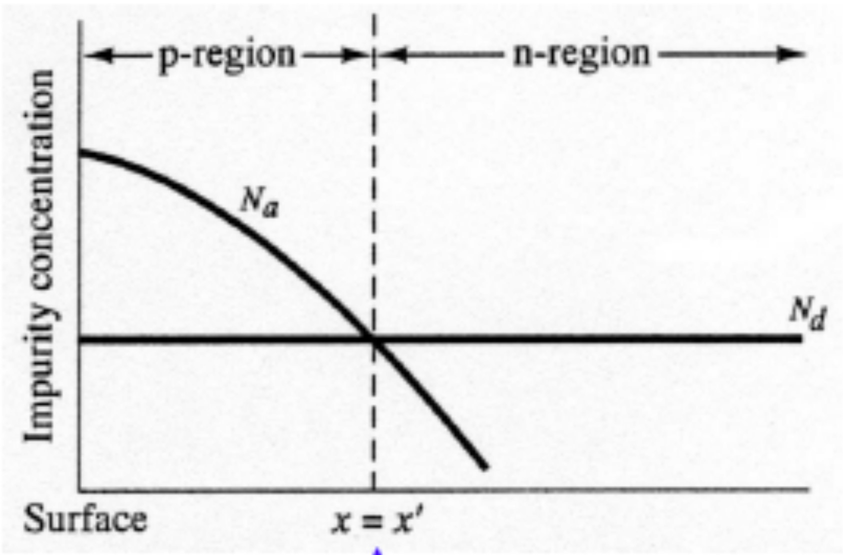


2. Non-uniformly Doped Junctions

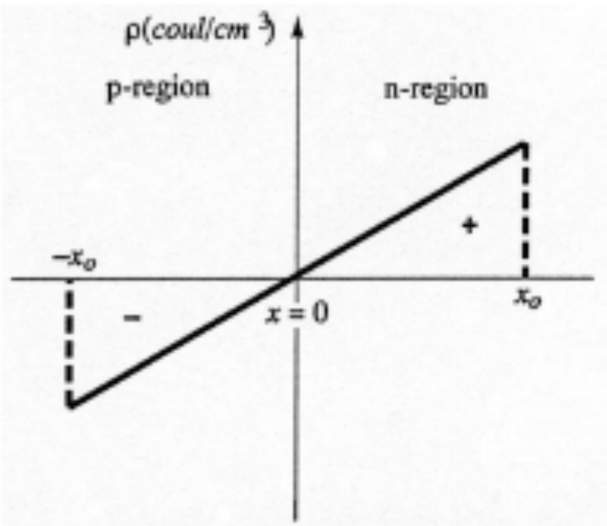
2.1 Linearly Graded Junctions

Linearly Graded Junctions

If acceptor atoms are diffused through the surface into a uniformly doped n-type semiconductor, linearly graded pn junctions will be obtained. These non-uniformly doped pn junctions will have special junction capacitance characteristics.



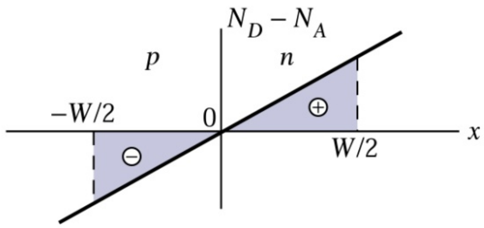
Metallurgical junction



$$\rho(x) = eax$$

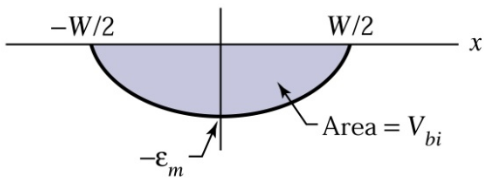
a is the gradient of the net impurity concentration.

Linearly Graded Junction - Overview



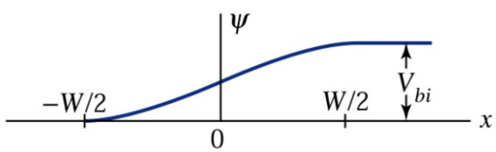
(a)

impurity distribution



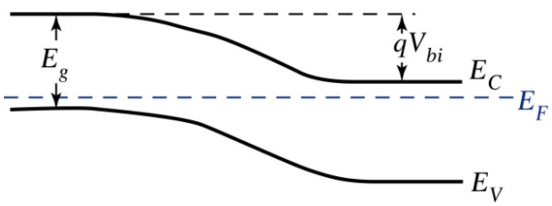
(b)

electric field distribution



(c)

potential distribution



(d)

energy band diagram

Linearly Graded Junction - Evaluation

Linearly Graded Junctions

$$\frac{dE}{dx} = \frac{\rho(x)}{\varepsilon_s} = \frac{eax}{\varepsilon_s}$$

$$E = \int \frac{eax}{\varepsilon_s} dx = \frac{eax^2}{2\varepsilon_s} + C_1$$

Assume $E = 0$ in the bulk region:

E=0 at x=x_o=W/2

$$C_1 = \frac{-eax_0^2}{2\varepsilon_s}$$

$$E = \frac{ea}{2\varepsilon_s}\left(x^2 - x_0^2\right)$$

Quadratic dependence on x

$$\phi(x) = -\int E dx = -\frac{ea}{2\varepsilon_s}\int\left(x^2 - x_0^2\right)dx = -\frac{ea}{2\varepsilon_s}\left(\frac{x^3}{3} - x_0^2 x\right) + C_2$$

Linearly Graded Junctions

Set $\phi = 0$ at $x = -x_0$:

$$C_2 = \frac{ea}{3\varepsilon_s} x_0^3$$

Cube dependence on x

$$\phi(x) = \frac{-ea}{2\varepsilon_s}\left(\frac{x^3}{3} - x_0^2 x\right) + \frac{ea}{3\varepsilon_s} x_0^3$$

ϕ will be the built-in potential at $x = +x_0$:

$$\boxed{V_{bi} = \frac{2}{3}\frac{eax_0^3}{\varepsilon_s}}$$

Under a reverse bias voltage:

$$x_0 = \left\{\frac{3}{2}\frac{\varepsilon_s}{ea}\left(V_{bi} + V_R\right)\right\}^{1/3}$$

$$\boxed{\frac{dx_0}{dV_R} = \frac{\varepsilon_s}{2ea} x_0^{-2}}$$

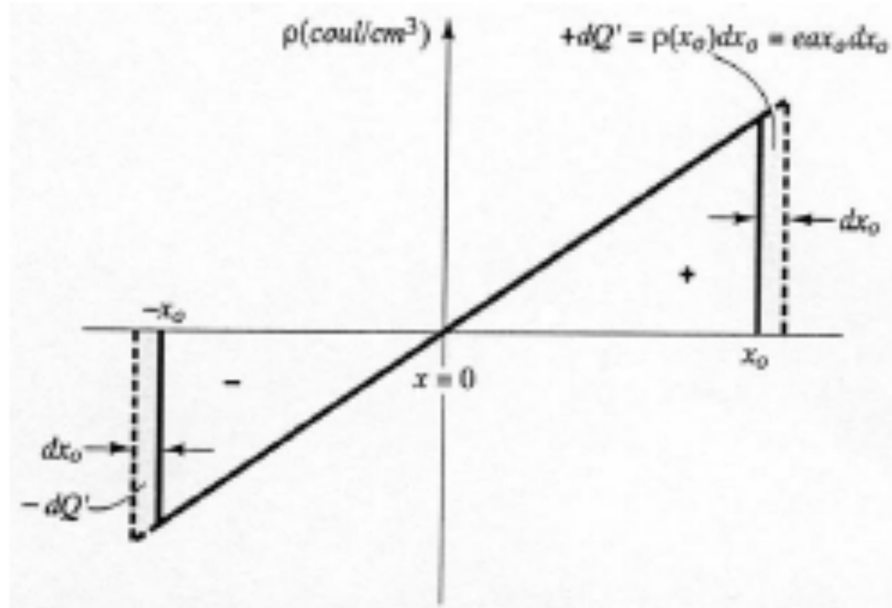
Linearly Graded Junctions

$$C' = \frac{dQ'}{dV_R} = (eax_0)\frac{dx_0}{dV_R}$$

$$= (eax_0)\frac{\varepsilon_s}{2ea}x_0^{-2}$$

$$= \frac{\varepsilon_s}{2}x_0^{-1}$$

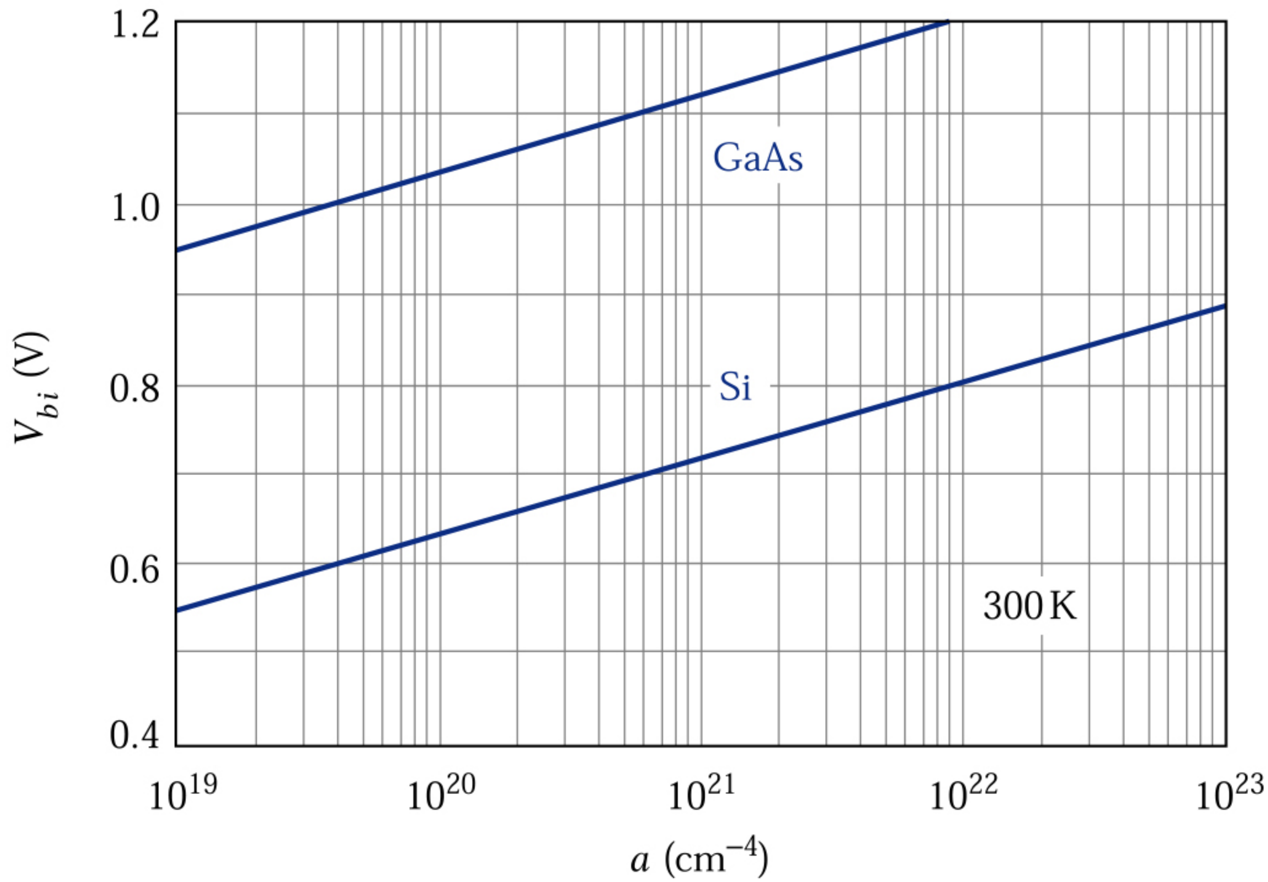
Still equivalent to plate-capacitance



$$C' = \left\{\frac{ea\varepsilon_s^2}{12(V_{bi}+V_R)}\right\}^{1/3}$$

$$C = \frac{\varepsilon_s}{2x_o} = \frac{\varepsilon_s}{W}$$

Built-in Voltage of **Linearly Graded** Junction



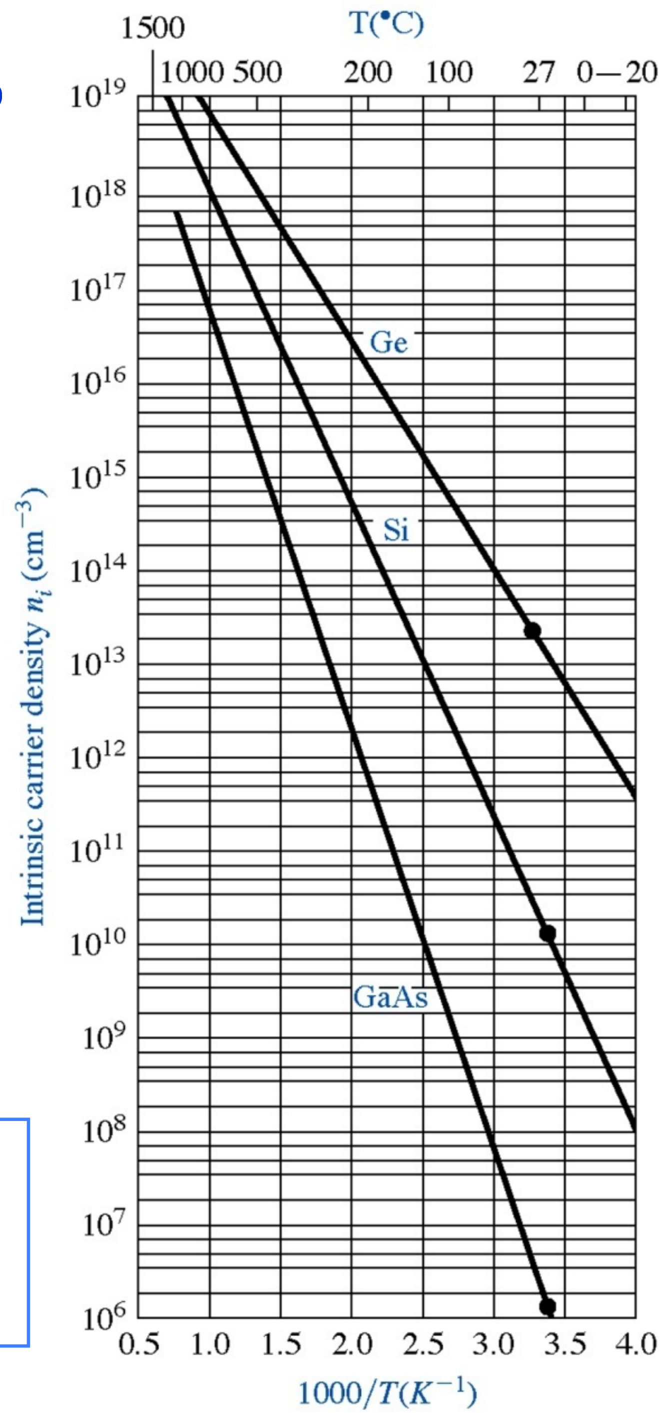
V_{bi} for GaAs much higher than for Si because of much lower intrinsic carrier concentration n_i for GaAs.

Dependence for abrupt junction

$$\boxed{V_{bi} = \psi_n - \psi_p = \frac{kT}{e}\ln\left(\frac{N_D N_A}{n_i^2}\right)}$$

n_i for Semiconductors

- Fig is a plot of form eq. (23) for silicon, gallium arsenide, and germanium as a function of temperature. As seen in the figure, the value of for these semiconductors may easily vary over several orders of magnitude as the temperature changes over a reasonable range.



Key Conclusions

- Built-in potential of p-n junction:

$$\phi_{bi} = \frac{kT}{q} \ln \frac{N_D N_A}{n_i^2}$$

- *Depletion approximation*: two quasi-neutral regions separated by a space-charge region. (SCR)

- In strongly asymmetric junction electrostatics dominated by region with lowest doping level; *i.e.* for p$^+$-n junction:

$$W \simeq \sqrt{\frac{2\epsilon\phi_{bi}}{qN_D}} \qquad |\mathcal{E}_{max}| \simeq \sqrt{\frac{2qN_D\phi_{bi}}{\epsilon}}$$

- Electrostatics out of equilibrium same as in TE if $\phi_{bi} \Rightarrow \phi_{bi} - V$.

- Depletion capacitance due to SCR width modulation:

$$C(V) = \frac{\varepsilon_s}{W(V)}$$