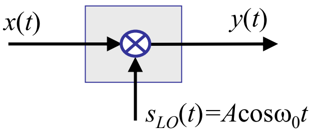
Active and Passive Mixers

Outline

- Basics of mixers
- Mixer circuits
  - active
  - passive
- Noise and nonlinearities
- Example of merged LNA and mixer
- Summary

Basics of Mixers



In Rx's for downconversion or AM demodulation,

In Tx's for upconversion or AM modulation

$$Y(\omega) = \frac{A}{2}\left[X(\omega - \omega_0) + X(\omega + \omega_0)\right]$$

Upconverted component

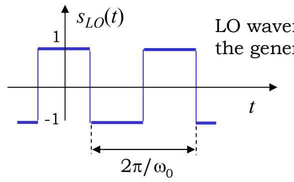
Downconverted component

In downconversion mixers the output stage at IF or BB

In upconversion mixers the output stage at RF

Basics of Mixers / Switching

In practice, LO provides a periodic switching rather than a sinusoidal waveform



LO waveform for the generic model

$$y(t) = K_0 \cdot x_{RF}(t) \cdot s_{LO}(t)$$

$$Y(\omega) = \frac{K_0}{2\pi} \int_{-\infty}^{\infty} X_{RF}(\omega - v) \cdot S_{LO}(v) dv$$

$$= \underbrace{\frac{2K_0}{\pi} X_{RF}(\omega + \omega_0)}_{\text{down-converted component at } IF} + \frac{2K_0}{\pi} X_{RF}(\omega - \omega_0) - \frac{2K_0}{3\pi} X_{RF}(\omega + 3\omega_0) + \frac{2K_0}{3\pi} X_{RF}(\omega - 3\omega_0) - \ldots$$

$$\boxed{s_{LO}(t) \;=\; \frac{4}{\pi}\cos\omega_0 t - \frac{4}{3\pi}\cos 3\omega_0 t + \frac{4}{5\pi}\cos 5\omega_0 t - \ldots}$$

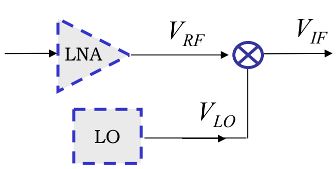
$$\boxed{K_C = \frac{V_{IF}}{V_{RF}} = \frac{2K_0}{\pi}}$$

Voltage conversion gain

$$\boxed{G_C = 20\log\frac{2K_0}{\pi} = 20\log K_0 - 3.9dB}$$

K_0 is RF gain when no switching

Basics of Mixers / feedthrough effects



Voltage conversion gain: $\Delta V_{IF}/\Delta V_{RF}$

Note that $V_{LO} >> V_{RF}$ $\rightarrow$ leakage

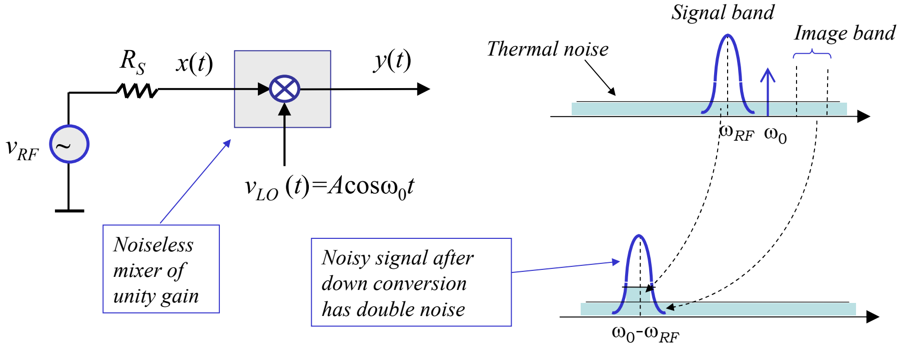
Isolation between each two ports LO, RF and IF is critical

$$v_{IF}(t) \propto \left(v_{RF}(t) \cdot A\cos\omega_0 t + \underbrace{a \cdot A\cos\omega_0 t}\right)$$

If a fraction of large V_{LO} appears at the output, the following stage can be desensitized

LO→RF leakage also critical in homodyne
- "low frequency beat"

Basics of Mixers
/SSB-NF



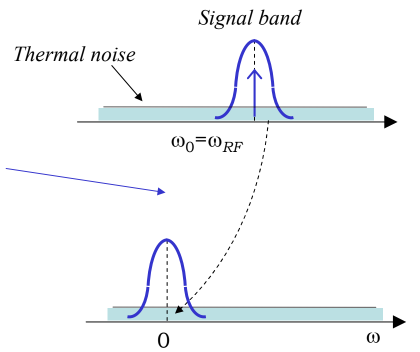
$\boxed{SNR_{out} = SNR_{in}/2 \;\rightarrow\; NF = 3\text{dB}}$

For noiseless mixer and SSB signal

Basics of Mixers
/DSB-NF

Double sideband case

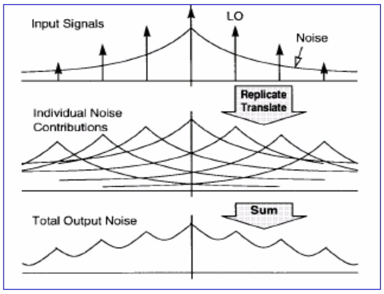
Zero-IF
conversion



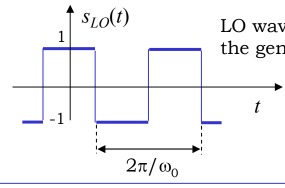
$\boxed{SNR_{out} = SNR_{in} \;\rightarrow\; NF = 0\text{dB}}$

If LO has harmonics more noise is produced at output unless suppressed by mixer gain at higher frequencies

Basics of Mixers/noise folding



In a mixer noise is replicated and translated by each harmonic of the LO that is referred to as **noise folding**



LO waveform for the generic model

$$\boxed{s_{LO}(t) \;=\; \frac{4}{\pi}\cos\omega_0 t - \frac{4}{3\pi}\cos 3\omega_0 t + \ldots}$$

$$y_n(t) = S_{LO}(t)\cdot x_n(t)$$

Input referred noise

$$= \sum_{k=-\infty}^{\infty} a_l\, e^{jk\omega_0 t}\cdot x_n(t)$$

convolution

$$Y_n(\omega) = \sum_{k=-\infty}^{\infty} a_k\, X_n(\omega + k\omega_0)$$

Power spectral density of stationary folded noise

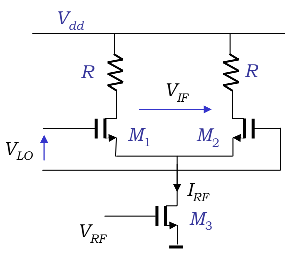
$$\boxed{S_{yn}(\omega) = \sum_{k=-\infty}^{\infty} |a_k|^2\, S_{xn}(\omega + k\omega_0)} \qquad |a_k| = \frac{2}{k\pi}$$

Mixer Circuits

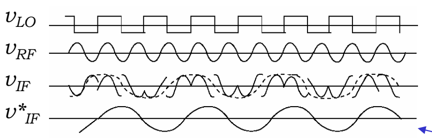
- Active mixers /some transistors provide gain/
  - balanced
  - double balanced
  - potentiometric
- Passive mixers
  /all transistors work as switches/
  - switching mixers
  - sampling mixers

Balanced mixer

Differential LO input



- V_{LO} is <u>a large symmetrical wave</u> switching ON/OFF the coupled M_1 and M_2 (~1V_{pp}), several harmonics $n\omega_0$ of LO contribute to mixing
- Differential output is preferred for higher gain and more immunity to RF→IF feedthrough (strong interferers undergo <u>intermodulation</u>)
- Main drawback is high LO→IF feedthrough
- Resistive load can be replaced with LC tanks, but it is impractical for downconversion (large LC needed !)



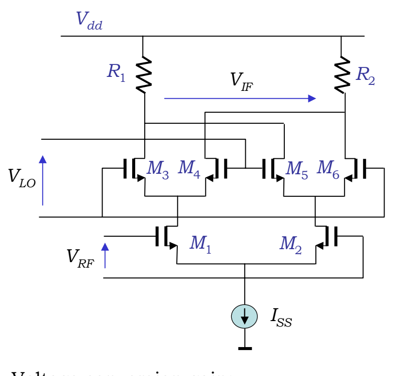
Effective transconductance: $G_c = 2g_{m3}/\pi$

Voltage conversion gain: $K_c = 2g_{m3}R/\pi$

After IF filter, LO→IF

Double balanced mixer

Both inputs differential (Gilbert's)

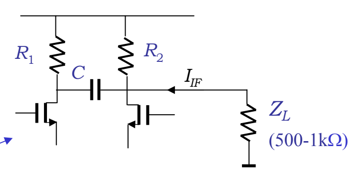


- Generates less even order distortions (~fully differential)
- Since LNA usually single ended, one RF terminal to bias → degrades symmetry, device <u>mismatch</u> as well
- IF filter (SAW) might require single-ended output
- LO feedthrough almost cancelled
- For perfect switching transistors M3..M6 much smaller than M1,M2

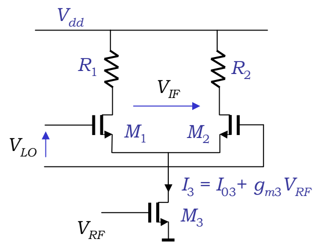
Voltage conversion gain:

$K_c = 2g_mR/\pi$

Conversion to single-ended output

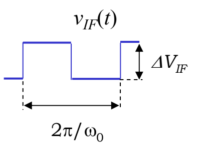


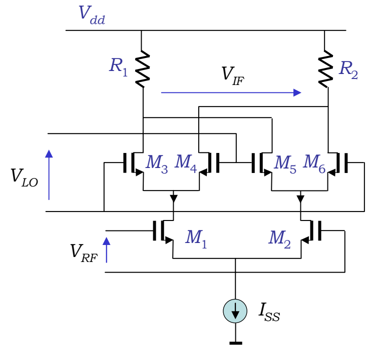
LO→IF feedthrough



$$\left.\begin{aligned} V_{LO}>0 &\rightarrow V_{IF} = R_1 I_{03} \\ V_{LO}<0 &\rightarrow V_{IF} = -R_2 I_{03} \end{aligned}\right\} \quad \Delta V_{IF} = 2RI_{03}$$

Large feedthrough





$V_{LO}>0 \rightarrow V_{IF} = R_1 I_{02} - R_2 I_{01} = R(I_{02} - I_{01})$

$V_{LO}<0 \rightarrow V_{IF} = R_1 I_{01} - R_2 I_{02} = R(I_{01} - I_{02})$

$\Delta V_{IF} = 2R(I_{01} - I_{02}) \approx 0$

small feedthrough

Linearity of Gilbert mixer

- For frequencies $\omega_{RF} << \omega_{-3dB}$
  upper MOSFET switches ON/OFF,
  do not introduce much distortions

Long channel model

$$I_{RF+} = \frac{k}{2}\left(V_{gs1} - V_t\right)^2, \quad I_{RF-} = \frac{k}{2}\left(V_{gs2} - V_t\right)^2$$

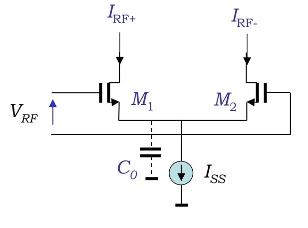
$$I_{RF+} = \frac{I_{SS}}{2} + \Delta I_{RF}, \quad I_{RF-} = \frac{I_{SS}}{2} - \Delta I_{RF}$$

$$V_{RF} = V_{gs1} - V_{gs2}$$

$$V_{RF} = \sqrt{\frac{I_{SS}}{k}} \cdot \left(\sqrt{1 + \frac{\Delta I_{RF}}{I_{SS}/2}} - \sqrt{1 - \frac{\Delta I_{RF}}{I_{SS}/2}}\right)$$

$$\Delta I_{RF} = \frac{I_{SS}}{2} \cdot \sqrt{\frac{kV_{RF}^2}{I_{SS}} \cdot \left(1 - \frac{kV_{RF}^2}{4I_{SS}}\right)}$$

Gilbert's mixer



$$\boxed{\Delta I_{RF} = a_1 V_{RF} + a_2 V_{RF}^2 + a_3 V_{RF}^3 + \ldots}$$

$$a_1 = \sqrt{\frac{kI_{SS}}{4}}, \quad a_2 = 0, \quad a_3 = -\frac{k}{16} \cdot \sqrt{\frac{k}{I_{SS}}}$$

IP3 of Gilbert mixer

$$a_1 = \sqrt{\frac{kI_{SS}}{4}}, \quad a_2 = 0, \quad a_3 = -\frac{k}{16}\cdot\sqrt{\frac{k}{I_{SS}}}$$

Nonlinearities tend to rise with frequency

$$A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{a_1}{a_3}\right|} \quad \Rightarrow \quad A_{IP3} = 4\sqrt{\frac{2}{3}\cdot\frac{I_{SS}}{k}} = 4\sqrt{\frac{2}{3}}\cdot\left(V_{GS0} - V_t\right)$$

IIP3 amplitude

Large power for good IP3

V_{gs} bias voltage should be large

3rd order harmonic distortion ratio

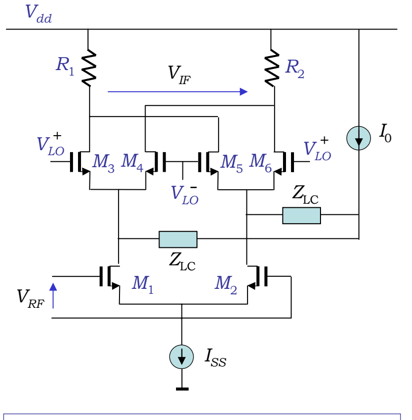
$$HD3 = \frac{\Delta I_{RF,3}}{\Delta I_{RF,1}} = \left|\frac{a_3}{4a_1}\right| A_{RF}^2 = \frac{A_{RF}^2}{32(V_{GS0} - V_t)^2}$$

Volterra series model when C_0 plays role

$$HD3(\omega) = \frac{A_{RF}^2}{32(V_{GS0} - V_t)^2}\left|1 - \frac{j\omega C_0}{k(V_{GS0} - V_t)}\right|$$

The model for short channel more complicated, but the tendency with IP3 is the same

Gilbert mixer with current injection



2×Z_{LC} prevent short circuit at RF

Larger bias I_{SS}

⇩

Larger IP3, but I_d's larger too

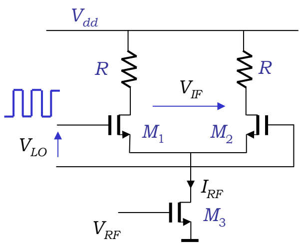
⇩

Less voltage headroom (voltage drop across R_1, R_2)

⇩

Bypassing with current injection, must be symmetrical, extra noise is introduced in common mode!

Noise in balanced mixer and Gilbert mixer



If M_1,M_2 switched by square wave then noise mainly from M_3 and $2\times R$

$$\frac{\overline{i_{nM3}^2}}{\Delta f} = 4kT\gamma g_{d0}, \quad \frac{\overline{e_{nR}^2}}{\Delta f} = 2\times 4kTR$$

Noise of M_3 modulated with square wave ±1

$$e_{nIF}(t) = R \cdot i_{nM3}(t) \cdot s_{LO}(t)$$

$$E_{nIF}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R \cdot I_{nM3}(\omega - \nu)\cdot S_{LO}(\nu)d\nu = \sum_{k=-\infty}^{\infty} a_k R I_{nM3}(\omega + k\omega_0)$$

$$S_{nIF}(\omega) = \sum_{k=-\infty}^{\infty}|a_k|^2 R^2 \cdot S_{nM3}(\omega + k\omega_0) = 4kT\gamma g_{d0}R^2 \sum_{k=-\infty}^{\infty}|a_k|^2$$

It holds $\sum|a_k|^2 = 1$, but in practice <1 because of band limitations

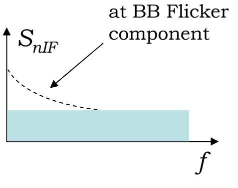
For Gilbert 2× larger

Noise in balanced and Gilbert mixer (cont'd)

PSD at IF output incl. noise of $2\times R$ and of R_S

$$S_{nIF}(\omega) = \left(4kTR_S g_{m3}^2 R^2 + 4kT\gamma g_{d0} R^2\right)\sum_k |a_k|^2 + 2\times 4kTR$$

$$G = (K_C)^2 = \left(\frac{2}{\pi} g_{m3} R\right)^2$$ power gain for signal



$$NF_{balanced} = \frac{S_{nIF}(\omega)}{G \cdot S_{nRs}(\omega)} = \frac{\pi^2}{4}\left(1 + \frac{\gamma g_{d0}}{g_{m3}^2 R_S} + \frac{2}{g_{m3}^2 R R_S}\right)$$

$$NF_{Gilbert} = \frac{\pi^2}{4}\left(1 + \frac{2\gamma g_{d0}}{g_{m3}^2 R_S} + \frac{2}{g_{m3}^2 R R_S}\right)$$

Using $\sum|a_k|^2 = 1$,

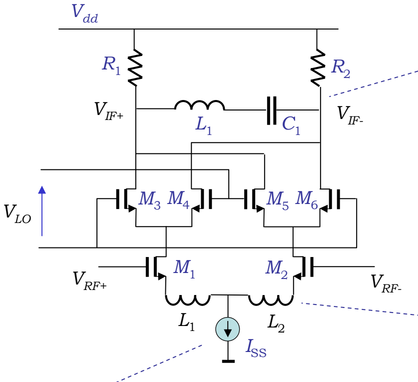
$|a_1|^2 + |a_{-1}|^2 = 0.81$

Observe that gain for R_S noise and for signal are different.

Flicker noise is omitted here but it does not fold since f_0 large

Imperfect switching also adds noise.

Other improvements



LC tank tuned to suppress the remaining LO feedthrough

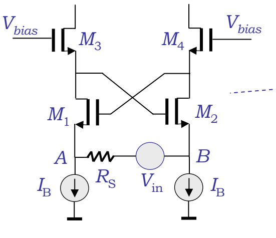
Inductors L_1L_2 provide negative feedback $\rightarrow$ IP3 would be improved

Inductors can also provide optimal noise matching (like in LNA), R_S should be considered of the LNA output

For low voltage application an LC tank can replace I_{SS}

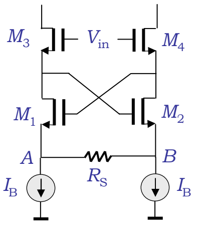
For a mixer linearity more critical than NF

Other improvements (cont'd)



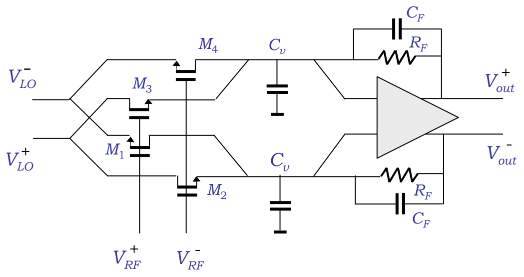
Cross-quad transconductor can greatly improve linearity of Gilbert mixer

If all transistors same
$V_A = V_B$ so $\Delta I_D = V_{in}/R_S$



Dual circuit
$V_A - V_B = V_{in}$ so $\Delta I_D = V_{in}/R_S$

Potentiometric downconversion mixer



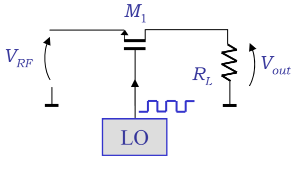
- MOSFETs biased in linear region – do not switch !
- OpAmp designed for low frequency
- C_v added to suppress high freq. and preserve virtual GND
- High IP3 possible but NF rather poor

$$I_D = k_n\left(V_{GS} - V_T - \frac{V_{DS}}{2}\right)V_{DS}$$ for each MOSFET

$$V_{out}^+ - V_{out}^- = R_F\left((I_{D1} - I_{D4}) + (I_{D3} - I_{D2})\right)$$
$$= k_n R_F\left(V_{RF}^+ - V_{RF}^-\right)\left(V_{LO}^+ - V_{LO}^-\right)$$

But when mismatch nonlinear distortions occur

Passive switching mixer

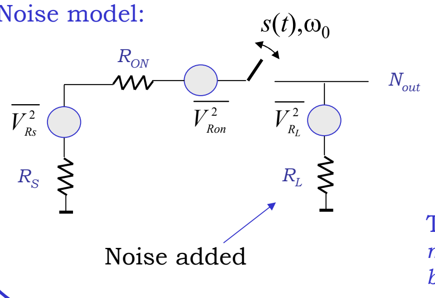


$$\boxed{G_C = \frac{1}{\pi}\cdot\frac{R_L}{R_L + R_{ON}}}$$

Usually $R_L >> R_{ON}$ for gain, so $G_C \approx 1/\pi$
and $R_{ON} = dV_{ds}/dI_d$ for linear region ($V_{ds} \approx 0$):

$$1/R_{ON} = k(V_{gs} - V_t) - kV_{ds} \approx k(V_{gs} - V_t)$$

Noise model:



Switch ON (for $T_0/2$):

$$S_n = 4kT\left[R_L \| (R_S + R_{ON})\right] \approx 4kT(R_S + R_{ON})$$

Switch OFF (for $T_0/2$):

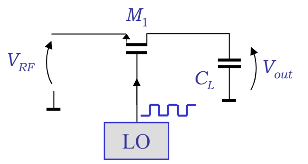
$$S_n = 4kTR_L$$

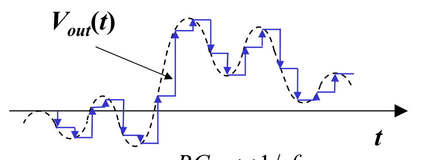
Total PSD: *might be dominated by R_L*

$$S_n = 4kT\frac{R_L + R_S + R_{ON}}{2}$$

Passive sampling mixer

Avoid noise of R_L, use C_L instead





$RC_L << 1/f_S$

Sampling mixer.
When switch OFF – in hold mode.
When ON – in track mode (*small time constant needed*).

Either Track- or Hold-segments can be used for down conversion

Hold values (samples) are preferred for ease in AD conversion.

for $f << |f_S|$:

$$S_{SH}(f) \cong 2kTR \times \frac{B_{eq}}{f_S}$$

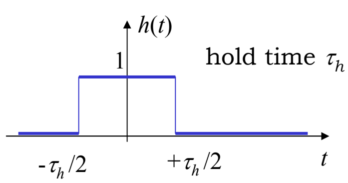
$$\boxed{S_{SH}(f) = (\tau_h f_S)^2 \frac{kT}{C_L f_S}}$$

Hold time $\tau_h < T_S$

Sampling mixer (cont'd)

$$v_{out}(t) = \sum_{k=-\infty}^{\infty} v_{RF}(kT_S) \cdot h(t - kT_S)$$

$$v_{out,\delta}(t) = \sum_{k=-\infty}^{\infty} v_{RF}(kT_S) \cdot \delta(t - kT_S)$$



$$v_{out}(t) = v_{out,\delta}(t) * h(t)$$

$$V_{out}(\omega) = \frac{1}{T_S} \sum_{k=-\infty}^{\infty} V_{RF}(\omega - k\omega_S) \times \tau_h \frac{\sin \omega\tau_h/2}{\omega\tau_h/2}$$

For simplicity we assume the track part to be negligible

$$\boxed{V_{out}(\omega) = (\tau_h f_S) \frac{\sin \omega\tau_h/2}{\omega\tau_h/2} \sum_{k=-\infty}^{\infty} V_{RF}(\omega - k\omega_S)}$$

Also useful to estimate noise

Replicas at higher freq. are suppressed by the sinc function

The conversion gain for $f_{IF} << f_S$ equals $(\tau_h f_S)$

Sampling mixer (cont'd)

Noise:

Switch permanently ON (no switching):

$R = R_S + R_{ON}$

Noise power at output:

$$N_{out} = \int_0^\infty |H(\omega)|^2 S_R \, df = \int_0^\infty \frac{4kTR}{1+(\omega R C_L)^2} df$$

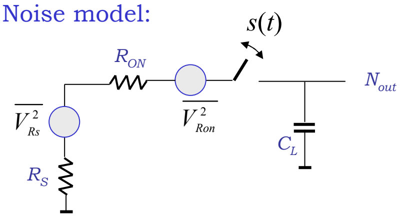
$$= \frac{4kTR}{2\pi R C_L} \cdot \mathrm{arctg}(\omega R C_L)\Big|_0^\infty = \frac{kT}{C_L}$$

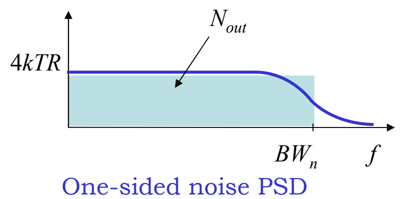
$$N_{out} = 4kTR \cdot BW_n$$

$$\boxed{BW_n = \frac{1}{4RC_L}}$$

Equivalent noise bandwidth

Noise model:





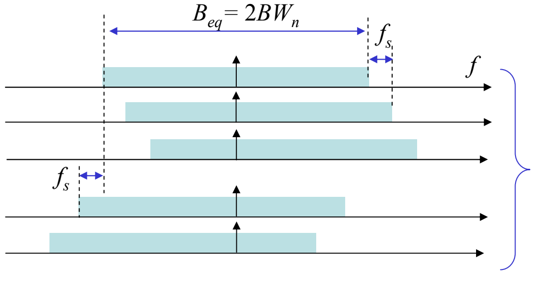
One-sided noise PSD (for no switching)

Sampling mixer (cont'd)

Noise (cont'd)

Switching case

$$\boxed{S_{SH}(f) = (\tau_h f_S)^2 \left(\frac{\sin(\pi \tau_h f)}{\pi \tau_h f}\right)^2 \sum_{n=-\infty}^{\infty} S_{n0}(f - n f_s)}$$



Folding of noise PSD, folding factor = B_{eq}/f_s

Two-sided PSD needed: $S_{n0}=2kTR$

for $f << |f_S|$:

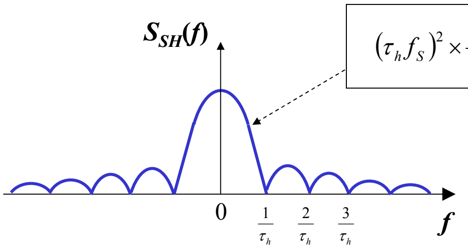
$$S_{SH}(f) \cong (\tau_h f_S)^2 \times 2kTR \times \frac{B_{eq}}{f_S}$$

$$\boxed{S_{SH}(f) = (\tau_h f_S)^2 \frac{kT}{C_L f_S}}$$

For higher frequency the shaping function prevails

Sampling mixer (cont'd)

$$\left(\tau_h f_S\right)^2 \times \frac{kT}{C_L f_S} \times \left(\frac{\sin(\pi \tau_h f)}{\pi \tau_h f}\right)^2$$

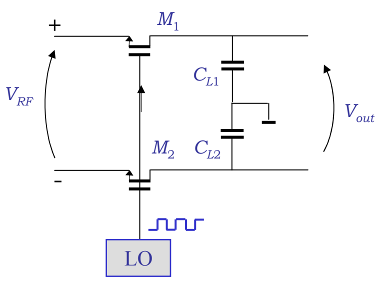


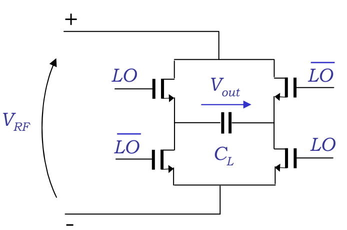
Noise PSD in a sampling mixer is inversely proportional to C_L and f_S, and shaped at higher frequencies

Noise figure for $f_{IF} << f_S$:

$$NF_{SH} = \frac{S_{in}/N_{SHin}}{S_{out}/N_{SHout}} = \frac{N_{SHout}}{N_{SHin}} = \left(\tau_h f_S\right)^2 \frac{kT}{C_L f_S} \times \frac{1}{2kTR_S} = \frac{\left(\tau_h f_S\right)^2}{2C_L R_S f_s}$$

Differential and balanced passive mixers



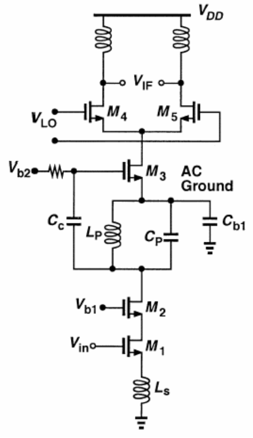


Balanced: has lower feedthrough but more prone for mismatch

Double gain (can be also bootstrap),
NF reduced by ~3dB,
even HD rejected when no mismatch,
but differential LNA needed (!)

High IP3 and 1dB point → dynamic range

Merged LNA and mixer *)



0.25µm CMOS, V_{DD}=2.5V, I_{DD}=2.5mA

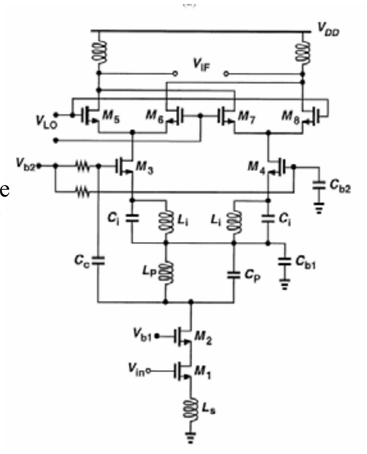
Stacked LNA and mixer in heterodyne Rx at 2.4GHz,

The bias current is reused to save power, L's can be used as mixer load since IF is high (800MHz)

LO=1.6GHz, L_pC_p tuned at 2.4GHz, and L_iC_i at 800MHz to reject image

Gilbert reduces LO feedthrough

NF = 3dB, IIP3 = -16dBm,
G = 29dBm, IR = 40dB



*) B.Razavi et al. IEEE J.Solid St.,No.2, 2003

Summary

- Mixers critical in RF design, trade-off linearity and power or NF and power
- Mixers more noisy than LNAs
- Feedthrough and leakage critical in mixers
- Mixers have more effect on linearity of front-end in Rx than LNAs
- Passive have usually worse NF (no gain) but provide better linearity than basic Gilbert's mixer
- LO jitter critical at high RF frequencies – much inherent noise produced by a mixer
- Passive sampling mixer useful as input stage for ADC (then more processing performed digitally)