ECE 5205
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MOSFET Operation Part 2

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MOSFET Operation

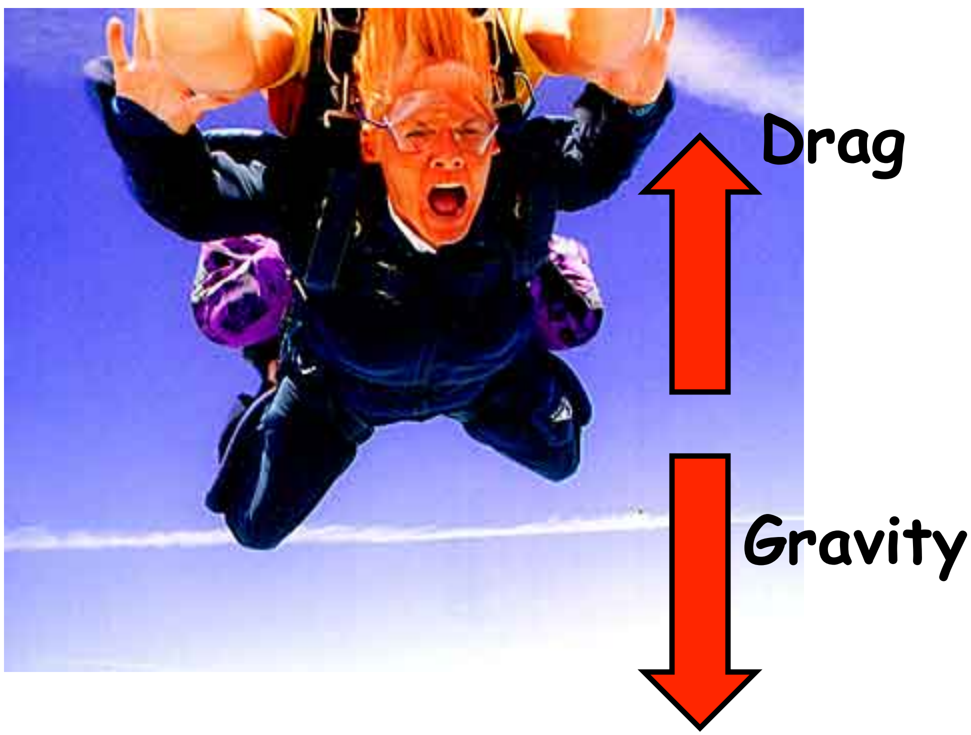
Contents

MOSFET Operation
Contents continued

- **Substrate Current**
- **Gate Current**
- **Oxide Degradation**
- **High Electric Fields at the Drain**
- **Lightly Doped Drain Engineering (LDD)**
- **Quantum Corrections in Inversion Channel**

Carrier Drift

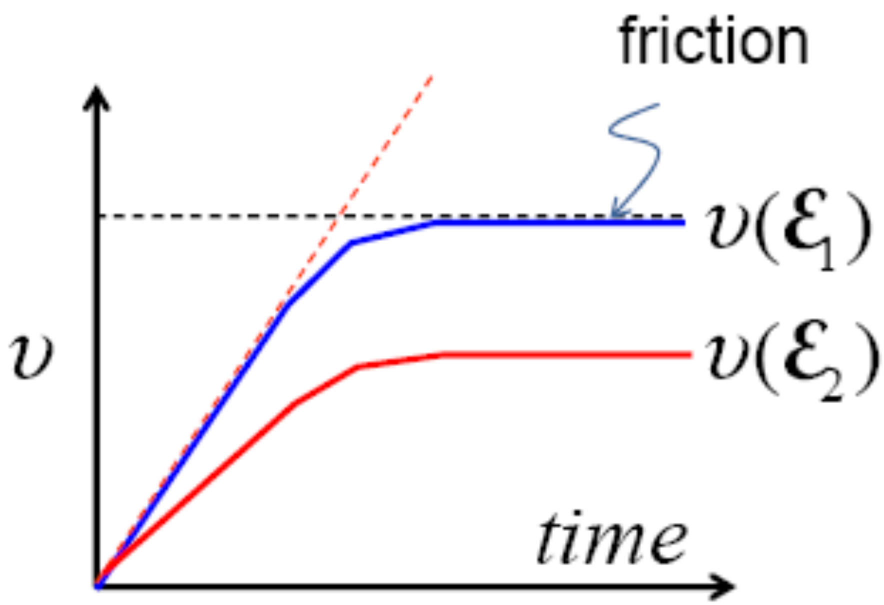
Why does a field create a velocity rather than an acceleration?



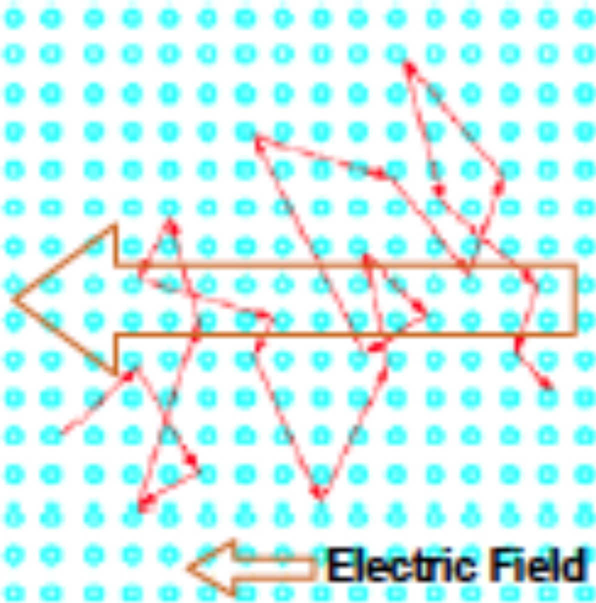
Terminal velocity

Carrier Drift

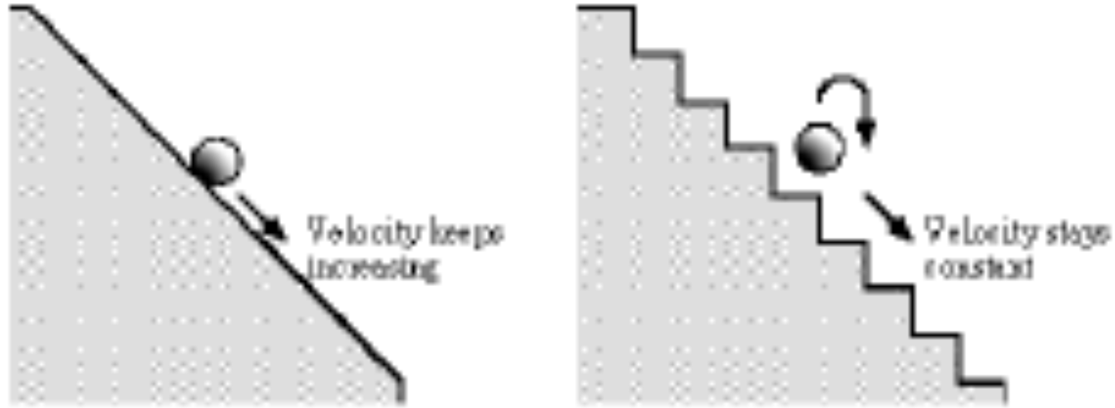
From accelerating charges to drift



Carrier Drift



Drift of electron in a solid



The ball rolling down the smooth hill speeds up continuously, but the ball rolling down the stairs moves with a constant average velocity.

Average drift velocity:

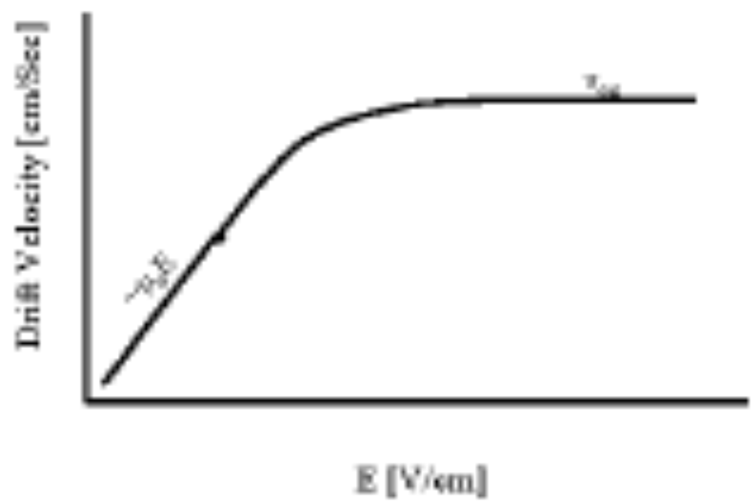
$$\langle \mathbf{v} \rangle \big|_{electron} = -\mu_n \mathbf{E}$$

$$\langle \mathbf{v} \rangle \big|_{hole} = \mu_p \mathbf{E}$$

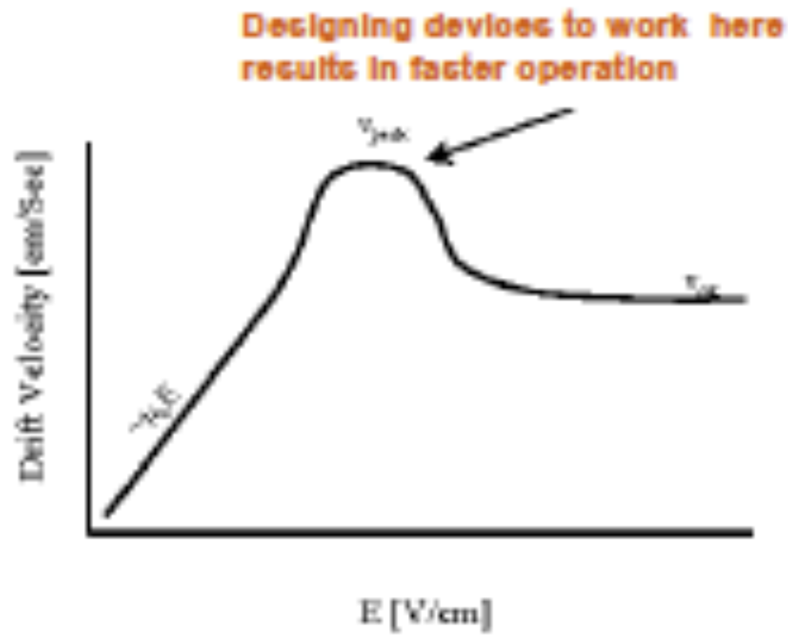
μ [$cm^2/Vsec$] : mobility

Carrier Drift

❑ Drift velocity vs. Electric field



Si and similar materials



GaAs and similar materials

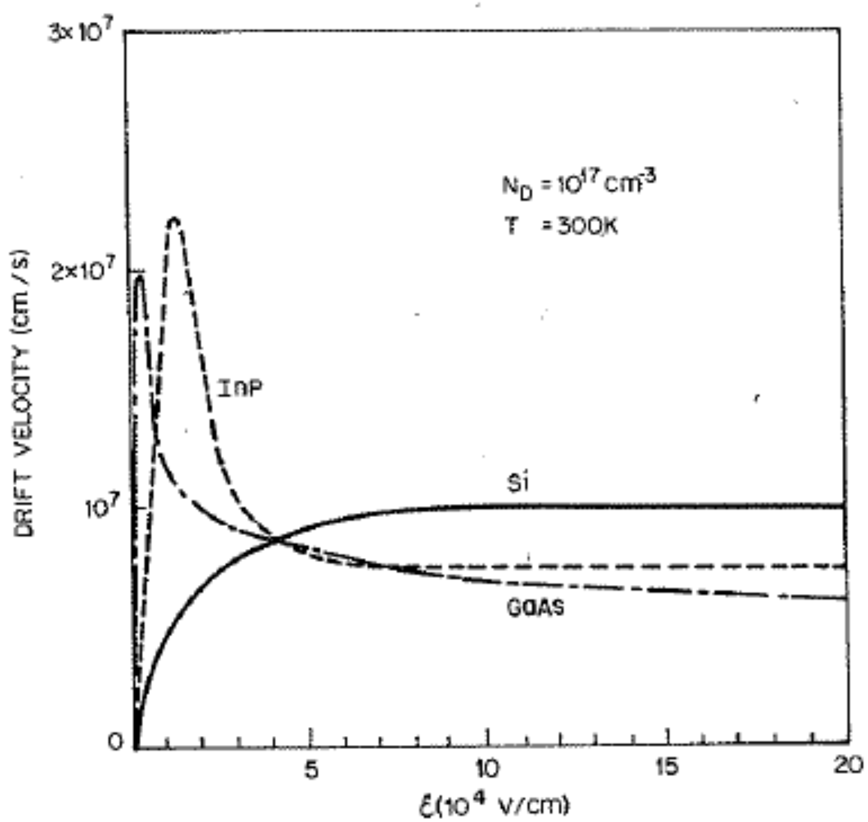
- Ohm's law is valid only in the low-field region where drift velocity is independent of the applied electric field strength.
- Saturation velocity is approximately equal to the thermal velocity (10^7 cm/s).

Saturation velocity

So far, to describe the drift velocity we have used the relation:

$$v_{drift} = \mu \cdot E$$

where mobility μ is a constant. This is only valid for relatively small electric fields and for long distances over which the drift velocity is being measured. At high fields or over a short distances the drift velocity cannot exceed a maximum value called saturation velocity. The actual drift velocities as a function of electric field are shown below:



Si

Velocity Saturation

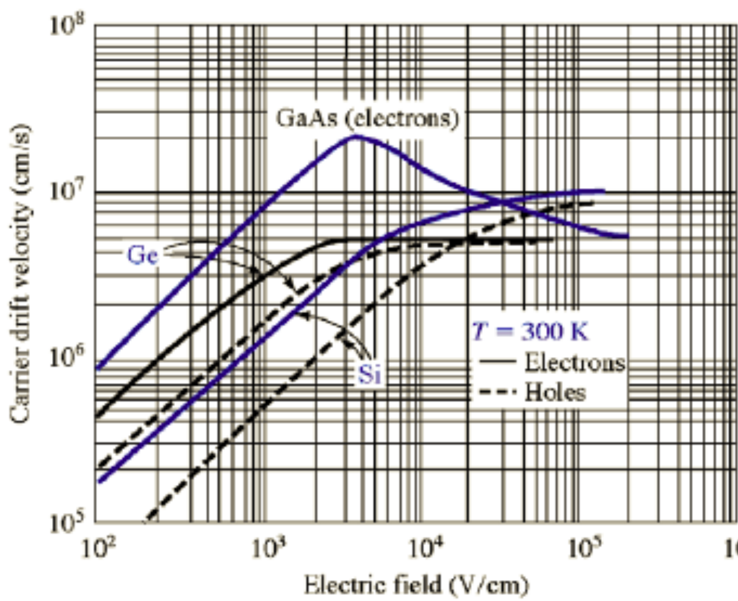
$$v_{dp} = \mu_p E$$

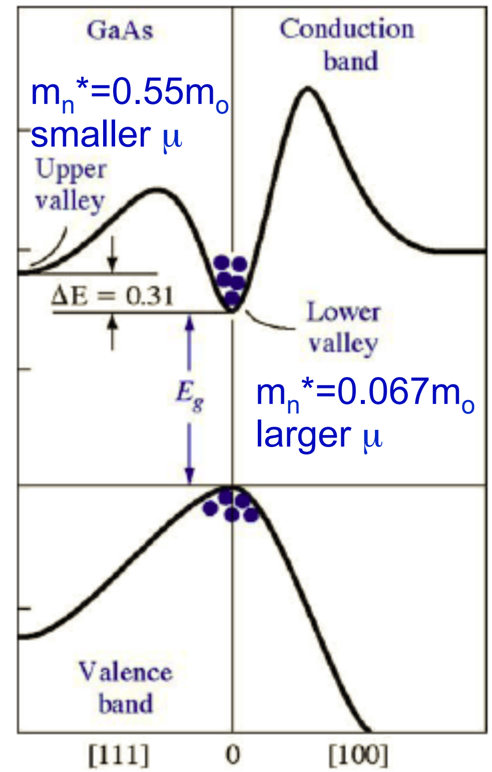
Linear dependence at fields < 30000 V/cm

At E>30K V/cm velocity saturation!

Thermal electron kinetic energy 3/2kT=0.039 eV
Si electron velocity 10^7 cm/s

Electric field 75V/cm → drift velocity $v_d=10^5$ cm/s
(1% of thermal velocity)





lower valley → large mobility
upper valley → small mobility

The negative differential mobility of GaAs produces a negative differential resistance; this characteristics is used in design of oscillators

Saturation velocity

One good (empirical) formula which describes this behavior for Si is given by

$$v_{drift} = \frac{\mu \cdot E}{1 + \mu \cdot E / v_{sat}}$$

Where v_{sat} is called the saturation velocity

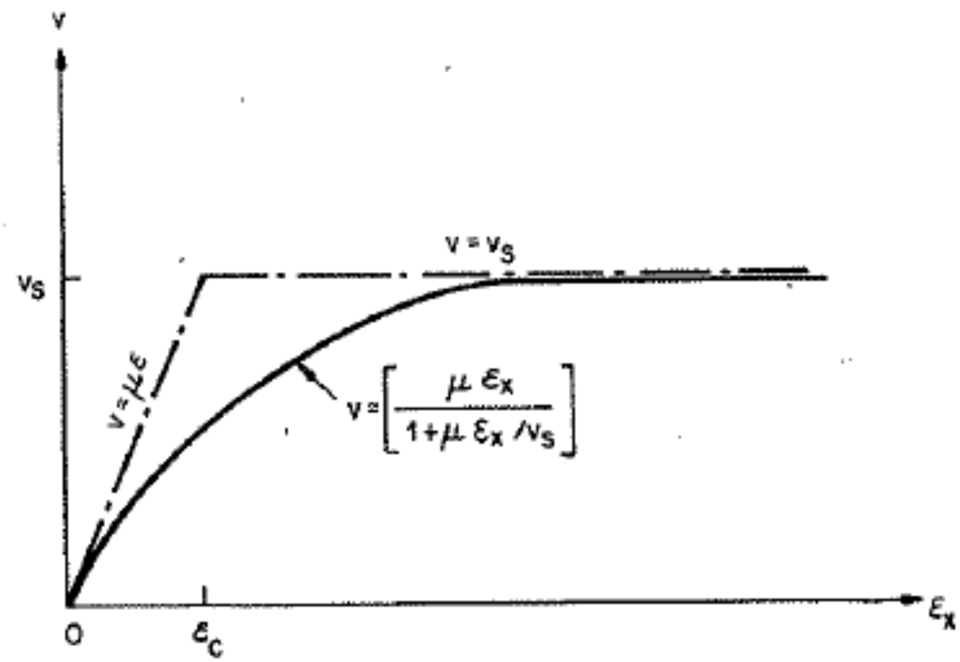


Fig. 8 Approximations for the velocity-field curves.

Saturation velocity

Saturation velocities of electrons and holes in a MOSFET channel (at the Si/SiO_2 interface) are slightly lower than their bulk values.

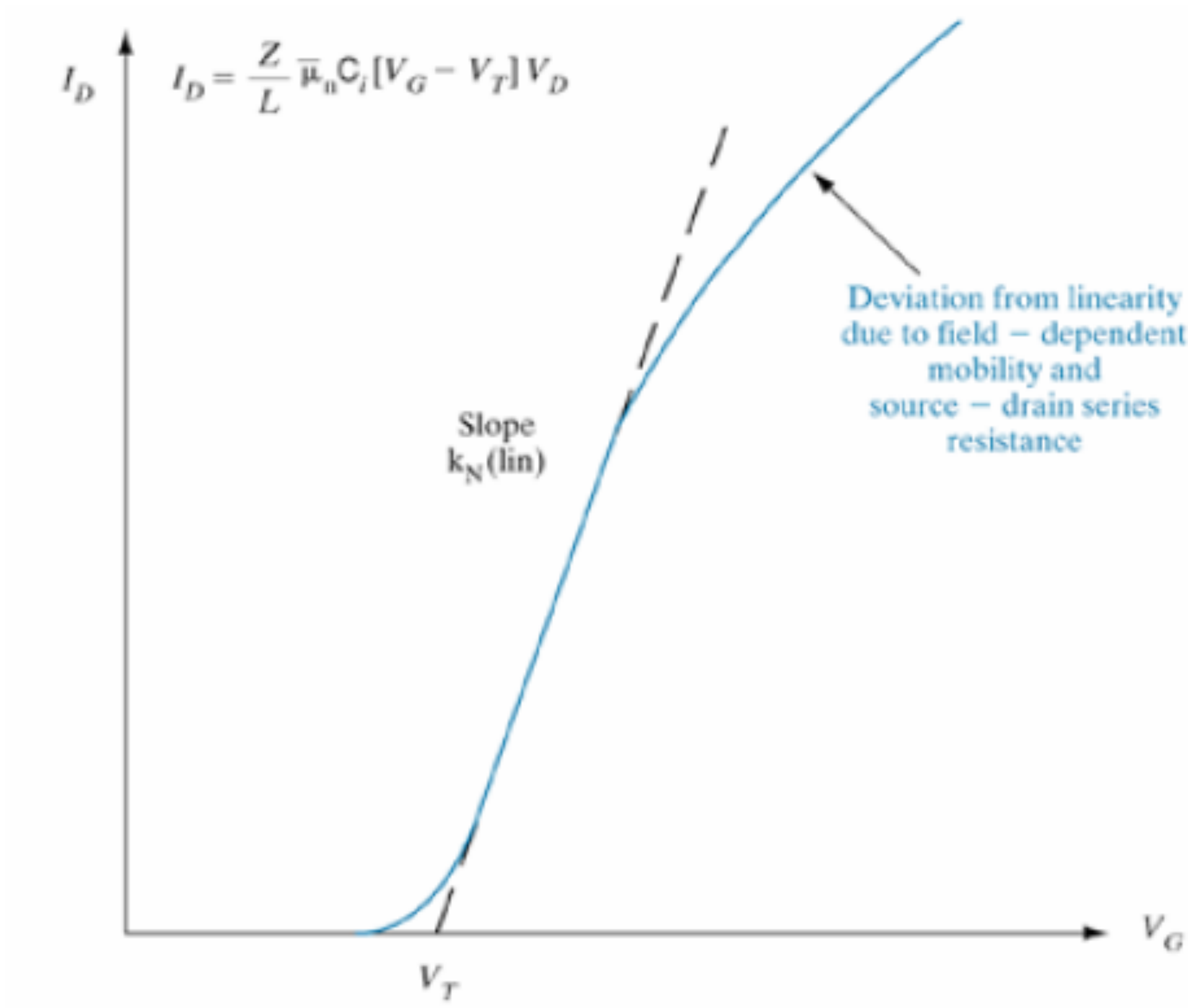
For electrons $v_{sat} \approx (7\text{-}8) \times 10^6$ cm/sec

For holes $v_{sat} \approx (6\text{-}7) \times 10^6$ cm/sec

$$v_e = \frac{\mu_e \mathcal{E}}{\left(1 + \left|\frac{\mu_e \mathcal{E}}{v_{sat}}\right|^n\right)^{1/n}}$$

	electrons	holes
$v_{sat}\ (cm/s)$	$8 \times 10^6\ cm/s$	$6 \times 10^6\ cm/s$
n	2	1

Saturation velocity’s Impact on I-V Characteristics



Various Velocity Saturation Models

a) Two – piece Model:

$$\upsilon(E) = \begin{cases} \mu_n E & \text{for } E < E_{sat} \\ \upsilon_{sat} & \text{for } E \geq E_{sat} \end{cases}$$

b) Sodini Velocity Model:

$$\upsilon(E) = \begin{cases} \dfrac{\mu_n E}{1 + E/2E_{sat}} & \text{for } E < 2E_{sat} \\ \upsilon_{sat} & \text{for } E \geq 2E_{sat} \end{cases}$$

c) continuous $\upsilon - E$ Model:

$$\upsilon(E) = \frac{\mu E}{\left[1 + \left(E/E_{sat}\right)^m\right]^{1/m}} \quad (m = 2 \text{ for NMOS; } m = 1 \text{ for PMOS})$$

Substituting $\upsilon(E)$ into the I_{ds} current model.

MOSFET *I-V* with Velocity Saturation

$$I_{DS} = \frac{\frac{W}{L} C_{oxe} \mu_{eff} (V_{GS} - V_T - \frac{m}{2} V_{DS}) V_{DS}}{1 + \frac{V_{DS}}{\mathcal{E}_{sat} L}}$$

$$\boxed{I_{DS} = \frac{long\text{-}channel\ I_{DS}}{1 + V_{DS} / \mathcal{E}_{sat} L}}$$

$$v = \frac{\mu \mathcal{E}}{1 + \frac{\mathcal{E}}{\mathcal{E}_{sat}}}$$

$$\mathcal{E} << \mathcal{E}_{sat} : v = \mu \mathcal{E}$$

$$\mathcal{E} >> \mathcal{E}_{sat} : v = \mu \mathcal{E}_{sat}$$

Correction of V_{dsat} due to Velocity Saturation

Solving for $\boxed{\dfrac{dI_{DS}}{dV_{DS}} = 0,}$

$$V_{Dsat} = \frac{2(V_{GS} - V_T)}{1 + \sqrt{1 + 2(V_{GS} - V_T)/\mathcal{E}_{sat} L}}$$

A simpler and more accurate V_{Dsat} is

$$\boxed{\frac{1}{V_{Dsat}} = \frac{m}{V_{GS} - V_T} + \frac{1}{\mathcal{E}_{sat} L}}$$

$m = 1 + 3T_{oxe}/W_{dm}$

$$\mathcal{E}_{sat} \equiv \frac{2v_{sat}}{\mu}$$

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$$\mathcal{E}_{sat} \equiv \frac{2v_{sat}}{\mu}$$

$$\frac{1}{V_{Dsat}} = \frac{m}{V_{GS} - V_{Tn}} + \frac{1}{\mathcal{E}_{sat} L}$$

- If $\mathcal{E}_{sat} L >> V_{GS}$-$V_{Tn}$ then the MOSFET is considered "long-channel". This condition can be satisfied when
  - L is large, or
  - V_{GS} is close to V_T

I_{dsat} with Velocity Saturation

Substituting V_{Dsat} for V_{DS} in I_{DS} equation gives:

$$\boxed{I_{Dsat} = \frac{W}{2mL} C_{oxe}\mu_{eff} \frac{(V_{GS}-V_T)^2}{1+\dfrac{V_{GS}-V_T}{\mathcal{E}_{sat}L}} = \frac{long\text{-}channel\ I_{Dsat}}{1+\dfrac{V_{GS}-V_T}{\mathcal{E}_{sat}L}}}$$

Very short channel case: $\mathcal{E}_{sat}L << V_{GS}-V_T$

$$I_{Dsat} = \frac{W}{2m} C_{oxe}\mu_n \mathcal{E}_{sat}(V_{GS}-V_T)$$
$$= Wv_{sat}C_{oxe}(V_{GS}-V_T)/m$$

- I_{Dsat} is proportional to $V_{GS}-V_T$ rather than $(V_{GS}-V_T)^2$

Note that there is now no dependence on channel length

Summary of the Impact of Velocity Saturation

- Linear region:

$$I_{DS} = \frac{\frac{W}{L} C_{oxe} \mu_{eff} (V_{GS} - V_{Tn} - \frac{m}{2} V_{DS}) V_{DS}}{1 + \frac{V_{DS}}{\mathcal{E}_{sat} L}}$$

$$\mathcal{E}_{sat} = 2 v_{sat} / \mu_{eff}$$

$$v_{sat} = \begin{cases} 8 \times 10^6 \text{ cm/s for electrons} \\ 6 \times 10^6 \text{ cm/s for holes} \end{cases}$$

- Saturation region:

$$I_{DS} = I_{Dsat} = \frac{\frac{W}{2mL} C_{oxe} \mu_{eff} (V_{GS} - V_{Tn})^2}{1 + \frac{(V_{GS} - V_{Tn})}{\mathcal{E}_{sat} L}}$$

Summary : Very Short Channel with Velocity Saturation

- If $\mathcal{E}_{sat}L << V_{GS}\text{-}V_{Tn}$:

$$V_{Dsat} \cong \mathcal{E}_{sat}L < \frac{(V_{GS} - V_{Tn})}{m}$$

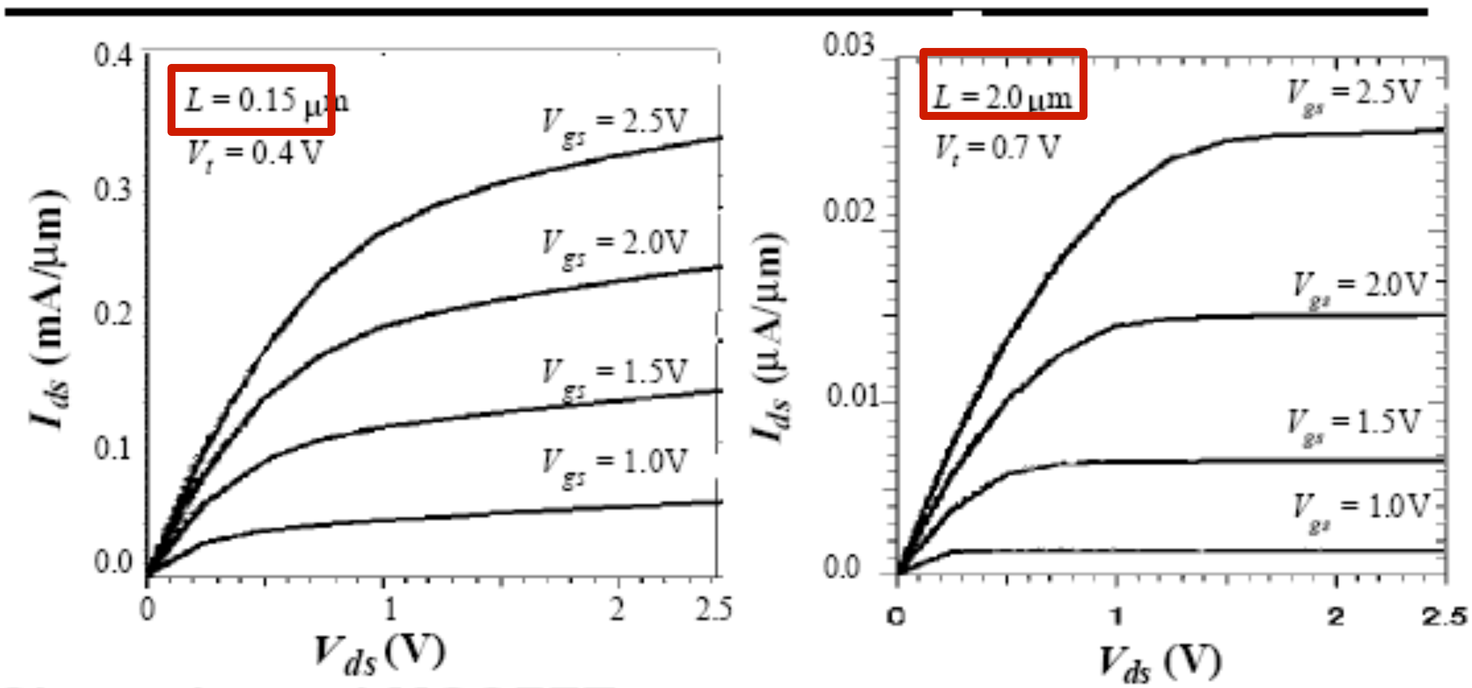
$$I_{Dsat} = \frac{W}{2m} C_{oxe} \mu_{eff} \mathcal{E}_{sat} (V_{GS} - V_{Tn})$$

$$= \frac{W}{m} C_{oxe} v_{sat} (V_{GS} - V_{Tn})$$

$\Rightarrow$ I_{Dsat} is not sensitive to L

- **To increase I_{Dsat}** (for faster circuit operation), we must increase $C_{oxe}(V_{GS}\text{-}V_{Tn})$, *i.e.* **reduce T_{oxe} and V_{Tn}**

Short vs Long-Channel MOSFET



Short-channel MOSFET:

- I_{Dsat} is proportional to V_{GS}-V_{Tn} rather than $(V_{GS}-V_{Tn})^2$
- V_{Dsat} is lower than for long-channel MOSFET
- Channel-length modulation is apparent to be addressed later In a quantitative way

Velocity Overshoot

- When L is comparable to or less than the mean free path, some of the electrons travel through the channel without experiencing a single scattering event

  $\rightarrow$ projectile-like motion ("**ballistic transport**")

$\Rightarrow$ The average velocity of carriers exceeds v_{sat}

e.g. 35% for L = 0.12 μm NMOSFET

$\Rightarrow$ Effectively, v_{sat} and $\mathcal{E}_{sat}$ increase when L is very small

Carrier Drift and Mobility in General

Total drift current density:

$$\boxed{J_{drf} = e(\mu_n n + \mu_p p)\mathrm{E}} \quad [\mathrm{A/cm^2}]$$

Table 5–1 Typical mobility values at $T = 300°\mathrm{K}$ and low doping concentrations

	μ_n (cm²/V-sec)	μ_p (cm²/V-sec)
Silicon	1350	480
Gallium arsenide	8500	400
Germanium	3900	1900

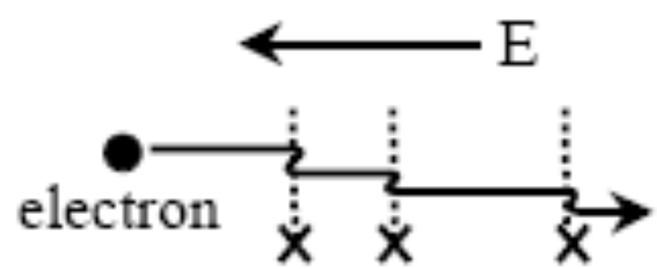
Electron and Hole Acceleration in Electric Field

Thermal Motion: Note that the carrier thermal-velocity is very large (~10^7 cm/s at 300 K), but this does not contribute to current transport. Why?

Random thermal motion of carriers

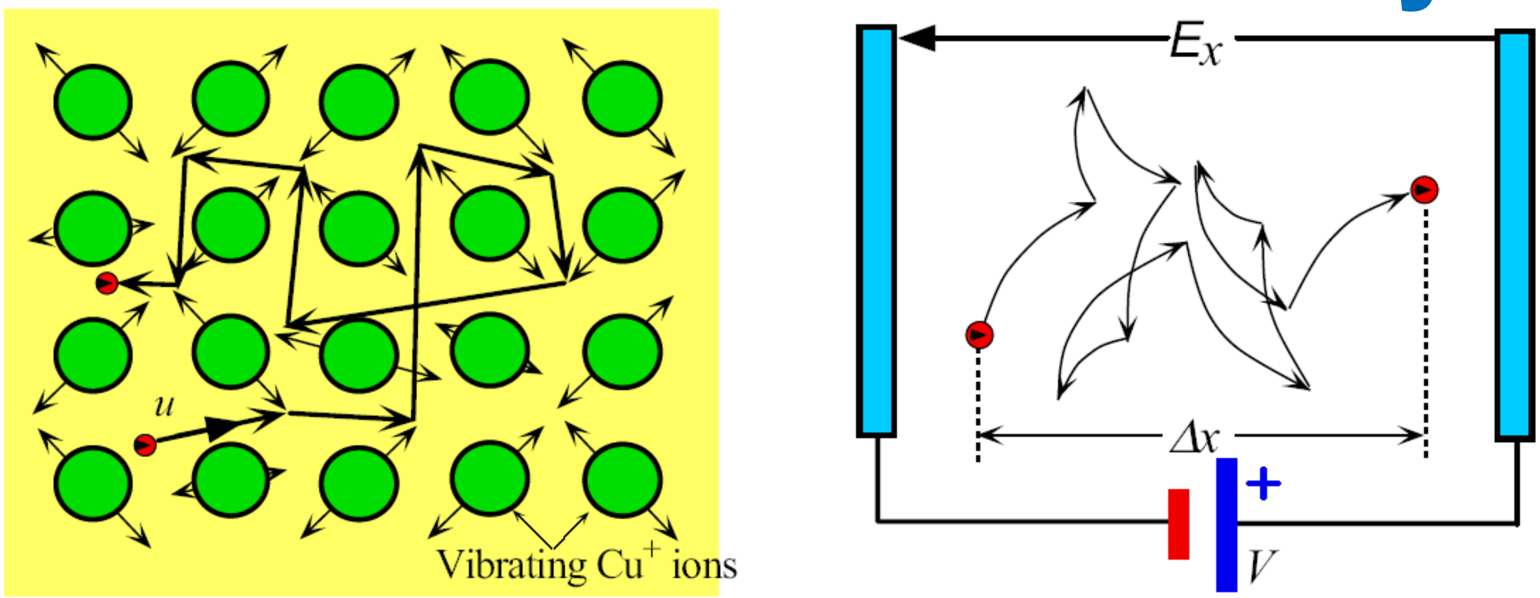
$$\frac{1}{2} m^* v_{th}^2 = \frac{3}{2} kT$$

$$v_{th} = \sqrt{3kT / m^*}$$



collision with lattice

Carrier Drift and Mobility



(a) (b)

(a) A conduction electron in the electron gas moves about randomly (with a mean speed *u*) being frequently and randomly scattered by thermal vibrations of the atoms. In the absence of an applied field there is no net drift in any direction.

(b) In the presence of an applied field, E_x, there is a net drift along the *x*-direction. This net drift along the force of the field is superimposed on the random motion of the electron. After many scattering events the electron has been displaced by a net distance, Δx, from its initial position toward the positive terminal.

Electron and Hole Acceleration in Electric Field

$$F = m_p^* a = e\mathrm{E} \quad \Longrightarrow \quad F = m_p^* \frac{dv}{dt} = e\mathrm{E}$$

Integrate (mass and electric field are time-independent)

v : particle velocity due to electric field (not include the random thermal velocity)

$$v = \frac{e\mathrm{E}t}{m_p^*}$$

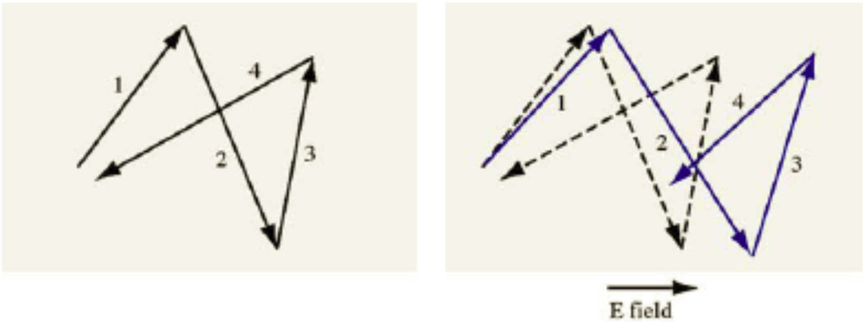
mean peak velocity : $v_{d|\mathrm{peak}} = \left(\dfrac{e\tau_{cp}}{m_p^*}\right)\mathrm{E}$

Just before collision

τ_{cp} ; mean time between collision

average drift velocity :

$$\langle v_d \rangle = \frac{1}{2}\left(\frac{e\tau_{cp}}{m_p^*}\right)\mathrm{E}$$



Fundamental Expression for Mobility

$$v_{drift} = \frac{1}{2}\left(\frac{e\tau_p}{m_p^*}\right)E \quad \rightarrow \quad v_{drift} = \mu \cdot E$$

$$\mu = \frac{q\tau}{m_{eff}}$$

Mean time between collisions

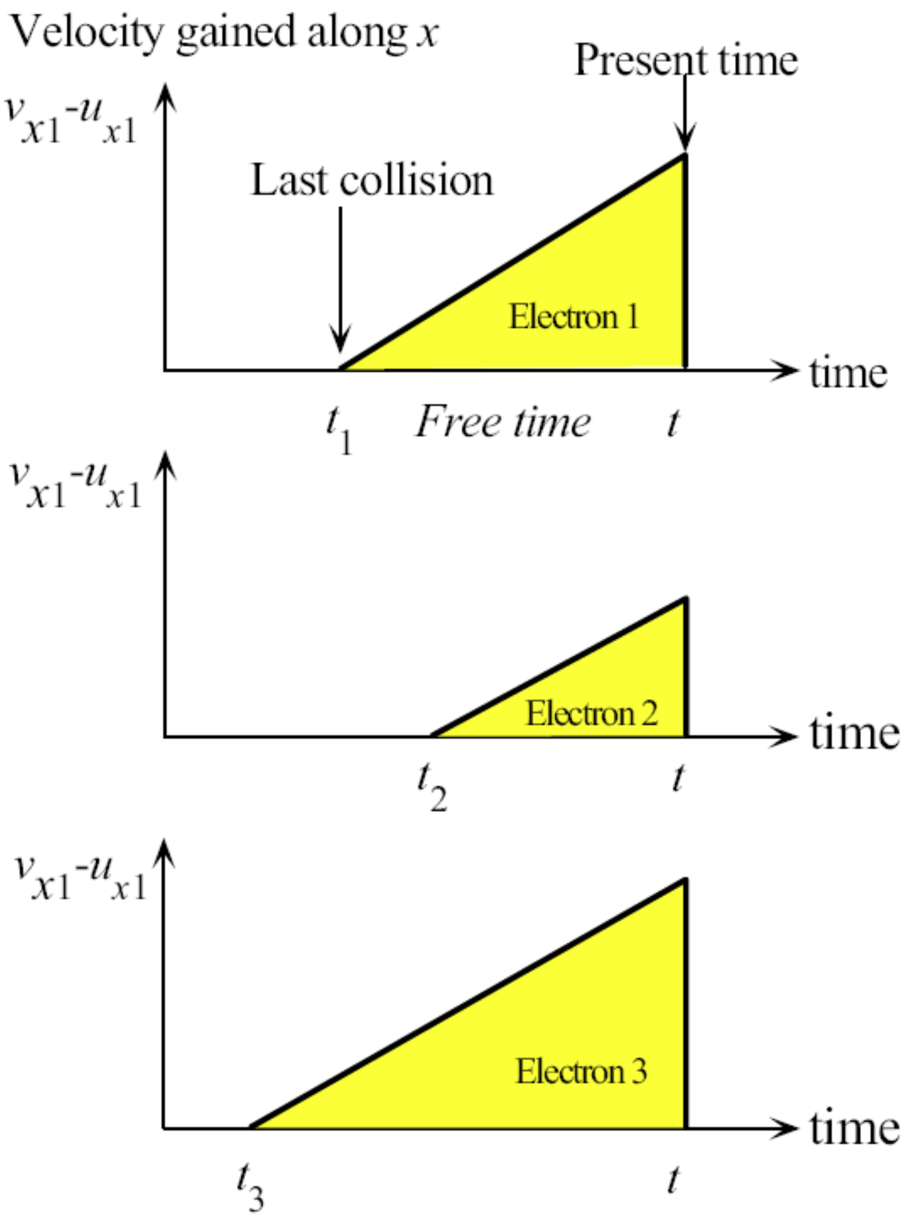
Electron or hole effective mass

This is a material issue

$$\tau = \frac{\lambda}{V_{th}}$$

λ *mean free path*

Acceleration in Electric Field



$$v_{xi} = u_{xi} + \frac{e\mathcal{E}_x}{m_e}(t - t_i)$$

Velocity gained in the *x* direction at time t from the electric field (E_x) for three electrons. There will be *N* electrons to consider in the semiconductor.

Matthiessen's Rule

4-3-2014

In general, an electron may be scattered by several mechanisms, so the effective mean free time between any two scattering events will be less than the individual scattering times τ_i and τ_j. Since in unit time, $1/\tau$ is the net probability of scattering, $1/\tau_i$ is the probability of scattering from mechanism i alone, and $1/\tau_j$ is the probability of scattering from mechanism j alone, then according to the probability laws, we have

$$\frac{1}{\tau} = \frac{1}{\tau_i} + \frac{1}{\tau_j}$$

or in general

$$\frac{1}{\tau} = \sum_i \frac{1}{\tau_i}$$

Therefore for mobility we obtain

$$\frac{1}{\mu} = \frac{1}{\mu_i} + \frac{1}{\mu_j}$$

σ $conductivity$

The last equation can be rewritten in the following way

$$\frac{1}{\sigma} = \frac{1}{en\mu} = \frac{1}{en\mu_i} + \frac{1}{en\mu_j} = \frac{1}{\sigma_i} + \frac{1}{\sigma_j}$$

Since $\rho_i = \frac{1}{\sigma_i} = \frac{1}{en\mu_i}$ and $\rho_j = \frac{1}{\sigma_j} = \frac{1}{en\mu_j}$

ρ $resistivity$

We can write $\rho = \rho_i + \rho_j$ or in general: $\rho = \sum_i \rho_i$

The summation rule of resistivities from different scattering mechanisms is called **Matthiessen's Rule**.

Background: Probability of Scattering

If τ_L is the mean time between collisions due to lattice scattering, then dt/τ_L is the probability of a lattice scattering event occurring in a differential time dt. Likewise, dt/τ_I is the probability of an ionized impurity scattering event occurring in the differential time dt with τ_I being the mean time between collisions due to ionized impurity scattering. If these two scattering processes are independent, then the total probability of a scattering event occurring in the differential time dt is the sum of the individual events:

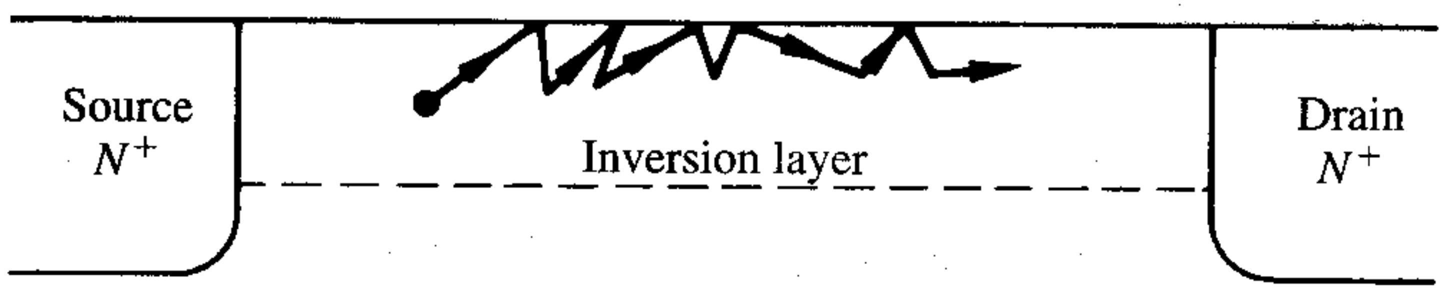
$$\frac{dt}{\tau} = \frac{dt}{\tau_I} + \frac{dt}{\tau_L}$$

τ is the mean time between any scattering event. From the definitions of mobility, we can write:

$$\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L}$$

With two or more independent scattering mechanisms, the inverse mobilities add, which means that the net mobility decreases.

Inversion Channel Mobility



At the Si/SiO2 interface we have an additional scattering mechanism: surface scattering!

Mobility and Carrier Scattering: Dependence on Temperature and Doping

Mobility of hole : $\boxed{\mu_p = \dfrac{v_{dp}}{\mathrm{E}} = \dfrac{e\tau_{cp}}{m_p^*}}$ (including the effect of a statistical distribution)

Mobility of electron : $\boxed{\mu_n = \dfrac{e\tau_{cn}}{m_n^*}}$ τ_{cn}; mean time between collision for an electron

Scattering mechanism

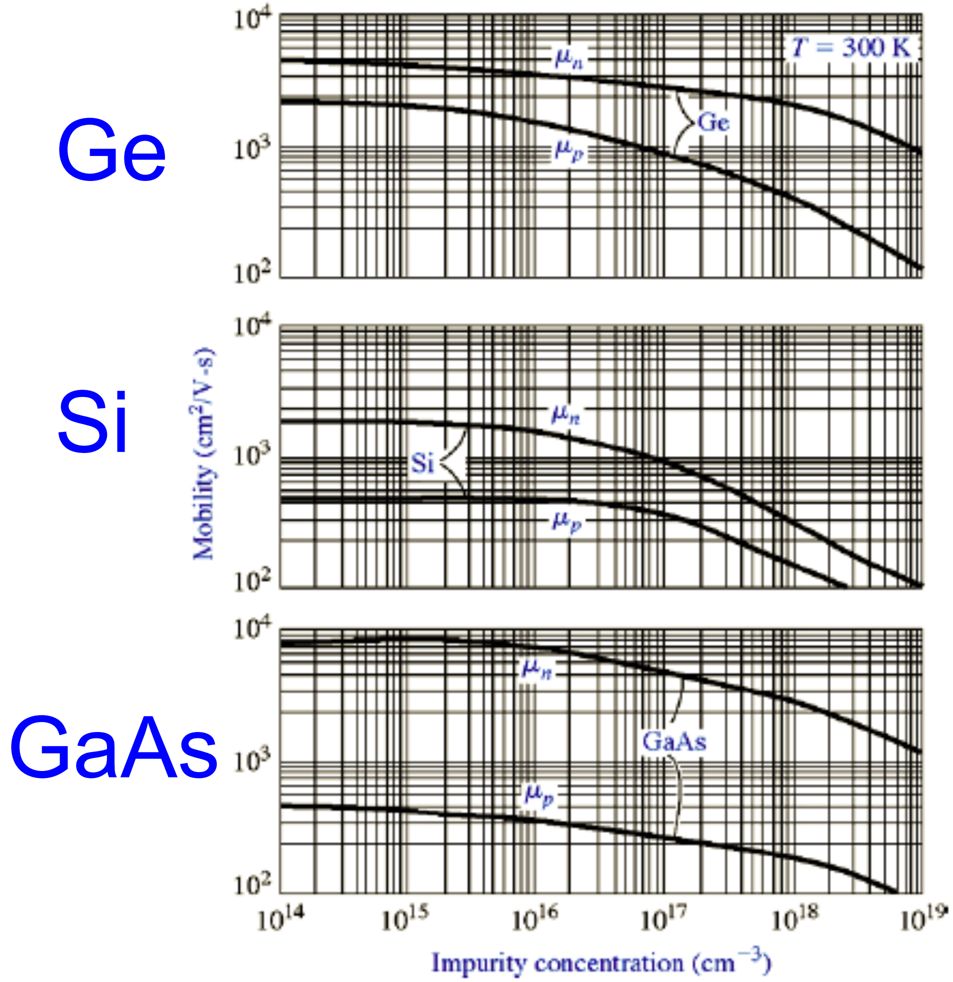
Phonon(lattice) scattering : $\mu_L \propto T^{-3/2}$ theory, experiment T^{-n}

Ionized impurity scattering : $\mu_I \propto \dfrac{T^{+3/2}}{N_I}$ $(N_I = N_d^+ + N_a^-)$

At higher T, v_{th} increases and on average electrons spend less time in the vicinity of ionized impurity.

$$\mu = \mu(N_I, T)$$

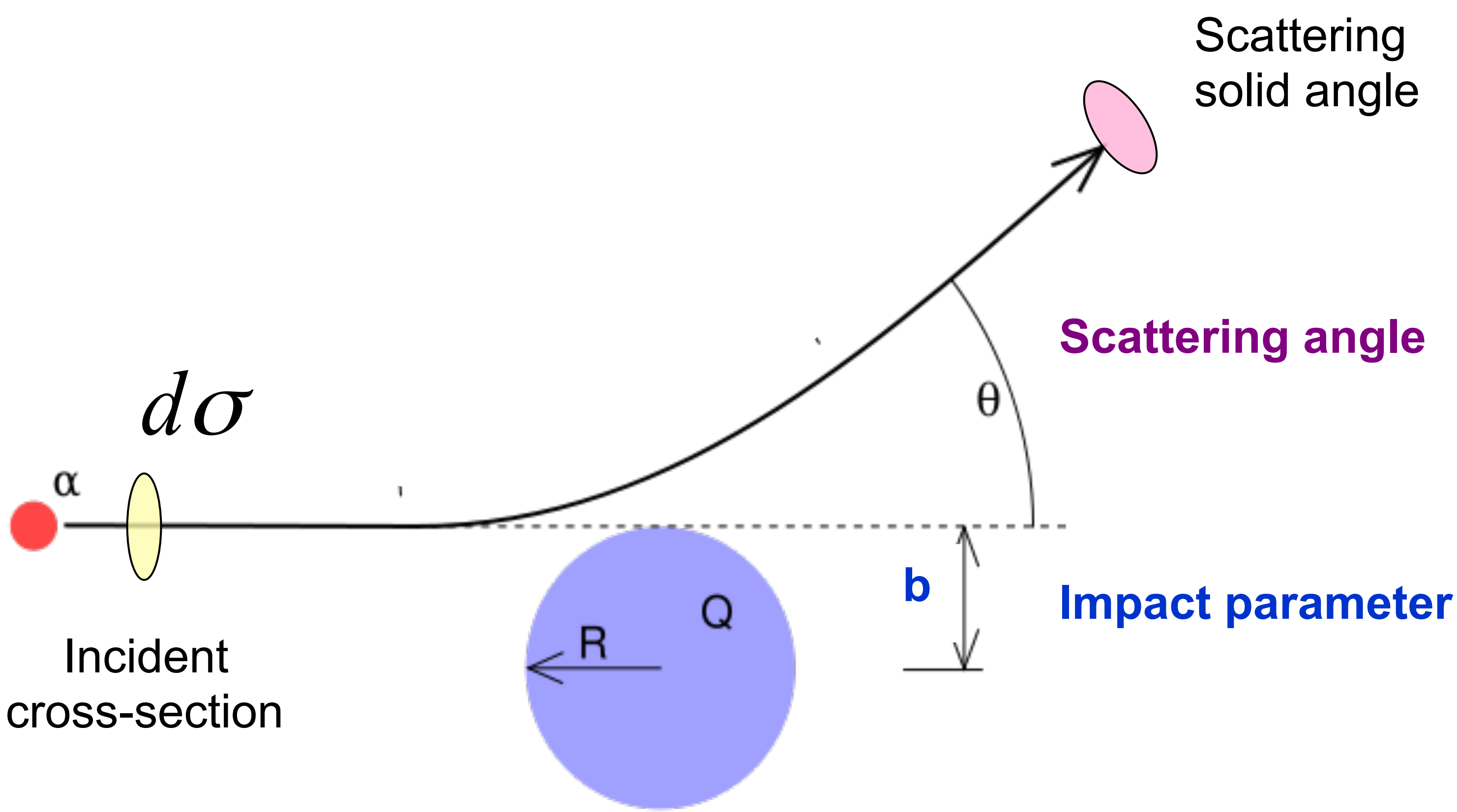
Mobility versus Impurity Conc. @T=300K
For different semiconductors



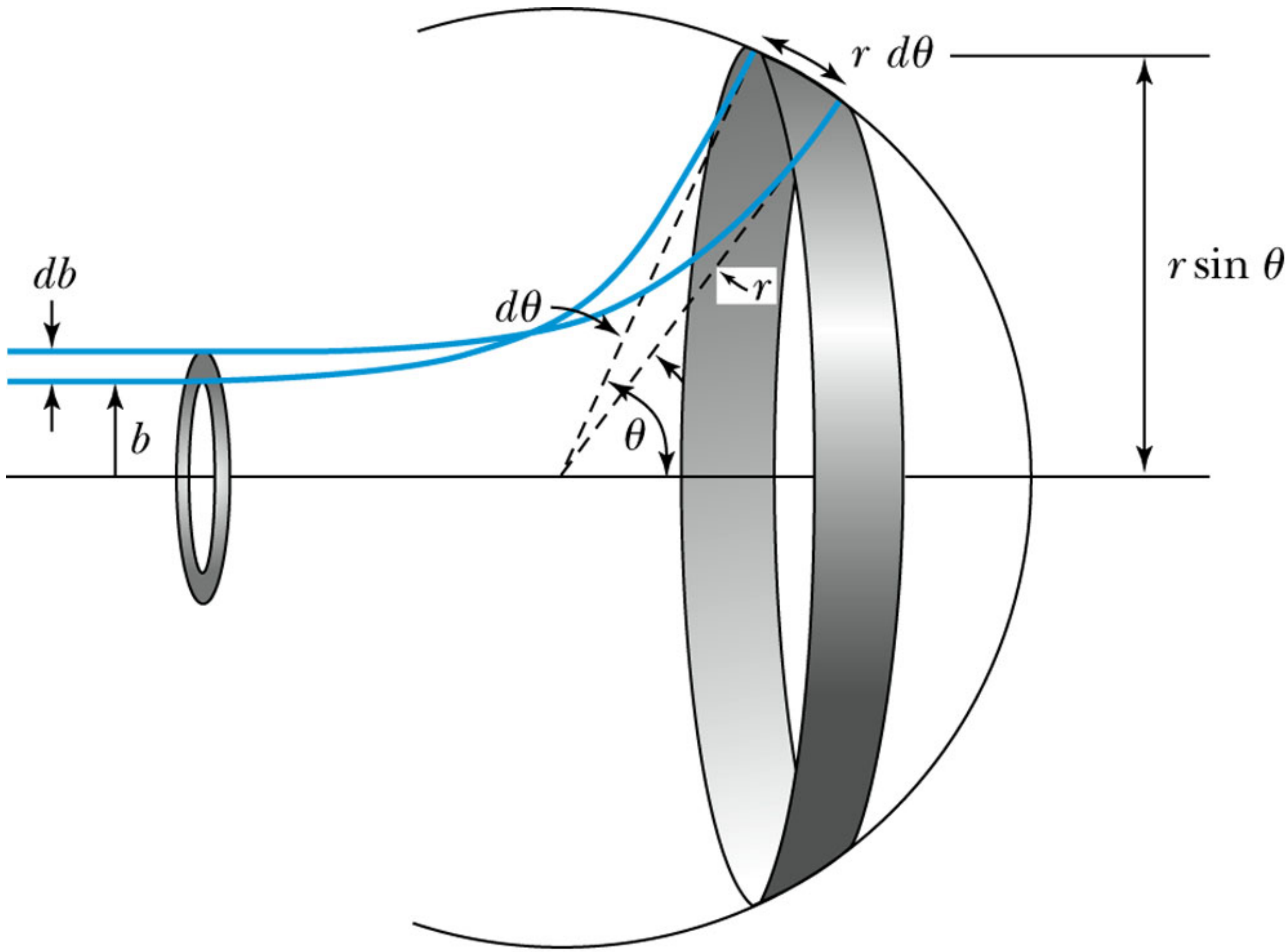
Electron and hole mobilities versus impurity concentrations.

As the *impurity concentrations increases*, the number of impurity scattering centers increases, thus *reducing mobility*.

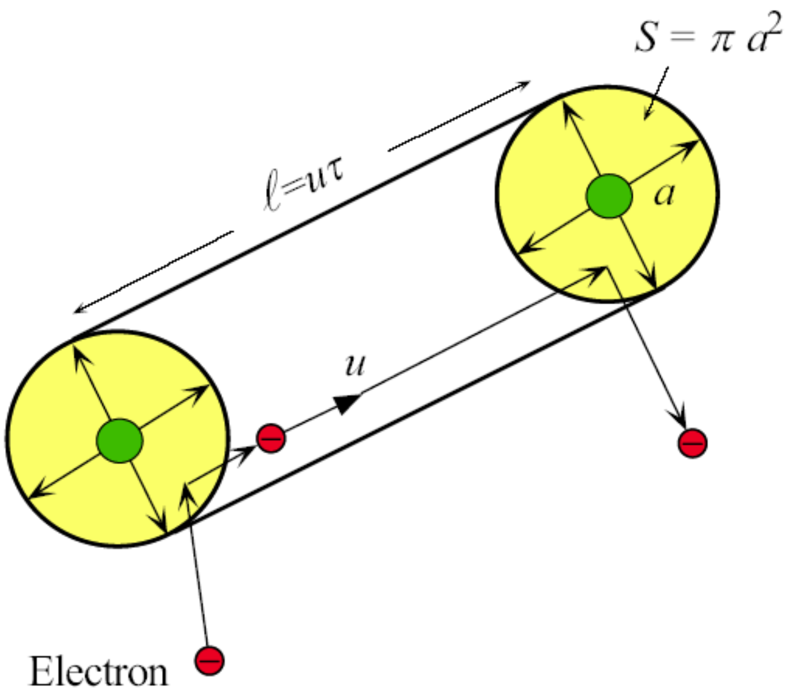
Scattering characterized by a Cross Section



Cross Section in Scattering



Scattering Mechanisms and Collision Time τ



Scattering of an electron from a scattering center.

The electron travels a mean distance $\ell = u\tau$ between collisions. Since the scattering cross-sectional area is S, in the volume $S\ell$ there must be at least one scatterer, $N_s\,(Sv_{th}\tau) = 1$.

$$\tau = \frac{1}{S v_{th} N_s}$$

cross section $S=\pi r_c^2$
v_{th} mean speed, thermal velocity
N_s number of scatterers

Temperature Dependence of Mobility due to lattice vibrations

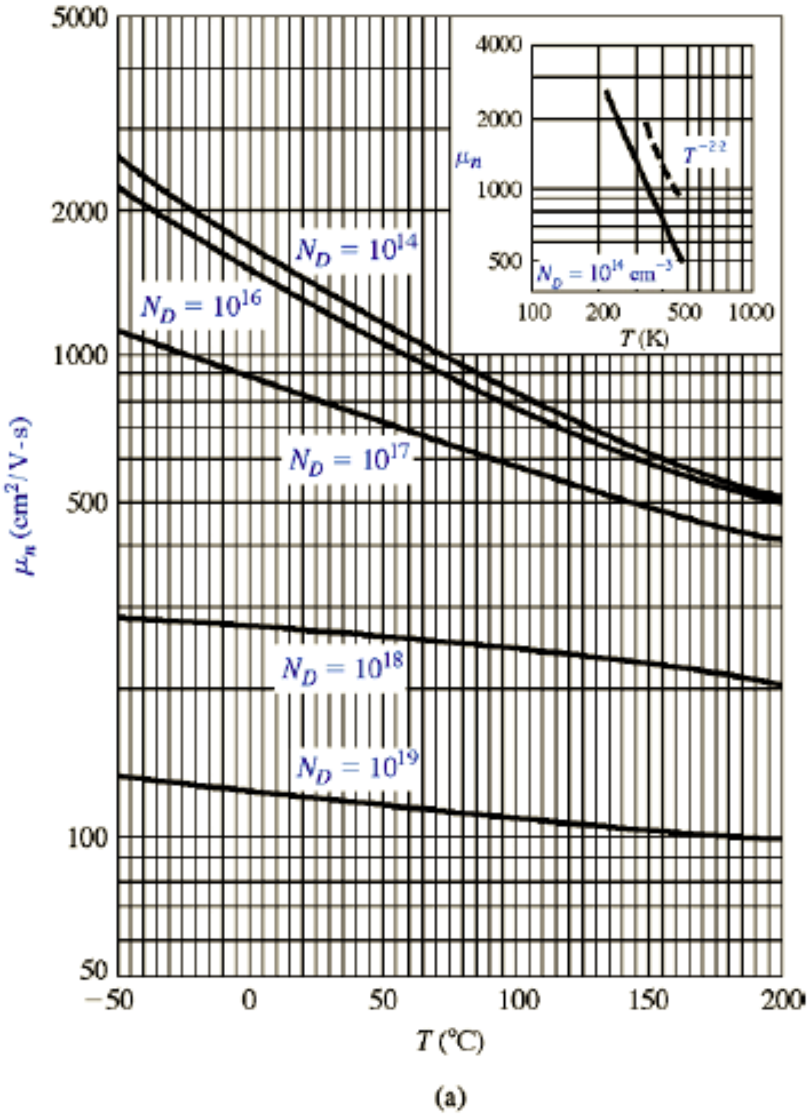
$$\mu_L = \frac{e\tau}{m_{eff}}$$

Quantized lattice vibration = = phonons

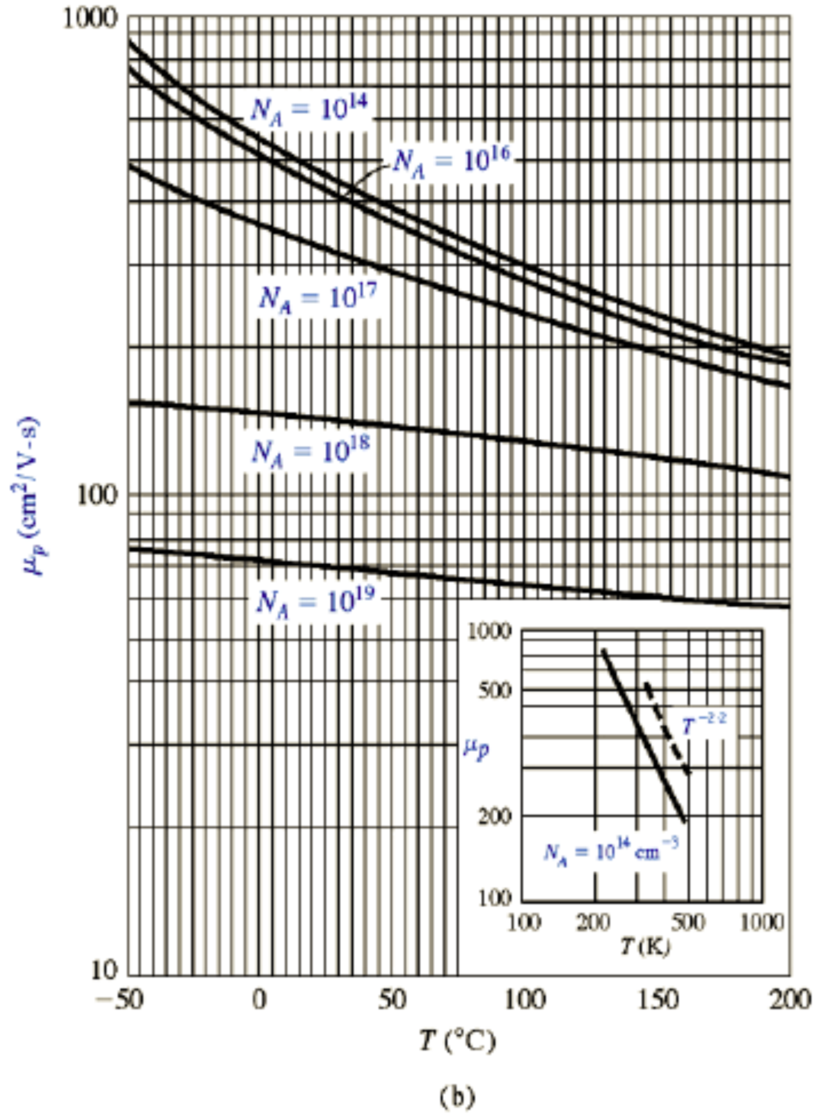
The mean free time τ is inversely proportional to the speed of the electrons and to the scattering cross section πa^2. The cross section radius a is given by the amplitude of lattice vibrations a. Recall that that potential energy of an oscillator is given by **$(1/2)k_s x^2 \sim (1/2)kT$**. Here k_s is the spring constant. Therefore $a^2 \sim T$. The thermal velocity is proportional to $T^{1/2}$ so overall we have:

$$\tau \propto \frac{1}{v_{th}(\pi a^2)} \propto T^{-3/2} \longrightarrow \mu_L \propto T^{-3/2}$$

Mobility as a Function of Temperature and Doping



(a) electron mobility



(b) hole mobility

Temperature Dependence of Mobility **due to impurity scattering**

$$\mu = \frac{e\tau}{m_{eff}} \qquad \tau = \frac{1}{S v_{th} N_s}$$

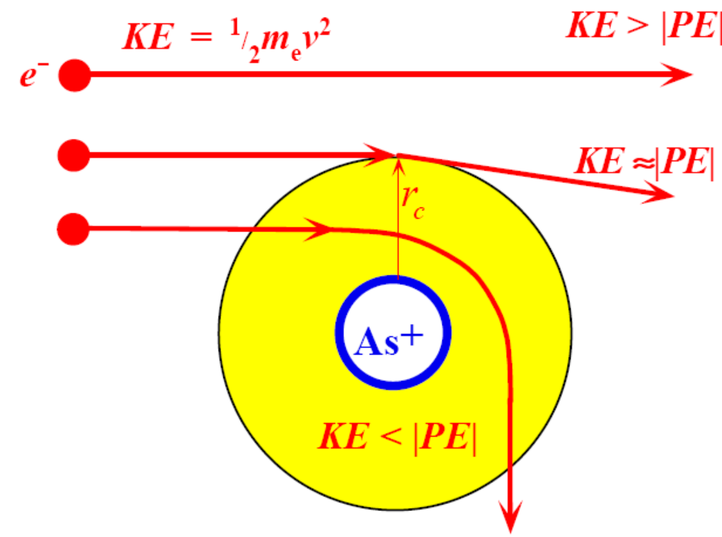
cross section $S=\pi r_c^2$
v_{th} mean speed, thermal velocity
N_s number of scatterers / volume unit

Consider scattering caused by As^+ ion. The Coulombic potential energy is:

$$|PE| = \frac{e^2}{4\pi\varepsilon_r\varepsilon_o r}$$

For high KE the electron does not feel the Coulombic pull. However, if KE is smaller than PE, then the Coulombic force will lead to a strong deflection. The critical distance from the As^+ is r_c at which KE=PE(r_c). But KE=3/2kT. So we obtain:

$$\frac{3}{2}kT = |PE(r_c)| = \frac{e^2}{4\pi\varepsilon_r\varepsilon_o r_c}$$



We can solve this equation for r_c.
to get the scatterin cross-section

Temperature Dependence of Mobility due to impurity scattering

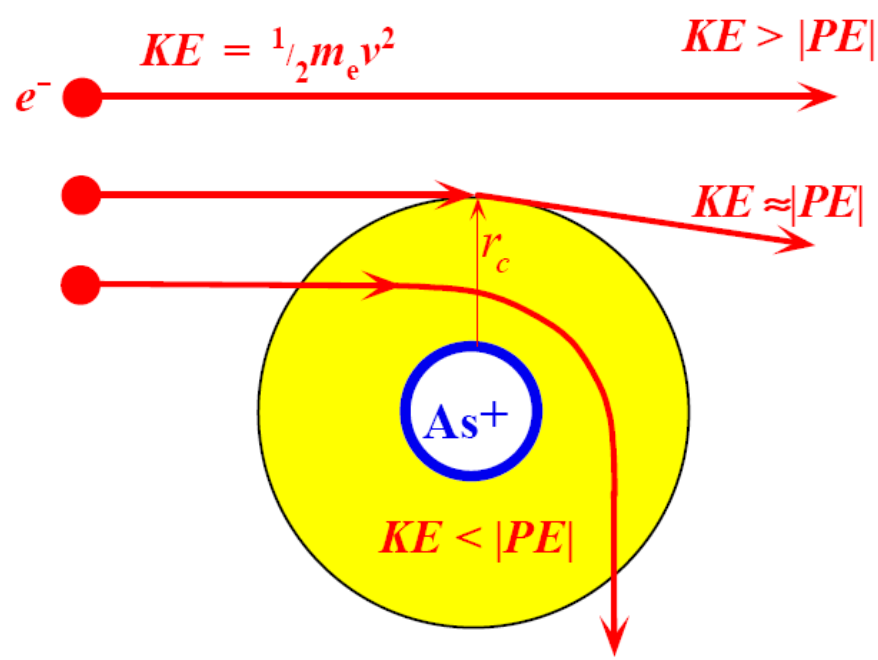
$$\mu = \frac{e\tau}{m_{eff}}$$

So for a scattering cross section $S=\pi r_c^2$ we obtain:

$$S = \frac{\pi e^4}{(6\pi\varepsilon_r\varepsilon_o kT)^2} \propto T^{-2}$$

So for the scattering time we obtain:

$$\tau = \frac{1}{S v_{th} N_I} \propto \frac{1}{(T^{-2})(T^{1/2})N_I} \propto \frac{T^{3/2}}{N_I}$$



Therefore for impurities:

$$\mu_I \propto \frac{T^{3/2}}{N_I}$$

Mobility and Carrier Phonon and Impurity Scattering

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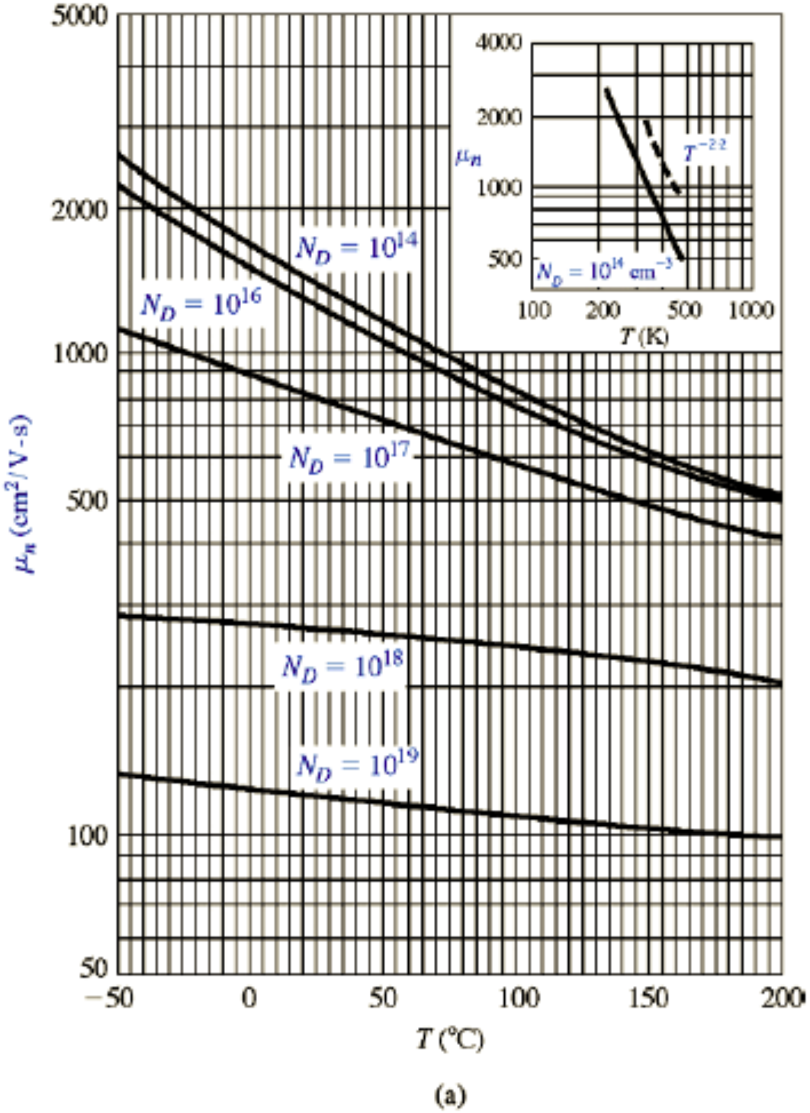
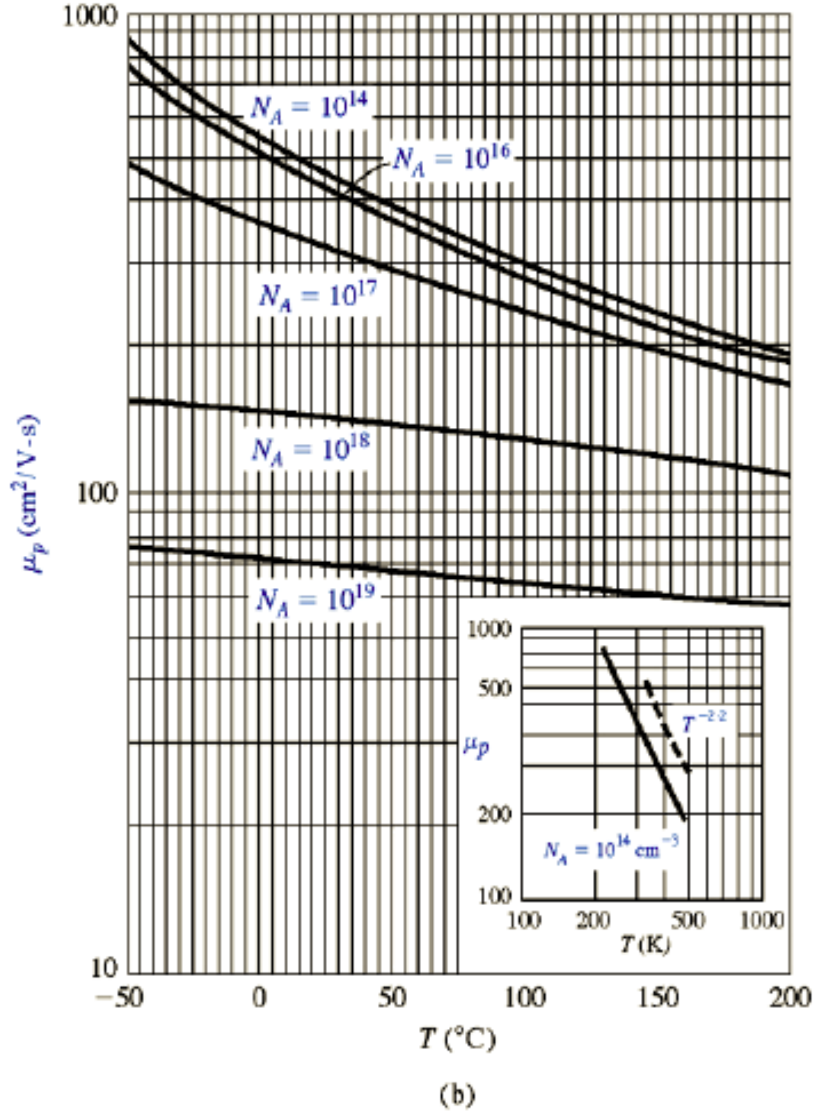
Net mobility : $\dfrac{1}{\mu_{total}} = \dfrac{1}{\mu_L} + \dfrac{1}{\mu_I}$

At higher T, v_{th} increases and on average electrons spend less time in the vicinity of ionized impurity.

Mobility as a Function of Temperature and Doping

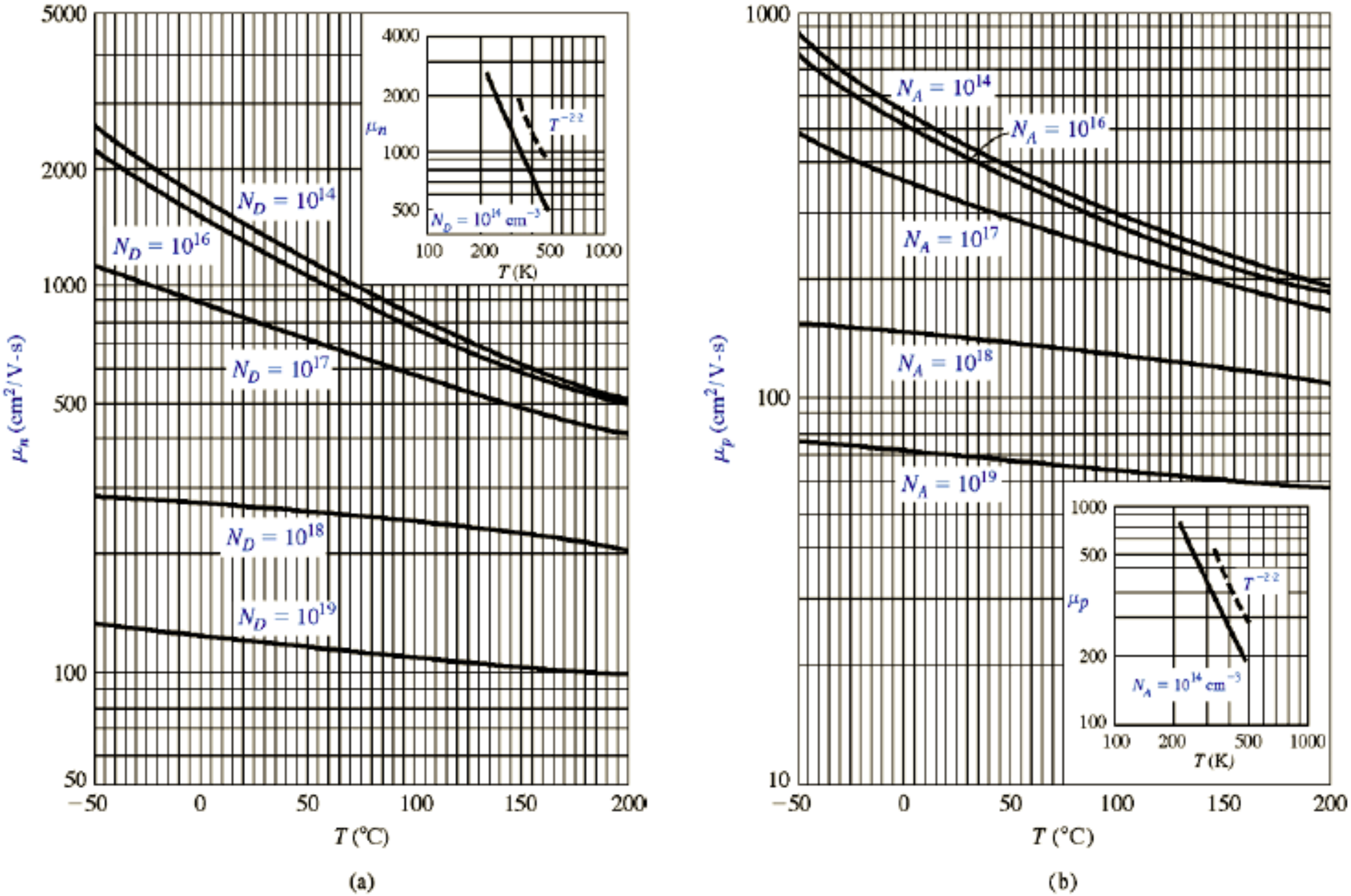


(a) electron mobility



(b) hole mobility

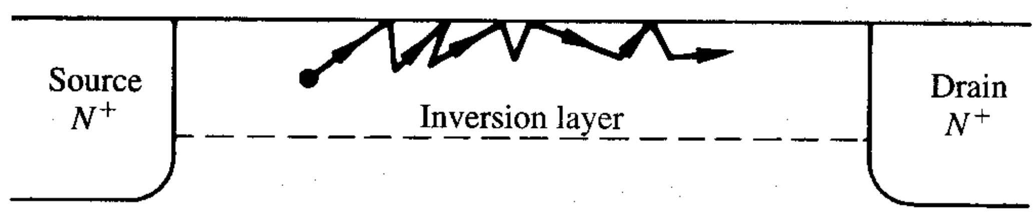
Mobility as a Function of Temperature and Doping



(a) electron mobility (b) hole mobility

Mobility in an Inversion Channel

- **Drain current model assumed constant mobility in channel**
- **Mobility of channel less than bulk – surface scattering**
- **Mobility depends on gate voltage – carriers in inversion channel are attracted to gate – increased surface scattering – reduced mobility**



Mathiessen Rule

$$\frac{1}{\mu} = \frac{1}{\mu_{Coul}} + \frac{1}{\mu_{ph}} + \frac{1}{\mu_{sr}}.$$

$$\mu_{sr} = \mu_{sr}(E_{ave}^{vert})$$

$$E_{ave}^{vert} = -\frac{Q_{dep} + \eta \cdot Q_n}{\varepsilon_s}$$

Vertical electric field in the inversion channel

Surface Mobility for Electrons and Holes

$$\mu_{nsr} \propto E_{ave}^{-2}$$

Electrons

$$\mu_{psr} \propto E_{ave}^{-1}$$

Holes

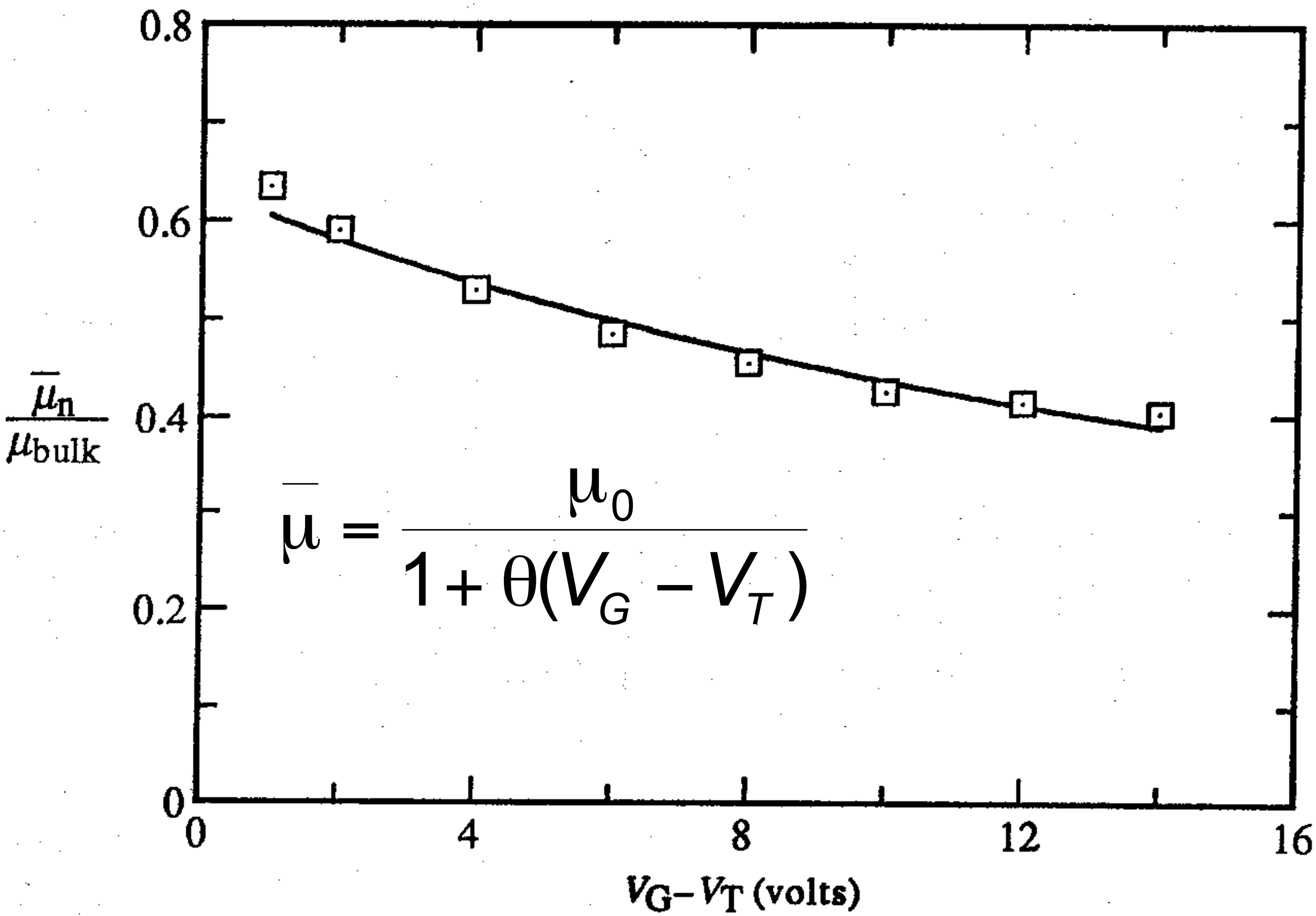
Temperature Dependence of mobility

$$\mu_0(T) = \mu_0(T_0)(T/T_0)^{-m}$$

m=1.5 theory, in practice…

m= 1.4 – 1.6 for n-channel transistors
1.2 – 1.4 for p-channel transistors

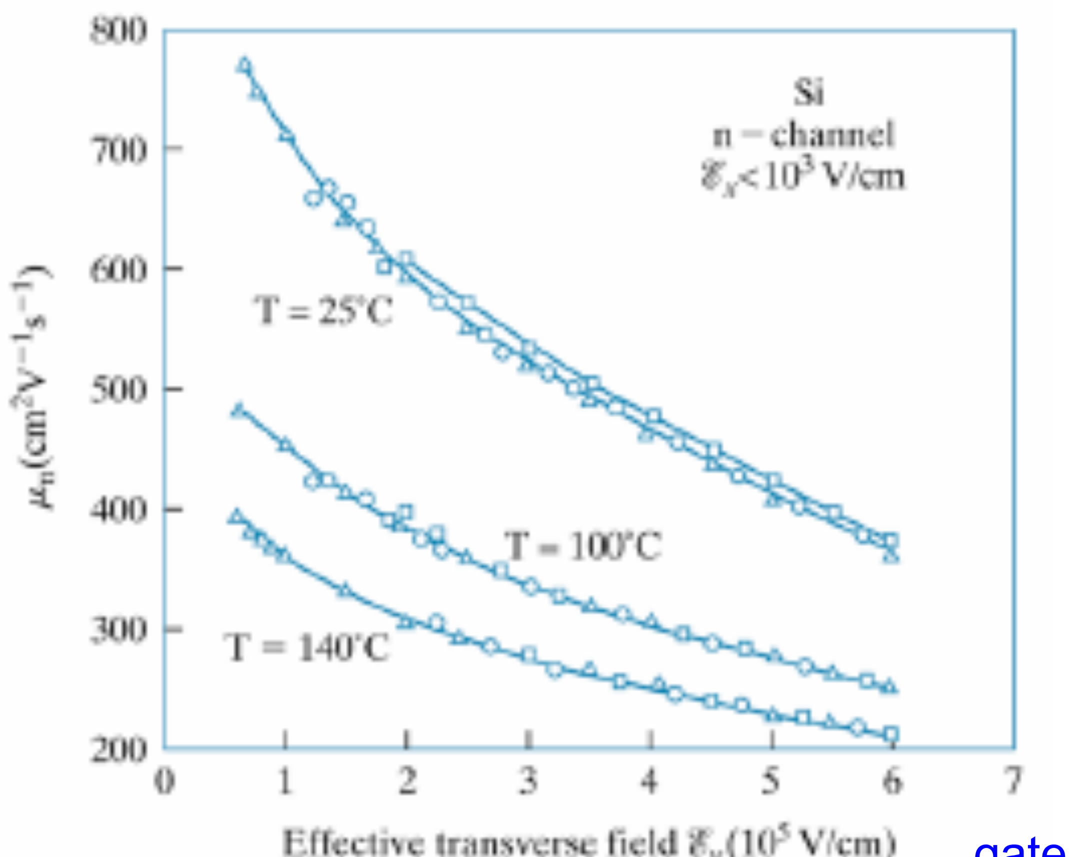
Surface scattering
Mobility Degradation with high V_g



$$\bar{\mu} = \frac{\mu_0}{1 + \theta(V_G - V_T)}$$

Inversion Mobility Degradation with Temperature

6.5.3 Mobility Model



gate field
vertical field

Figure 6—30
Inversion layer electron mobility versus effective transverse field, at various temperatures. The triangles, circles and squares refer to different MOSFETs with different gate oxide thicknesses and channel dopings. (After Sabnis and Clemens, IEEE IEDM, 1979).

Effective Inversion Mobility μ_{eff}

$$\mu(y) = \frac{\mu_o}{1 + \alpha \cdot E_{ave}(y)}$$

$$\mu_{eff} = \frac{\mu_o}{\frac{1}{L}\int_0^L \left[1 - \alpha\left(\frac{Q_d + \eta Q_n}{\varepsilon_s}\right)\right] dy}$$

Dependence on y along the channel

$$\mu_{eff} = \frac{1}{\frac{1}{L}\int_0^L \frac{1}{\mu(y)} dy} = \frac{1}{\frac{1}{L}\int_0^L \left[\frac{1}{\mu_{Coul}} + \frac{1}{\mu_{ph}} + \frac{1}{\mu_{sr}}\right] dy}$$

MOSFET
Channel Conductance
and
Transconductance

Channel Conductance and Transconductance

$$g_{dlin} = \left(\frac{\partial I_d}{\partial V_d}\right)_{V_g = const} = \frac{W}{L}\mu_{eff} C_{ox}(V_g - V_T)$$

Channel conductance

$$g_{dsat} = \left(\frac{\partial I_d}{\partial V_d}\right)_{V_g = const} = 0$$

Channel conductance called also output conductance is the conductance of a resistor modulated by the drain bias V_d. Note that channel conductance in the saturation regime is zero, because in saturation regime the drain current is independent of the drain bias.

In the discussion of the MOSFET so far, the series resistances associated with source and drain, were not considered. These series resistances impose ohmic drops between source and drain contacts and the channel. In the linear region, the effect of the series resistances can be seen in

$$\frac{1}{g_d(meas)} = \frac{1}{g_d} + R_s + R_d$$

Where R_s and R^d are the source and drain series resistances, respectively.

Channel Conductance and Transconductance

$$g_{mlin} = \left(\frac{\partial I_d}{\partial V_g} \right)_{V_d = const} = \frac{W}{L} \mu_{eff} C_{ox} V_d$$

Channel transconductance

$$g_{msat} = \left(\frac{\partial I_d}{\partial V_g} \right)_{V_d = const} = \frac{W}{L} \mu_{eff} C_{ox} (V_g - V_T)$$

Channel transconductance measures the sensitivity of the drain current to the gate voltage. In linear region the transconductance is linear function of drain bias and independent of the gate bias V_g. In the saturation region the transconductance is linear in gate voltage and independent of the drain bias.

Note that by plotting g_{msat} as a function of V_g we can determine the threshold voltage as an intercept with the V_g-axis. By the way, this is a third possibility to determine the V_T from I-V characteristics.

In the saturation region the measured transconductance is given by:

$$\frac{1}{g_m(meas)} = \frac{dV_g}{dI_d} = \frac{1}{g_{msat}} + R_s$$

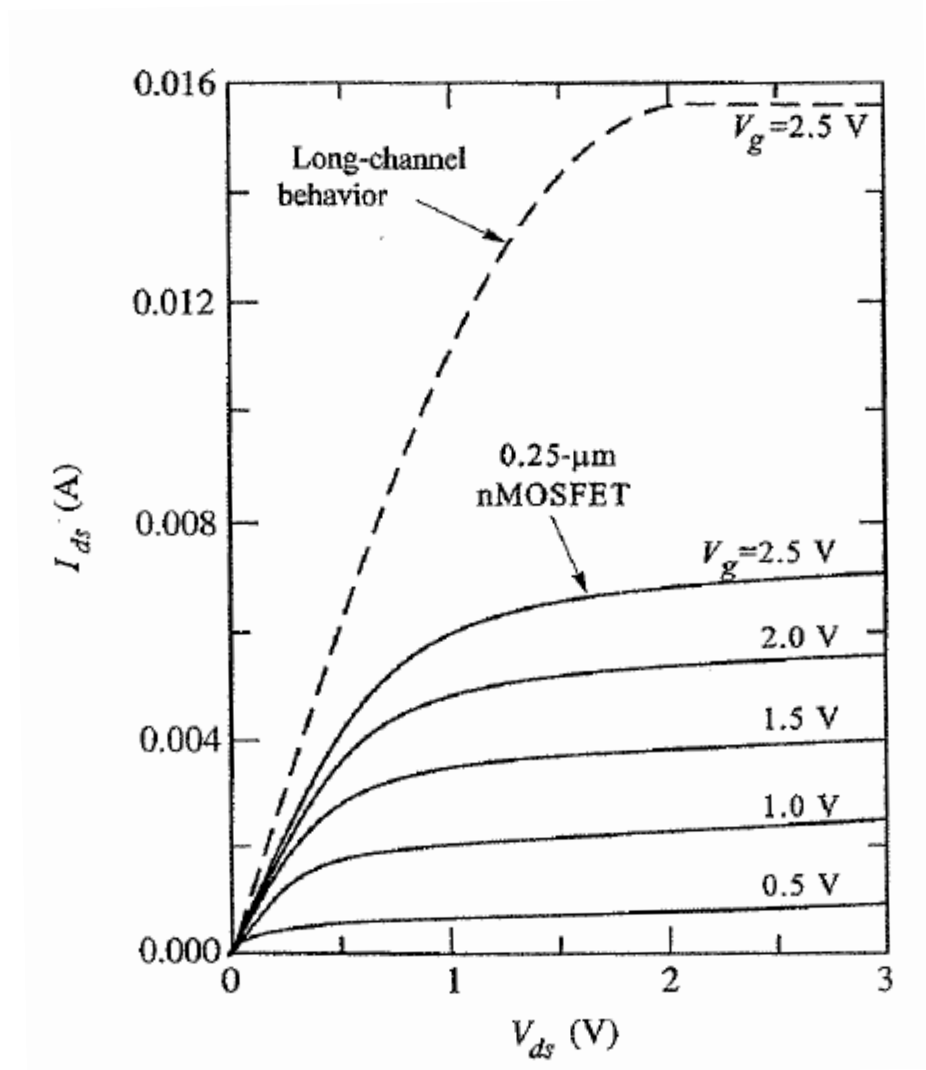
R_d does not appear in this equation (saturation!) because the change of V_d caused by IR drop has no effect on saturation current.

Channel Transconductance and Velocity Saturation

$$I_{ds} = I_{dsat} = \mu_{eff} C_{ox} \frac{W}{2L} (V_g - V_T)^2 \qquad V_{dsat} = V_g - V_T$$

In a short channel device, the saturation of the v_{drift} and, by extension, the drain current may occur at a much lower voltage due to velocity saturation. This causes the I_{dsat} to deviate form the 1/L dependence.

The figure to the right shows the experimentally measured I-V curves for a 0.25 um MOSFET. The dashed line represents the long-channel current for V_g=2.5V. Due to the velocity saturation, the I_d saturates at a drain voltage much lower than (V_g-V_T), thus severely limiting the saturation current of short channel device.



Channel Transconductance Modeling

Local dependence of drift velocity

E_c has the dimension of electric field and serves as a fitting parameter.

$$v_{drift} = \mu_n \frac{E_y}{E_y / E_c + 1}$$

$$v_{drift}(y) = \mu_n \frac{dV(y)/dy}{1 + \frac{1}{E_c}\frac{dV(y)}{dy}}$$

Recall

$$I_d = -W\mu_n Q_{inv}(y)\frac{dV(y)}{dy}$$

Channel Transconductance

We now replace μ_n by: $$\frac{\mu_n}{1+\frac{1}{E_c}\frac{dV(y)}{dy}}$$

$$I_d(1+\frac{1}{E_c}\frac{dV(y)}{dy}) = -W\mu_n Q_{inv}(y)\frac{dV(y)}{dy}$$

After integration we obtain:

$$I_d = \mu_n C_{ox}\frac{W}{L(1+\frac{V_{ds}}{LE_c})}\{(V_G - V_T)V_{ds} - \frac{1}{2}mV_{ds}^2\}$$

Channel Transconductance

By imposing $dI_d/dV_{ds}=0$ the drain saturation voltage can be found:

$$V_{dsat} = LE_c\left[\sqrt{1+\frac{2(V_g - V_T)}{mLE_c}} - 1\right]$$

Therefore, when velocity saturation is taken into account it is equivalent to making the channel longer, and to reducing the drain saturation voltage and drain saturation current.

$$I_d = \mu_n C_{ox}\frac{W}{L(1+\frac{V_{ds}}{LE_c})}\{(V_G - V_T)V_{ds} - \frac{1}{2}mV_{ds}^2\}$$

Channel Conductance and Transconductance

$$I_{ds} = I_{dsat} = \mu_{eff} C_{ox} \frac{W}{2L} (V_g - V_T)^2 \qquad V_{dsat} = V_g - V_T$$

When $$V_g - V_T << \frac{2 v_{sat} L}{\mu_{eff}}$$

the saturation current for small channel lengths L becomes velocity-saturation-limited drain current

$$I_{dsat} = W C_{ox} v_{sat} (V_g - V_T)$$

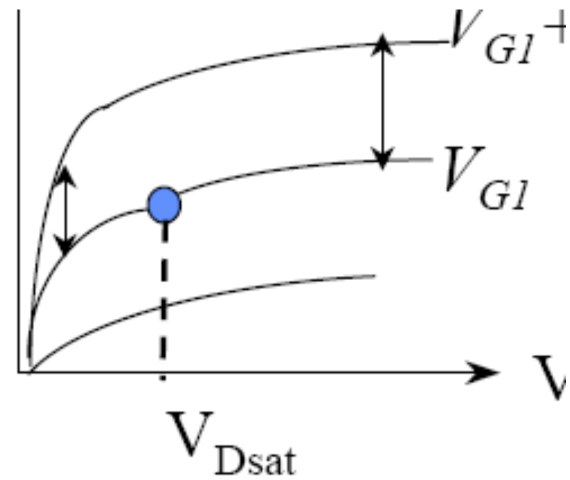
In this case the saturation transconductance becomes

$$g_{msat} = \left(\frac{\partial I_d}{\partial V_g} \right)_{V_d = const} = W C_{ox} v_{sat}$$

The measurement of saturation transconductance allows us to extract the saturation velocity.

Channel Transconductance - Summary

$$g_m \equiv \left.\frac{\partial I_D}{\partial V_G}\right|_{fixed\ V_D}$$



(a) For $V_{DS}<V_{Dsat}$

$$I_D = \frac{\mu_n W}{L} C_{OX}\left(V_G - V_T - \frac{V_{DS}}{2}\right)V_{DS}$$

$$\therefore \frac{\partial I_D}{\partial V_G} = \mu_n C_{OX}\frac{W}{L}\cdot V_{DS}$$ [g_m varies with V_{DS}]

(b) For $V_{DS} > V_{Dsat}$

$$I_D = I_{Dsat} = \frac{\mu_n W}{2L} C_{OX}\left(V_G - V_T\right)^2$$

$$\frac{\partial I_D}{\partial V_G} = \frac{\mu_n W}{L} C_{OX}\cdot\left(V_G - V_T\right)$$ [g_{msat} varies with V_G]

Channel Conductance and Transconductance Summary

Channel transconductance in linear and saturation regime

$$g_{mlin} = \left(\frac{\partial I_d}{\partial V_g}\right)_{V_d = const} = \frac{W}{L}\mu_{eff} C_{ox} V_d$$

$$g_{msat} = \left(\frac{\partial I_d}{\partial V_g}\right)_{V_d = const} = \frac{W}{L}\mu_{eff} C_{ox} (V_g - V_T)$$

Channel Length Modulation

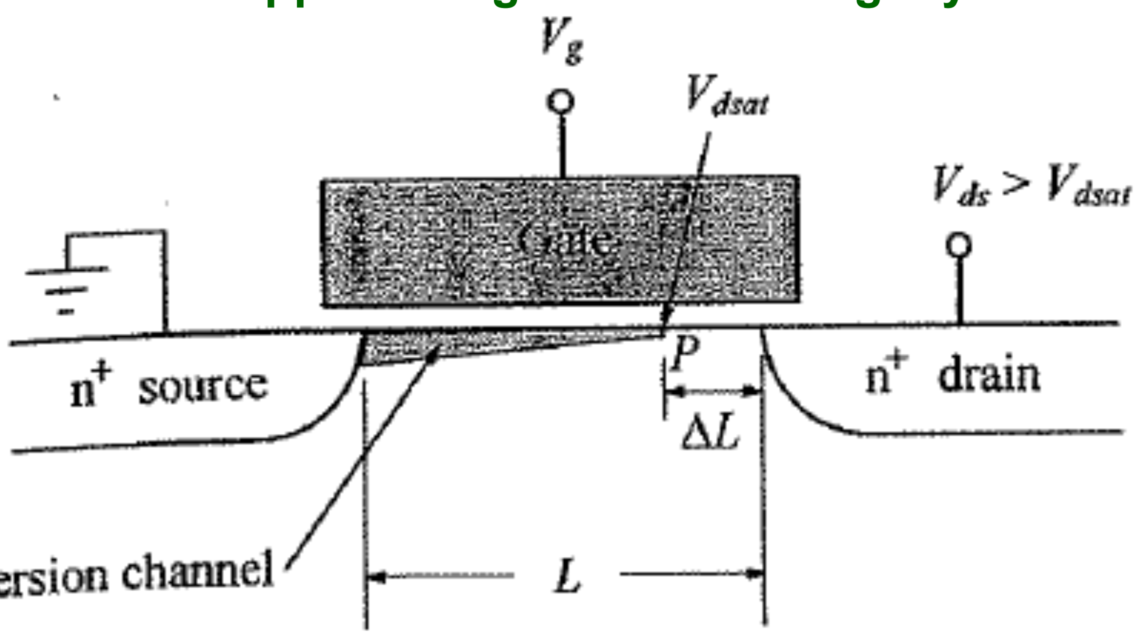
In a long channel device (or ideal MOSFET), the drain current stays constant for $V_d > V_{dsat}$. The output conductance is zero. In short channel device I_d can still increase beyond the pinch-off. This arises from two factors:

1) the threshold voltage decreases for short channel devices (to be discussed later)

and

2) channel length modulation.

As V_d increases beyond the saturation voltage V_{dsat} the saturation point where the inversion channel disappears begins to move slightly toward the source as shown below.



Channel Length Modulation intuitive picture

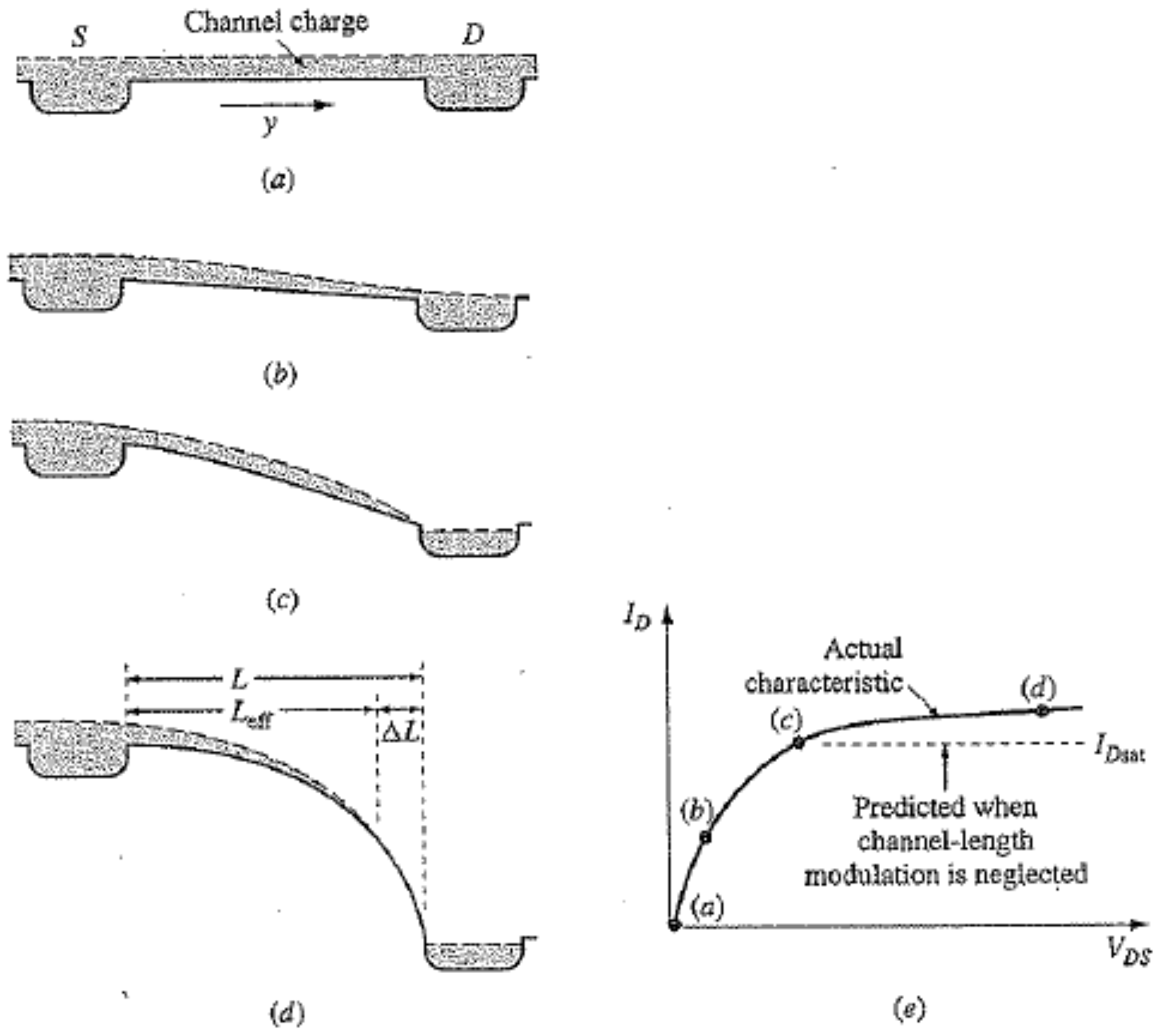
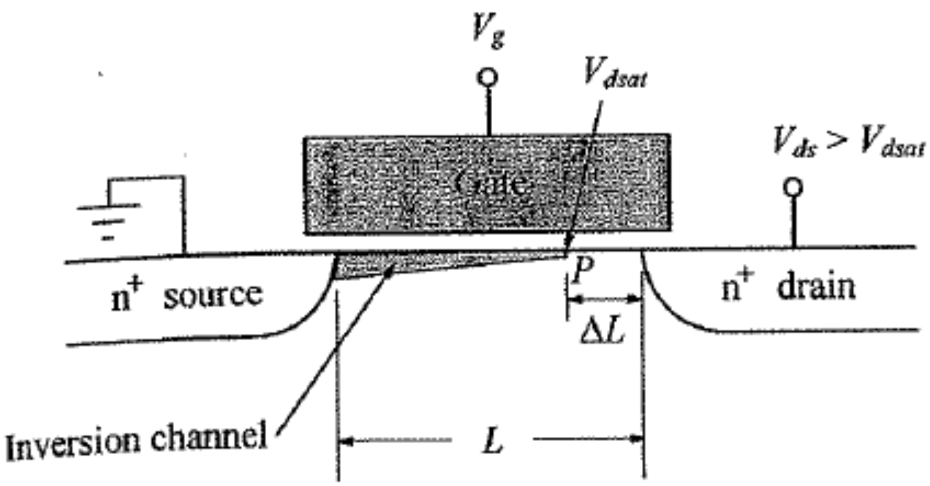


Figure 7.22 Qualitative explanation for channel-length modulation. Parts (a) to (c) repeat the explanation of the simple long-channel model. In (d), as the drain voltage continues to increase, the point at which the channel charge approaches 0 (shaded region), or the point at which $V_{ch} = V_{GS} - V_T$, moves along the channel toward the source. The channel becomes effectively shorter. (e) The corresponding points on the I_D-V_{DS} characteristics. From point (c) on, the simple model predicts constant current (dashed line).

Channel Length Modulation

The point of saturation is still at V_{dsat}, independent of V_d. So V_d-V_{dsat} drops between the saturation point and the drain. The corresponding distance ΔL is referred to as the amount of channel length modulation. ΔL increases with V_d>V_{dsat}. The device acts as if its channel length were shortened by ΔL. The drain current is then obtained by replacing L with L- ΔL.



$$I_{ds} = \frac{I_{dsat}}{1 - \Delta L / L}$$

We can have an estimate on ΔL by assuming that the pinch-off region is dominated by the lateral drain field E_y , that the mobile charges are negligible compared with the fixed charges, and that we have an abrupt junction. In this case we obtain:

$$\Delta L = \sqrt{\frac{2\varepsilon_s}{qN_A}(V_d - V_{dsat})}$$

Channel Length Modulation

ΔL can be also estimated from the following argument:

ΔL is the total lateral space charge width minus the space charge width that exists when V_d=V_{dsat}.

$$\Delta L = \sqrt{\frac{2\varepsilon_s}{qN_A}(\psi_B + V_d)} - \sqrt{\frac{2\varepsilon_s}{qN_A}(\psi_B + V_{dsat})}$$

for $V_d > V_{dsat}$

Channel Length Modulation

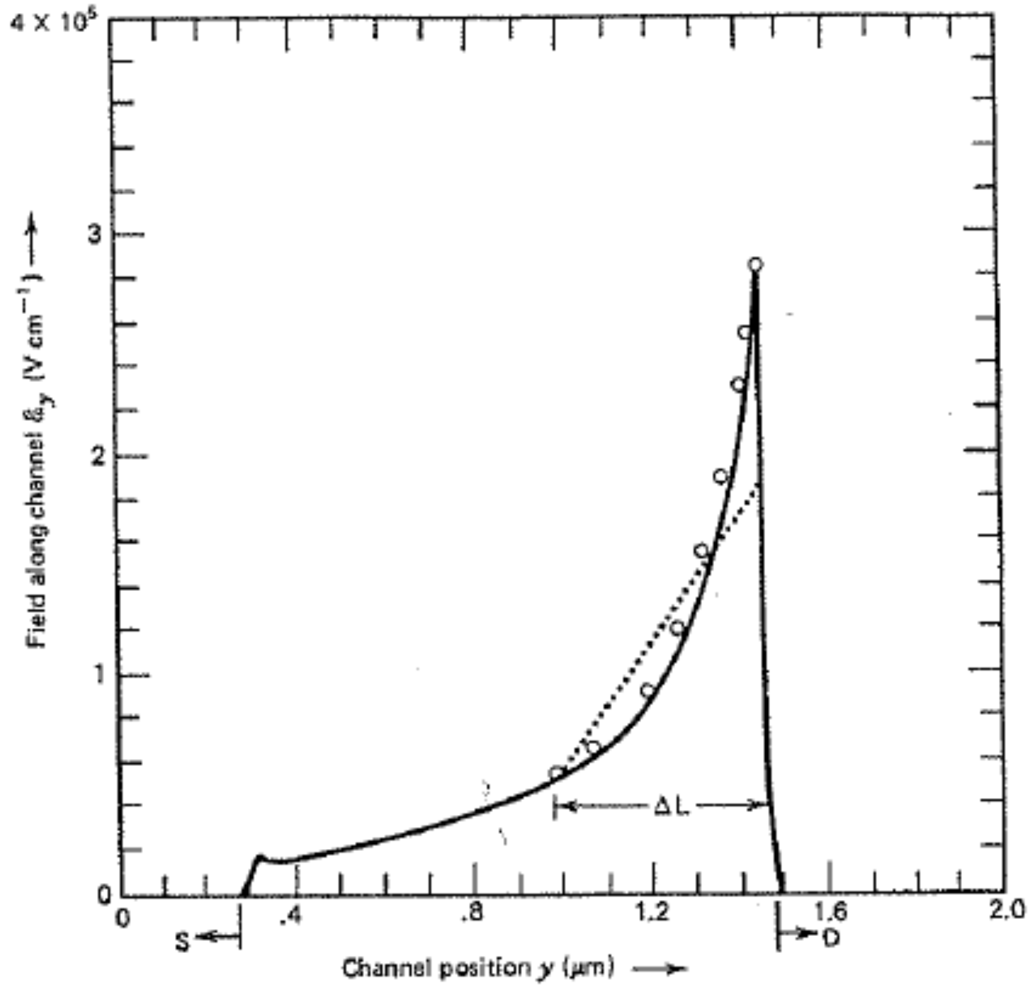
$$\Delta L = \sqrt{\frac{2\varepsilon_s}{qN_A}(V_d - V_{dsat})}$$

A still better approximation can be obtained when the mobile charges are included

$$\frac{dE_y}{dy} = \frac{qN_A}{\varepsilon_s} + \frac{qN_{mob}}{\varepsilon_s}$$

$$qN_{mob} = A\frac{I_d}{\varepsilon_s r_j v_{sat} W}$$

where r_j is the s/d junction depth.
The result of the integration of the above equation is shown by dotted line compared with full numerical solution of the problem by solving a 2D Poisson equation in the saturation region.



Channel Length Modulation

Example of channel length modulation

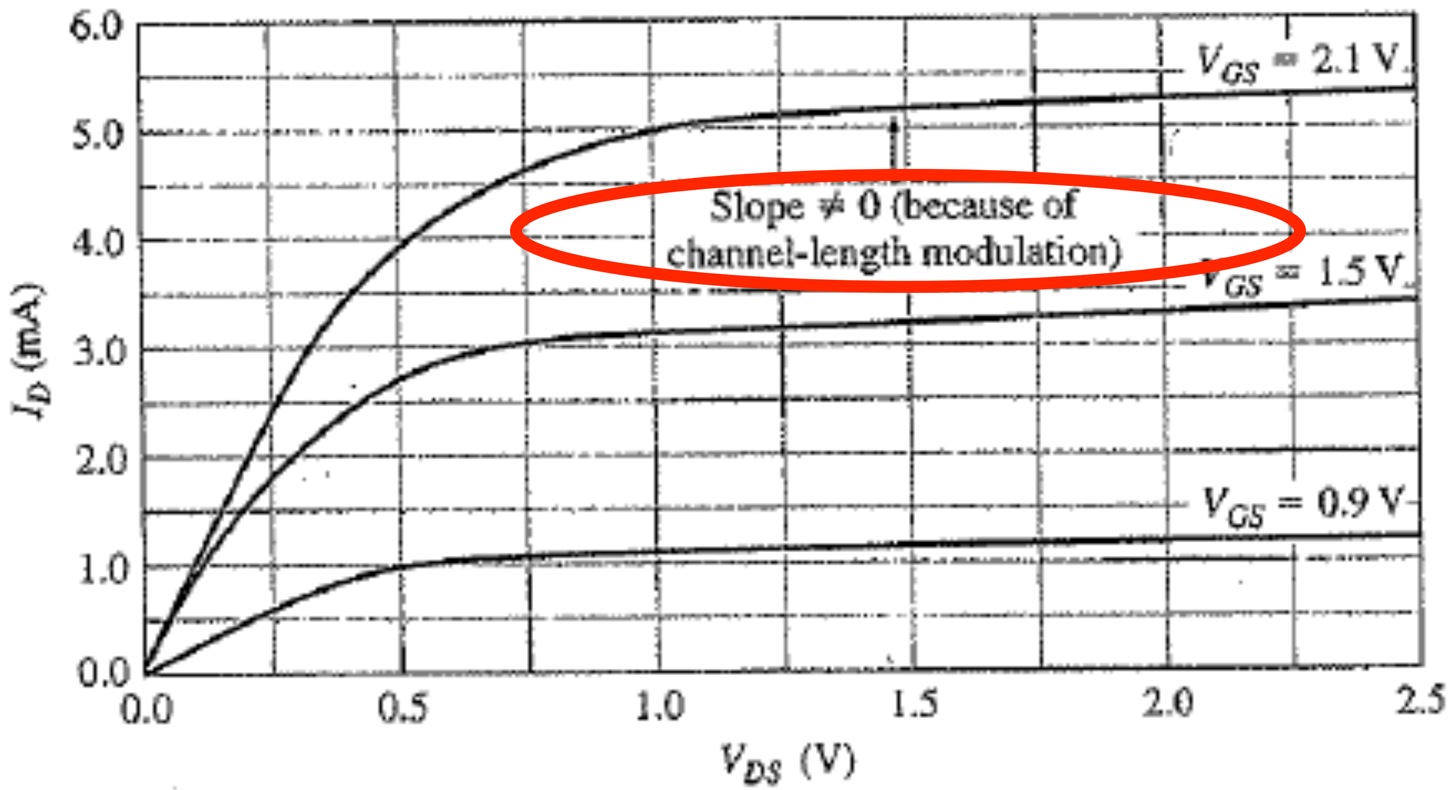


Figure 7.21 Experimental I_D-V_{DS} characteristics for an n-channel MOSFET for three values of gate voltage. The current actually increases with increasing V_D in the "current saturation" region because of channel-length modulation. For this device, $t_{ox} = 4.7$ nm, $L = 0.27\ \mu$m, $W = 8.6\ \mu$m, and $V_T = 0.3$ V.

Channel Length Modulation

Very often the channel length modulation is modeled by the following equation:

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$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_T)^2 \cdot (1+\lambda V_{DS})$$

Sometimes $1/V_A$ is used instead of λ, V_A is called the Early voltage, similar to the neutral zone variation in bipolar transistor.

$$V_A = {}^{1}\!/_{\lambda}$$

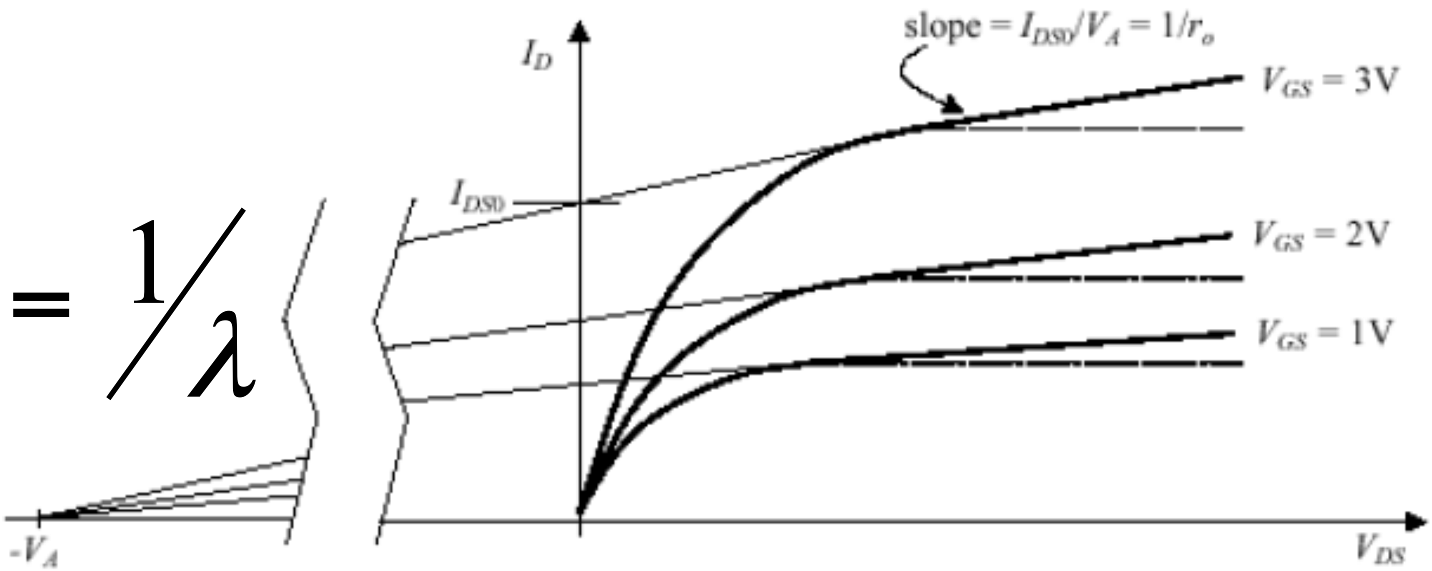
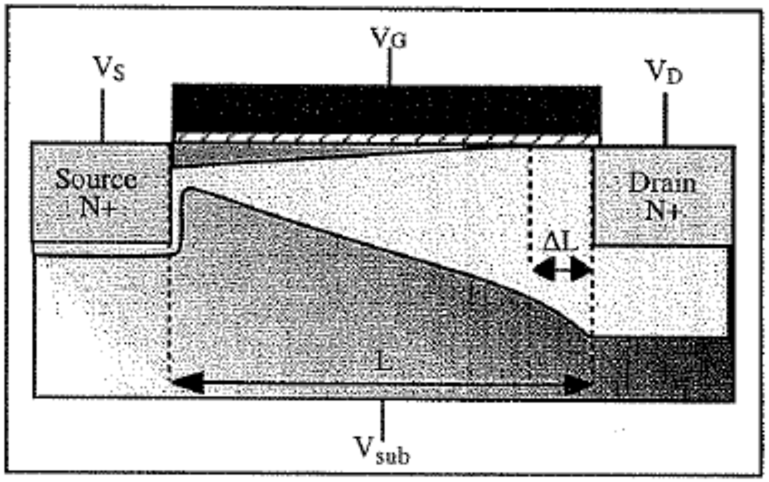


Figure 14. $V_{DS} - I_D$ plot for long channel nMOS

Channel Length Modulation

Another approach to modeling
similar to modeling of the Early voltage in bipolar transistors



$$I'_{Dsat} = I_{Dsat}\left(1 + \frac{V_D - V_{Dsat}}{V_A + V_{Dsat}}\right)$$

Linear dependence of I_{dsat}
as a function of V_d