Homework-3, ECE 5654, Spring 2014, Due: March 7, 2014

Problem 1 (3 points)

Consider a four-phase PSK signal represented by the equivalent lowpass signal

$$u(t) = \sum_n I_n g(t - nT)$$

where I_n takes on one of the four possible values $\sqrt{\frac{1}{2}}(\pm 1 \pm j)$ with equal probability. The sequence of information symbols $\{I_n\}$ is statistically independent.

1. Determine and sketch the power density spectrum of $u(t)$ when

$$g(t) = \begin{cases} A & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

2. Repeat Part 1 when

$$g(t) = \begin{cases} A \sin(\pi t/T) & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

3. Compare the spectra obtained in Parts 1 and 2 in terms of the 3-dB bandwidth and the bandwidth to the first spectral zero.

Problem 2 (3 points)

The lowpass equivalent representation of a PAM signal is

$$u(t) = \sum_n I_n g(t - nT)$$

Suppose $g(t)$ is a rectangular pulse and

$$I_n = a_n - a_{n-2}$$

where $\{a_n\}$ is a sequence of uncorrelated binary-valued $(1, -1)$ random variables that occur with equal probability.

1. Determine the autocorrelation function of the sequence $\{I_n\}$.
2. Determine the power density spectrum of $u(t)$.
3. Repeat (2) if the possible values of the a_n are $(0, 1)$.

Problem 3 (4 points) A binary digital communication system employs the signals

$$\begin{aligned} s_0(t) &= 0 \qquad 0 \le t \le T \\ s_1(t) &= A \qquad 0 \le t \le T \end{aligned}$$

for transmitting the information. This is called *on-off signaling*. The demodulator cross-correlates the received signal $r(t)$ with $s(t)$ and samples the output of the correlator at $t + T$.

a. Determine the optimum detector for an AWGN channel and the optimum threshold, assuming that the signals are equally probable.

b. Determine the probability of error as a function of the SNR. How does on-off signaling compare with antipodal signaling?

Problem 4 (4 points) Consider a signal detector with an input

$$r = \pm A + n$$

where $+A$ and $-A$ occur with equal probability and the noise variable n is characterized by the (Laplacian) PDF shown in Figure P4.7.

1. Determine the probability of error as a function of the parameters A and σ.
2. Determine the SNR required to achieve an error probability of 10^{-5}. How does the SNR compare with the result for a Gaussian PDF?

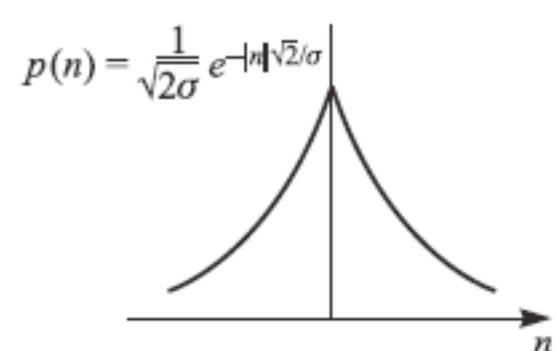
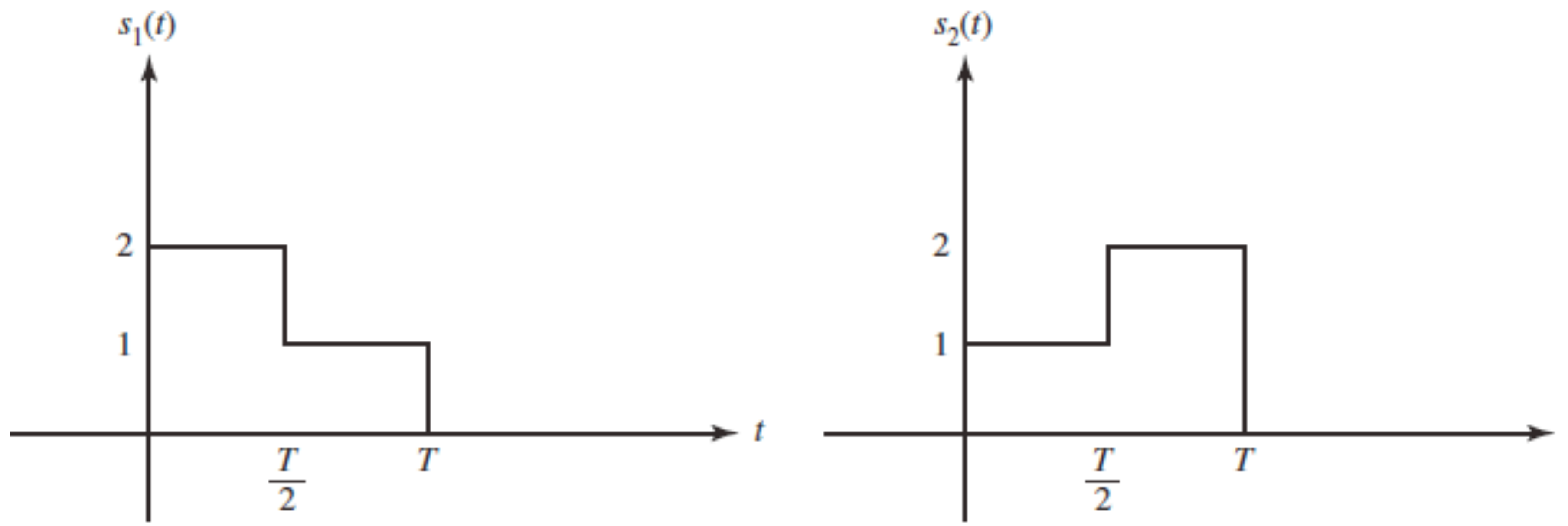


FIGURE P4.7

Problem 5 (5 points) A binary signaling scheme over an AWGN channel with noise power spectral density of $\frac{N_0}{2}$ uses the equiprobable messages shown in Figure P4.26 and is operating at a bit rate of R bits/s.



1. What is $\frac{\mathcal{E}_b}{N_0}$ for this system (in terms of N_0 and R)?
2. What is the error probability for this system (in terms of N_0 and R)?
3. By how many decibels does this system underperform a binary antipodal signaling system with the same $\frac{\mathcal{E}_b}{N_0}$?
4. Now assume that this system is augmented with two more signals $s_3(t) = -s_1(t)$ and $s_4(t) = -s_2(t)$ to result in a 4-ary equiprobable system. What is the resulting transmission bit rate?
5. Using the union bound, find a bound on the error probability of the 4-ary system introduced in part 4.