This situation represents an interesting contrast to systems such as RAKE where a time diversity improvement is obtained by having several independently fading paths instead of a single fading path with the same total average power. Obviously, the system of Fig. 3 performs best when S_0 arrives via a single rather than several paths. There are two main reasons for this difference. First, as far as time diversity is concerned, the system of Fig. 3 is making full use of this signal characteristic since $T \gg \tau_0$. Additional paths give no additional diversity advantage but result in lowered output SNR for a given S_0. Secondly, systems such as RAKE require relatively high information rates concerning medium variations in order to properly process the received signal so as to take advantage of the time diversity characteristics of multipath. Obviously, such medium information is unavailable under the conditions assumed for the system of Fig. 3.

Doppler Shift

If the signal is received with a frequency error spread of Δf cps due to Doppler or other causes, a loss in output SNR will result. If the received signal frequency is changing during the transmission period or if the precise frequency cannot be determined, then multiple processing of the type shown in Fig. 3 must be used and the outputs combined before the decision circuit. The processors would be designed to operate at center frequencies with a spacing of about $1/4\tau$ cps. The loss factor due to Doppler or frequency error which would be applied to (53) would be about $4\tau\Delta f$.

V. Conclusions

The communications problem for which a solution was attempted in this paper involved a fading channel under conditions of intense interference. The solutions obtained are valid only for low SNR's but it is believed that important practical applications exist which can make use of the results obtained. At sufficiently low SNR's nearly all media must be considered as fading or time-variant due to the long symbol periods required for operation. The results show that relatively low capacities result under the conditions assumed and that other capacity formulas, based on more favorable channel conditions, may give results that are in error by several orders of magnitude, if such formulas are mistakenly applied to the problem stated.

Acknowledgment

The writer is grateful for the time and effort given so willingly by his friends in support of the personal, unsponsored work reported in this paper. Special credits are due Dr. C. A. Stutt and Dr. G. M. White of the General Electric Company Research Laboratory.

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Gate Noise in Field Effect Transistors at Moderately High Frequencies*

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Summary—At higher frequencies the gate noise of a field effect transistor increases rapidly with increasing frequency. This effect is here attributed to the thermal noise of the conducting channel and is caused by the capacitive coupling between the channel and the gate. The noise is represented by gate and drain noise current generators i_g and i_d, respectively; an approximation method is developed that allows calculation of $\overline{i_g^2}$, $\overline{i_d^2}$ and $\overline{i_g^* i_d}$ for moderately high frequencies. The correlation coefficient of i_g and i_d is imaginary and amounts to about 0.40j under saturated conditions. $\overline{i_g^2}$ can be expressed in terms of the noise resistance R_n and the gate-source capacitance C_{gs}. It is shown that the correlation has only a slight influence on the noise figure F and that $(F_{\min}-1)$ varies as $\omega C_{gs} R_n$ over a wide frequency range.

* Received July 23, 1962; revised manuscript received, October 22, 1962. This work was supported by a U. S. Signal Corps Contract.
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I. Introduction

The noise behavior of field effect transistors at low frequencies is at present well understood. Shockley [1] has calculated the dc characteristic and the small signal ac parameters of the device. The noise can be represented by two current generators i_g and i_d at the input and the output, respectively. Van der

Ziel [2] has shown that the output noise current generator is caused by the thermal noise of the conducting channel, modified by modulation effects. He finds

$$\overline{i_d^2} = 4kTg_{\max}Q(V_g, V_d)\Delta f \tag{1}$$

where V_g is the gate bias, V_d the drain bias, $g_{\max}$ the transconductance in the saturated part of the characteristic for the given gate bias V_g, and $Q(V_g, V_d)$ is a factor that is slightly less than unity. The input noise current generator [2], [3] is caused by the shot noise of two currents I_1 and I_2; I_1 is the current due to electrons arriving at the gate and holes leaving the gate and I_2 is the current due to electrons leaving the gate and holes arriving at the gate. Hence

$$\overline{i_g^2} = \overline{i_{g0}^2} = 2e(I_1 + I_2)\Delta f. \tag{2}$$

Since the two noise sources are independent, i_g and i_d are uncorrelated.

At higher frequencies the noise can again be represented by the two current generators i_g and i_d, but the physical cause of i_g is somewhat different. Fig. 1 shows the full equivalent circuit of a field effect transistor for arbitrary frequencies. It consists of the two current generators mentioned above and the admittances Y_{11}, Y_{12} and Y_{22}; the signal transfer properties of the circuit are described by the generator $Y_{21}v_g$, where v_g is the ac gate voltage. It is seen from the figure that i_g and i_d are, respectively, the short-circuit noise currents at the input and output, if both input and output are short-circuited.

That i_g will now have a component that is partially correlated with i_d can be seen as follows. Due to the distributed thermal noise EMF's in the channel, the noise current i_d will flow through the short-circuited output. But in addition, noise voltages will be developed along the channel and, because of the capacitive coupling between the channel and the gate, a capacitive noise current will flow through the short-circuited input. Since this component of i_g comes from the same noise EMF's as i_d, one would expect i_g and i_d to become partly correlated. This thermal noise component of i_g may be considerably larger than the shot noise component given by (2).

In a rigorous theory the transistor should be treated as a distributed active line. But at moderately high frequencies it may be assumed that the capacitive currents flowing to the gate are small in comparison with the conductive currents flowing through the channel. In that case suitable approximation methods can be developed that allow to calculate $\overline{i_g^2}$, $\overline{i_g^* i_d}$ and $\overline{i_d^2}$ in a straightforward manner. It is the aim of this paper to treat such a method in detail.

Experimentally it is found that $\overline{i_d^2}$ is independent of frequency [4] whereas $\overline{i_g^2}$ contains a term [3], [4] varying as ω^2, both over a wide frequency range. Even without a detailed calculation it is easily seen that the proposed model can explain this noise behavior. For as long as capacitive currents are small, i_d will be independent of frequency and i_g will vary as $j\omega$. As a consequence $\overline{i_d^2}$ is frequency independent, $\overline{i_g^2}$ varies as ω^2 and $\overline{i_g^* i_d}$ varies as $j\omega$. The same is brought out by the detailed calculation of Section III.

Fig. 2 shows a field effect transistor, with bias voltages applied, that is short-circuited ac-wise at both input and output. Let the uniform, conducting channel be p-type and let the device have planar geometry. Let the device have unit width and let $2a$ be the thickness of the p-type layer and L the length of the conducting channel. Let $2b(x)$ be the thickness of the conducting channel at a distance x from the drain contact. In the figure the conducting channel is divided into small sections Δx and independent noise EMF's are assumed in each section.

The approximation method mentioned above proceeds as follows. First, one calculates the short-circuited output noise current Δi_d due to the thermal noise EMF ΔW_x in particular section Δx, determines the fluctuating channel width $2\Delta b(x)$ as a function of the distance x to the source contact and evaluates the fluctuating charge Δq stored in the space charge region because of the above noise EMF in the section Δx (Fig. 2). This allows one to determine $\overline{\Delta i_d^2}$, $\overline{\Delta_q^2}$ and $\overline{\Delta q^* \Delta i_d}$. By summing over all sections Δx, which amounts to integrating over the length of the channel, one obtains $\overline{i_d^2}$, $\overline{q^2}$ and $\overline{q^* \Delta i_d}$, where i_d is the short-circuit noise current at the output and q the total fluctuating charge in the space charge region. Bearing in mind that the fluctuating gate current i_g is related to q by the relation

$$i_g = j\omega q \tag{3}$$

one may calculate $\overline{i_d^2}$, $\overline{i_g^2}$ and $\overline{i_g^* i_d}$.

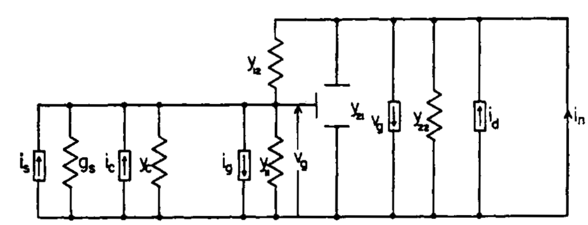

Fig. 1—Equivalent circuit of the device for arbitrary frequencies.

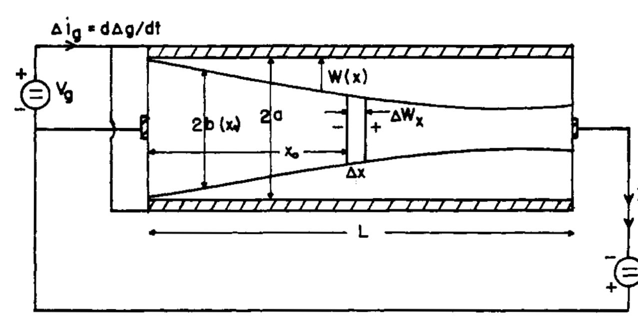

Fig. 2—Cross section of a planar field-effect transistor showing the source, the gate, the drain, the channel of width $2b$ and the space-charge regions of width $(b-a)$. $W(x)$ is the bias between the conducting channel and the gate.

II. Calculation of Δi_d and $\Delta b(x)$ due to the Noise EMF ΔW_x

Let $W(x)$ be the bias voltage between the gate and the channel at a distance x from the source contact. If V_g is the gate bias, $-V_d$ the drain bias and V_{dif} the diffusion potential of the gate-channel junction, then $W=W_s=(V_g+V_{\text{dif}})$ at the source side of the gate contact and $W=W_d=(V_g+V_{\text{dif}}-V_d)$ at the drain side of the gate contact.

According to Shockley [1] the following relations hold. If W_{00} is the bias needed for channel cutoff, then the relation between $W(x)$ and $b(x)$ is

$$W = W_{00}\left(1 - \frac{b}{a}\right)^2. \quad (4)$$

The current I is given by the equation

$$I = g(W)\frac{dW}{dx}; \quad g(W) = g_0\left[1 - \left(\frac{W}{W_{00}}\right)^{1/2}\right] \quad (5)$$

where $g(W)$ is the conductance per unit length at a distance x from the source contact, and g_0 is the conductance per unit length of the open channel. Under equilibrium conditions $W=W_0$, $b=b_0$ and $I=I_0$,

$$W_0 = W_{00}\left(1 - \frac{b_0}{a}\right)^2 \quad (4a)$$

$$I_0 = g(W_0)\frac{dW_0}{dx}; \quad g(W_0) = g_0\left[1 - \left(\frac{W_0}{W_{00}}\right)^{1/2}\right]. \quad (5a)$$

According to Shockley,

$$I_0 = \frac{g_0}{L}\left[W_d - W_s - \tfrac{2}{3}\left(\frac{W_d^{3/2} - W_s^{3/2}}{W_{00}^{1/2}}\right)\right] = \frac{g_0 W_{00}}{L} f_1(y, z) \quad (6)$$

where $y=W_d/W_{00}$ and $z=W_s/W_{00}$ and

$$f_1(y, z) = \left[(y - z) - \tfrac{2}{3}(y^{3/2} - z^{3/2})\right]. \quad (6a)$$

Under saturated conditions $W_d=W_{00}$ or $y=1$.

It is also convenient to know the capacitances C_{gs} and and C_{gd} between gate and source, and gate and drain respectively. These capacitances are calculated in the Appendix. If $-\rho_0$ is the (negative) space charge density in the space charge region, then

$$C_{gs} = \frac{2\rho_0 a L}{W_{00}} f_2(y, z) \quad (7)$$

$$C_{gd} = \frac{2\rho_0 a L}{W_{00}} f_3(y, z) \quad (7a)$$

where the functions $f_2(y, z)$ and $f_3(y, z)$ are defined in the Appendix. The total gate capacitance C_{gg} is, therefore,

$$C_{gg} = (C_{gs} + C_{gd}) = \frac{2\rho_0 a L}{W_{00}}\left[f_2(y, z) + f_3(y, z)\right]. \quad (7b)$$

If a noise EMF ΔW_x is generated between x_0 and $(x_0+\Delta x)$, then anywhere along the channel

$$W = W_0 + \Delta W, \qquad b = b_0 + \Delta b, \qquad I = I_0 + \Delta i_d,$$

where $\Delta W(x)$ and $\Delta b(x)$ are functions of x but Δi_d is independent of x. The function $\Delta W(x)$ satisfies the following conditions:

a) $\Delta W(x)=0$, at $x=0$, and at $x=L$, since the gate and drain bias voltages are fixed and the output of the device is short-circuited.

b) $[\Delta W(x_0+\Delta x) - \Delta W(x_0)] = -\Delta W_x$, for if ΔW_x is the EMF in the section Δx, then the bias at $(x_0+\Delta x)$ is ΔW_x lower than at x_0.

The first step is to calculate $\Delta W(x)$ and Δi_d. According to (5)

$$(I_0 + \Delta i_d) = g(W_0 + \Delta W)\frac{d(W_0 + \Delta W)}{dx}.$$

Substituting (5a) one obtains

$$\Delta i_d = -g_0\left(\frac{1}{2W_0^{1/2}W_{00}^{1/2}}\right)\frac{dW_0}{dx}\Delta W + g_0\left[1 - \left(\frac{W_0}{W_{00}}\right)^{1/2}\right]\frac{d\Delta W}{dx}$$
$$= g_0\frac{d}{dx}\left\{\left[1 - \left(\frac{W_0}{W_{00}}\right)^{1/2}\right]\Delta W\right\}. \quad (8)$$

The solution of this differential equation is

$$\left[1 - \left(\frac{W_0}{W_{00}}\right)^{1/2}\right]\Delta W = \frac{\Delta i_d}{g_0}x \qquad \text{for } 0 < x < x_0, \quad (9)$$

satisfying the condition $\Delta W=0$ at $x=0$ and

$$\left[1 - \left(\frac{W_0}{W_{00}}\right)^{1/2}\right]\Delta W = \frac{\Delta i_d}{g_0}(x - L) \qquad \text{for } (x_0 + \Delta x) < x < L, \quad (10)$$

satisfying the conditions $\Delta W=0$ at $x=L$. Consequently there is a jump $-\Delta i_d L/g_0$ in the right-hand side of the equation between x_0 and $(x_0+\Delta x)$. Hence

$$-\left[1 - \frac{W_0^{1/2}(x_0)}{W_{00}^{1/2}}\right]\Delta W_x = -\frac{\Delta i_d L}{g_0},$$

or

$$\Delta i_d = \frac{g_0}{L}\left[1 - \frac{W_0^{1/2}(x_0)}{W_{00}^{1/2}}\right]\Delta W_x. \quad (11)$$

This result agrees with van der Ziel's earlier calculation [2] and indicates that $\overline{i_d^2}$ is equal to its low frequency value in this approximation.

The next step is to calculate the fluctuating width $2\Delta b(x)$ of the channel. From (4)

$$\frac{\Delta b}{a} = -\frac{\Delta W}{2W_{00}^{1/2}W_0^{1/2}} \tag{12}$$

so that according to (9) and (10)

$$\frac{\Delta b}{a} = -\frac{\Delta i_d}{g_0}\frac{x}{2W_{00}^{1/2}W_0^{1/2}[1-(W_0/W_{00})^{1/2}]} \tag{12a}$$

for $0<x<x_0$, and

$$\frac{\Delta b}{a} = -\frac{\Delta i_d}{g_0}\frac{(x-L)}{2W_{00}^{1/2}W_0^{1/2}[1-(W_0/W_{00})^{1/2}]} \tag{12b}$$

for $(x_0+\Delta x)<x<L$.

III. Calculation of $\overline{i_g^2}$ and $\overline{i_g^* i_d}$

The next step is to calculate the fluctuating charge Δq stored in the two space charge regions. The distributed negative charge $Q'dx$ per length dx stored in these regions, each of width $(a-b)$, is

$$Q'dx = -2(a-b)\rho_0 dx = -2a\rho_0\left(1-\frac{b}{a}\right)dx \tag{13}$$

since $-\rho_0$ is the negative space charge density in the space charge region. This is compensated by an equal and opposite charge Qdx per length dx at the gate contact

$$Qdx = 2a\rho_0\left(1-\frac{b}{a}\right)dx. \tag{13a}$$

Consequently the fluctuation Δb in b between x and $(x+dx)$ produces a fluctuating charge

$$d\Delta q = -2a\rho_0\left(\frac{\Delta b}{a}\right)dx \tag{13b}$$

at the gate. Applying (12a) and (12b), one obtains for the total fluctuating charge Δq at the gate caused by the noise EMF ΔW_x between x_0 and $(x_0+\Delta x)$

$$\begin{aligned}\Delta q &= \int_0^{x_0} -2a\rho_0\frac{\Delta b(x)}{a}dx + \int_{x_0}^{L} -2a\rho_0\frac{\Delta b(x)}{a}dx \\ &= \frac{2a\rho_0\Delta i_d}{g_0}\left\{\int_0^{L}\frac{xdx}{2W_{00}^{1/2}W_0^{1/2}[1-(W_0/W_{00})^{1/2}]}\right. \\ &\quad \left. -\int_{x_0}^{L}\frac{Ldx}{2W_{00}^{1/2}W_0^{1/2}[1-(W_0/W_{00})^{1/2}]}\right\}.\end{aligned} \tag{14}$$

Applying (5), one can eliminate x as follows:

$$dx = \frac{g_0}{I_0}\left[1-\left(\frac{W_0}{W_{00}}\right)^{1/2}\right]dW_0 \tag{15}$$

$$x = \frac{g_0 W_{00}}{I_0}\left\{\frac{W_0}{W_{00}}-\frac{2}{3}\left(\frac{W_0}{W_{00}}\right)^{3/2} - \left[\frac{W_s}{W_{00}}-\frac{2}{3}\left(\frac{W_s}{W_{00}}\right)^{3/2}\right]\right\}. \tag{15a}$$

Consequently

$$\begin{aligned}\Delta q &= \frac{2a\rho_0\Delta i_d}{g_0}\left\{\frac{g_0^2 W_{00}}{I_0^2}\cdot\int_{W_s}^{W_d}\left\{\frac{W_0}{W_{00}}-\frac{2}{3}\left(\frac{W_0}{W_{00}}\right)^{3/2}\right.\right. \\ &\quad \left.-\left[\frac{W_s}{W_{00}}-\frac{2}{3}\left(\frac{W_s}{W_{00}}\right)^{3/2}\right]\right\}\frac{dW_0}{2W_0^{1/2}W_{00}^{1/2}} \\ &\quad \left.-\frac{g_0 L}{I_0}\int_{W_0(x_0)}^{W_d}\frac{dW_0}{2W_0^{1/2}W_{00}^{1/2}}\right\} \\ &= \frac{2a\rho_0 L\Delta i_d}{I_0}\left[-p+\frac{W_0^{1/2}(x_0)}{W_{00}^{1/2}}\right]\end{aligned} \tag{16}$$

where

$$p = \frac{1}{f_1(y,z)}\left[-\tfrac{1}{3}(y^{3/2}-z^{3/2})+\tfrac{1}{6}(y^2-z^2) + (z-\tfrac{2}{3}z^{3/2})(y^{1/2}-z^{1/2})\right]+y^{1/2}. \tag{17}$$

Eq. (16) shows that Δq and Δi_d are fully correlated. The quantity p is generally positive.

Substituting (7), (16) may be written

$$\Delta q = \frac{-C_{gs}}{f_1(y,z)f_2(y,z)}\left[p-\frac{W_0^{1/2}(x_0)}{W_{00}^{1/2}}\right]\cdot\left[1-\frac{W_0^{1/2}(x_0)}{W_{00}^{1/2}}\right]\Delta W_x. \tag{18}$$

Substituting $u=W_0(x_0)/W_{00}$ and bearing in mind that

$$\overline{\Delta W_x^2} = 4kT\Delta f\frac{\Delta x}{g\{W_0(x_0)\}} = \frac{4kT\Delta f}{(I_0/W_{00})}\frac{\Delta W(x_0)}{W_{00}} = \frac{4kT\Delta f}{(g_0/L)f_1(y,z)}\Delta u, \tag{19}$$

one obtains

$$\overline{\Delta q^*\Delta i_d} = -\frac{4kT\Delta f C_{gs}}{f_1^2(y,z)f_2(y,z)}(p-u^{1/2})(1-u^{1/2})^2\Delta u \tag{20}$$

$$\overline{\Delta q^2} = \frac{4kT\Delta f}{(g_0/L)}\frac{C_{gs}^2}{f_1^3(y,z)f_2^2(y,z)}\cdot(p-u^{1/2})^2(1-u^{1/2})^2\Delta u. \tag{21}$$

Integrating with respect to u between the lower limit z and the upper limit y yields

$$\overline{q^* i_d} = -4kT\Delta f C_{gs}\frac{g_2(y,z)}{f_1^2(y,z)f_2(y,z)} \tag{22}$$

$$\overline{q^2} = \frac{4kT\Delta f}{g_{\max}}C_{gs}^2\frac{g_3(y,z)(1-z^{1/2})}{f_1^3(y,z)f_2^2(y,z)}, \tag{23}$$

where

$$g_{\max} = \frac{g_0}{L}(1-z^{1/2}) \tag{24}$$

is the transconductance under saturated conditions $(y=1)$,

$$h_1(y, z) = \int_z^y (1 - u^{1/2})^2 du = \left[(y - z) - \tfrac{4}{3}(y^{3/2} - z^{3/2}) + \tfrac{1}{2}(y^2 - z^2)\right], \quad (25)$$

$$h_2(y, z) = \int_z^y u^{1/2}(1 - u^{1/2})^2 du = \left[\tfrac{2}{3}(y^{3/2} - z^{3/2}) - (y^2 - z^2) + \tfrac{2}{5}(y^{5/2} - z^{5/2})\right]; \quad (26)$$

$$h_3(y, z) = \int_z^y u(1 - u^{1/2})^2 du = \left[\tfrac{1}{2}(y^2 - z^2) - \tfrac{4}{5}(y^{5/2} - z^{5/2}) + \tfrac{1}{3}(y^3 - z^3)\right], \quad (27)$$

and functions $g_1(y, z)$, $g_2(y, x)$ and $g_3(y, z)$ are defined as

$$g_1(y, z) = h_1(y, z); \qquad g_2(y, z) = p h_1(y, z) - h_2(y, z)$$
$$g_3(y, z) = p^2 h_1(y, z) - 2p h_2(y, z) + h_3(y, z). \quad (28)$$

In this notation van der Ziel's earlier result [2] for $\overline{i_d^2}$ may be written

$$\overline{i_d^2} = 4kTg_{\max}\Delta f \frac{g_1(y, z)}{(1 - z^{1/2}) f_1(y, z)} \cdot \quad (29)$$

Consequently

$$\overline{i_g^* i_d} = 4kT\Delta f j\omega C_{gs} \frac{g_2(y, z)}{f_1^2(y, z) f_2(y, z)} \quad (30)$$

$$\overline{i_g^2} = \overline{i_{g0}^2} + \frac{4kT\Delta f}{g_{\max}} \omega^2 C_{gs}^2 \frac{g_3(y, z)(1 - z^{1/2})}{f_1^3(y, z) f_2^2(y, z)} \quad (31)$$

where $\overline{i_{g0}^2}$ is the low-frequency value of $\overline{i_g^2}$, given by (2).

The theory presented here holds for the bias conditions $z \leqq y < 1$. Experimentally it is found that the noise parameters of field effect transistors approach limiting values at the onset of saturation ($y=1$ in our notation) which are maintained well into the saturation region of the characteristic. It is thus appropriate to define the "theoretical values" of the noise parameters in the saturation region as the values given by (29)–(31) in the limit $y \rightarrow 1$, even though the theory in and by itself does not apply inside the saturation region of the characteristic.

IV. Application of the Results

Having calculated $\overline{i_g^2}$, $\overline{i_g^* i_d}$ and $\overline{i_d^2}$, we now apply the results to the calculation of the noise figure F. The method for calculating F is the same as for the HF triode. Since we need our expression for F in a form adapted to the results of Section III we give here its derivation.

Let g_s be the signal source conductance and $Y_c = g_c + jb_c$ the admittance of the input circuit. The noise of the signal source and the input circuit are represented by current generators i_s and i_c, respectively, where

$$\overline{i_s^2} = 4kTg_s\Delta f; \qquad \overline{i_c^2} = 4kTg_c\Delta f. \quad (32)$$

By evaluating the short-circuit noise current i_n in the output for each of the noise sources, one obtains for the noise figure F, with the help of Fig. 1,

$$F = 1 + \frac{g_c}{g_s} + \frac{\overline{|i_g + i_d(g_s + Y_c + Y_{\text{in}})/Y_{\text{tr}}|^2}}{\overline{i_s^2}} \quad (33)$$

where $Y_{\text{in}} = Y_{11} + Y_{12} = g_{\text{in}} + jb_{\text{in}}$, and $Y_{\text{tr}} = Y_{21} - Y_{12} = g_{\text{tr}} + jb_{\text{tr}}$.

To simplify (33), i_g is split into a part i_g' that is fully correlated with i_d and a part i_g'' that is uncorrelated with i_d

$$i_g = i_g' + i_g'', \quad \text{or} \quad \overline{i_g^2} = \overline{i_g'^2} + \overline{i_g''^2}. \quad (34)$$

One now introduces the input noise conductance g_n, the equivalent noise resistance R_n and the correlation admittance Y_{cor} of the device with the help of the definitions

$$4kTg_n\Delta f = \overline{i_g''^2} = \overline{i_g^2} - \overline{i_g'^2} = \overline{i_g^2} - \frac{|\overline{i_g i_d^*}|^2}{\overline{i_d^2}}, \quad (35)$$

$$4kTR_n\Delta f = \frac{\overline{i_d^2}}{|Y_{\text{tr}}|^2}, \quad (36)$$

$$Y_{\text{cor}} = g_{\text{cor}} + jb_{\text{cor}} = Y_{\text{tr}} \frac{i_g'}{i_d} = Y_{\text{tr}} \frac{\overline{(i_g i_d^*)}}{\overline{i_d^2}} \cdot \quad (37)$$

Eq. (33) then becomes

$$F = 1 + \frac{g_c + g_n + R_n|g_s + Y_c + Y_{\text{in}} + Y_{\text{cor}}|^2}{g_s} \cdot \quad (38)$$

Considering F as a function of the circuit susceptance b_c, it is seen that (38) has a minimum value

$$F' = 1 + \frac{g_c + g_n + R_n(g_s + g_c + g_{\text{in}} + g_{\text{cor}})^2}{g_s} \quad (39)$$

for

$$(b_c + b_{\text{in}} + b_{\text{cor}}) = 0. \quad (39a)$$

Considered as a function of g_s, F' has a minimum value $F'_{\min}$

$$F'_{\min} = 1 + 2R_n(g_c + g_{\text{in}} + g_{\text{cor}}) + 2\sqrt{R_n(g_c + g_n) + R_n^2(g_c + g_{\text{in}} + g_{\text{cor}})^2}$$

for

$$g_s = (g_s)_{\text{opt}} = \sqrt{(g_c + g_n)/R_n + (g_c + g_{\text{in}} + g_{\text{cor}})^2}. \quad (41)$$

It often happens that $R_n^2(g_c + g_{\text{in}} + g_{\text{cor}})^2 \ll R_n(g_c + g_n)$. In that case, (40) and (41) may be written

$$F'_{\min} = 1 + 2\sqrt{R_n(g_c + g_n)} \quad (40a)$$

for

$$g_s = (g_s)_{\text{opt}} = \sqrt{(g_c + g_n)/R_n}. \quad (41a)$$

All reference to the correlation conductance has then disappeared from the equations.

It is convenient to introduce the (complex) correlation coefficient c, defined as

$$c = \frac{\overline{i_g{}^* i_d}}{\sqrt{(\overline{i_g{}^2} \cdot \overline{i_d{}^2})}}. \tag{42}$$

In that case

$$4kTg_n\Delta f = \overline{i_g{}^2}(1 - |c|^2) \tag{43}$$

$$Y_{\text{cor}} = c^* Y_{\text{tr}} \sqrt{\frac{\overline{i_g{}^2}}{\overline{i_d{}^2}}}. \tag{44}$$

Applying these results to the discussion of Section III, it is seen that for $\overline{i_{go}{}^2} \gg \overline{i_{go}{}^2}$:

$$c = j\frac{g_2(y, z)}{g_1(y, z)g_3(y, z)} \tag{45}$$

so that c is imaginary. At intermediate frequencies the value of Y_{tr} is practically real and equal to the low-frequency transconductance g_m of the device. As a consequence the correlation admittance Y_{cor} is reactive and the correlation conductance g_{cor} is negligible at those frequencies.

For a more detailed discussion the various noise parameters must be determined. Field effect transistors are generally used in the saturated region, so that $y=1$. Because the transconductance of the device is equal to $g_{\max}$ under that condition, the noise resistance R_n is

$$R_n = \frac{1}{g_{\max}} \frac{g_1(1, z)}{(1 - z^{1/2})f_1(1, z)} = \frac{1}{g_{\max}} \cdot \frac{1}{2} \frac{(1 + 3z^{1/2})}{(1 + 2z^{1/2})} \tag{46}$$

which is of the order of $1/g_{\max}$. The value of $\overline{i_g{}^2}$ is

$$\begin{aligned}\overline{i_g{}^2} &= \overline{i_{g0}{}^2} + 4kTR_n\Delta f\omega^2 C_{gs}{}^2 \cdot \frac{g_3(1, z)(1 - z^{1/2})^2}{f_1{}^2(1, z)f_2{}^2(1, z)g_1(1, z)} \\ &= \overline{i_{g0}{}^2} + 4kTR_n\Delta f\omega^2 C_{gs}{}^2 \cdot \frac{24(1 + 2z^{1/2})^2}{(1 + z^{1/2})^2(1 + 3z^{1/2})} \\ &\quad \cdot \left[\frac{1}{24}\frac{(1 + 3z^{1/2})^3}{(1 + 2z^{1/2})^2} - \frac{1}{10}\frac{(1 + 3z^{1/2})(1 + 4z^{1/2})}{(1 + 2z^{1/2})} + \frac{1}{15}(1 + 5z^{1/2})\right]. \end{aligned} \tag{47}$$

Neglecting $\overline{i_{g0}{}^2}$, the correlation coefficient is

$$c = j\frac{\left[\frac{1}{10}(1 + 4z^{1/2}) - \frac{1}{12}\frac{(1 + 3z^{1/2})}{(1 + 2z^{1/2})}\right]}{\sqrt{\frac{(1 + 3z^{1/2})}{6}\left[\frac{1}{24}\frac{(1 + 3z^{1/2})^3}{(1 + 2z^{1/2})^2} - \frac{1}{10}\frac{(1 + 3z^{1/2})(1 + 4z^{1/2})}{(1 + 2z^{1/2})} + \frac{1}{15}(1 + 5z^{1/2})\right]}}. \tag{48}$$

Figs. 3 and 4 show $|c|$ and

$$H(z) = (\overline{i_g{}^2} - \overline{i_{g0}{}^2})/(4kTR_n\Delta f\omega^2 C_{gs}{}^2)$$

as functions of z, respectively.

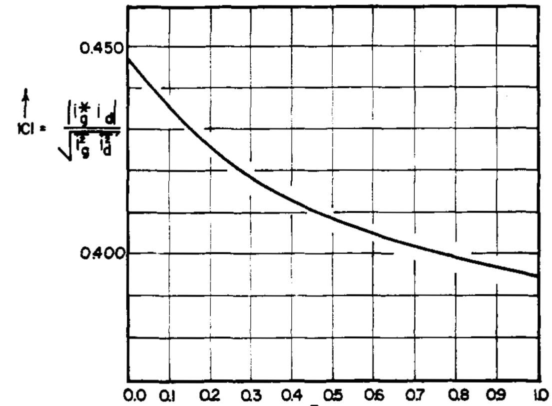

Fig. 3—$c = \overline{i_g{}^* i_d}/\sqrt{\overline{i_g{}^2} \cdot \overline{i_d{}^2}}$ plotted as a function of the normalized gate bias $z = (V_g + V_{\text{dif}})/W_{00}$ for the saturated condition and with $\overline{i_{g0}{}^2} \ll \overline{i_g{}^2}$.

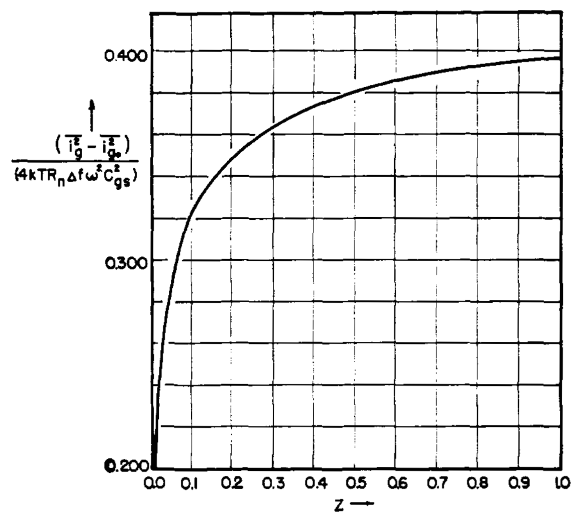

Fig. 4—Normalized gate noise $(\overline{i_g{}^2} - \overline{i_{g0}{}^2})/(4kTR_n\Delta f\omega^2 C_{gs}{}^2)$ plotted as a function of the normalized gate bias $z = (V_g + V_{\text{dif}})/W_{00}$ for the saturated condition.

It is seen from Fig. 3 that $|c|$ is of the order of 0.4; hence only a relatively small error is made in (43) if the factor $(1 - |c|^2)$ is omitted. This means that the effect of correlation can be ignored without great loss of accuracy and that little improvement in noise figure can be expected by properly detuning the input circuit.

It is seen from Fig. 4 that $\overline{i_g{}^2}$ may be written

$$\overline{i_g{}^2} = H(z) \cdot 4kTR_n\Delta f\omega^2 C_{gs}{}^2 + \overline{i_{g0}{}^2} \tag{49}$$

where $H(z)$ lies between 0.3 and 0.4 for practical bias conditions. Ignoring correlation, the input noise conductance g_n can thus be expressed as

$$g_n = H(z)R_n\omega^2 C_{gs}{}^2 + g_{n0} \tag{50}$$

where g_{n0} is the low-frequency value of g_n. Consequently the minimum noise figure $F_{\min}'$ can be written as

$$F'_{\min} \simeq 1 + 2\sqrt{R_n g_n} = 1 + 2\omega C_{gs} R_n \sqrt{H(z)} \qquad (51)$$

if the assumptions are made that $g_n \gg (g_c + g_g + g_{cor})^2 R_n$, $g_n \gg g_c$ and $g_n \gg g_{n0}$. *This means that the effect of the gate noise at intermediate frequencies can be considerably reduced by designing devices with a low noise resistance* R_n, *and a low capacitance* C_{gs}. In addition it is seen that $(F_{\min}' - 1)$ is approximately proportional to the frequency.

As a numerical example take $R_n = 2000$ ohms, $C_{gs} = 50$ $\mu\mu$f and $F(z) = 0.3$, a value expected for zero gate bias. Eq. (50) then gives $g_n \simeq 6 \times 10^{-5}$ mhos and (51) gives that $F_{\min}' \simeq 1.7$; the latter value is considerably larger than the minimum low frequency value of F.[1]

V. Conclusions

The approximation method outlined in Section I and worked out in detail in Section III thus allows one to calculate the noise performance of the field effect transistor at intermediate frequencies. The calculation ignores the effect of the series resistances r_s and r_d at the source and the drain side of the channel, respectively. The author intends to deal with this problem in a later paper.

$$f_2(y, z) = \frac{[\frac{2}{3}(y^{3/2} - z^{3/2}) - \frac{1}{2}(y^2 - z^2)](1 - z^{1/2}) - f_1(y, z)(z^{1/2} - z)}{[f_1(y, z)]^2} \qquad (57a)$$

$$f_3(y, z) = \frac{[-\frac{2}{3}(y^{3/2} - z^{3/2}) + \frac{1}{2}(y^2 - z^2)](1 - y^{1/2}) + f_1(y, z)(y^{1/2} - y)}{[f_1(y, z)]^2}. \qquad (57b)$$

Appendix

Field-Effect Transistor Capacitances

The space charge regions between x and $(x + dx)$ contain a charge

$$dQ = 2\rho_0(a - b)dx = 2\rho_0 a \left(\frac{W_0}{W_{00}}\right)^{1/2} dx. \qquad (52)$$

The total charge Q per unit width is therefore

$$\begin{aligned} Q &= \int_0^L 2\rho_0 a \left(\frac{W_0}{W_{00}}\right)^{1/2} dx \\ &= 2\rho_0 a \int_{W_s}^{W_d} \frac{g(W_0)(W_0/W_{00})^{1/2}}{g(W_0)(dW_0/dx)} dW_0 \\ &= \frac{2\rho_0 a g_0 W_{00}}{I_0} \int_z^y (1 - u^{1/2}) u^{1/2} du \\ &= \frac{2\rho_0 a L}{f_1 y, z)} [\tfrac{2}{3}(y^{3/2} - z^{3/2}) - \tfrac{1}{2}(y^2 - z^2)]. \qquad (53) \end{aligned}$$

where $u = W_0/W_{00}$, $y = W_d/W_{00}$, $z = W_s/W_{00}$ and $g(W_0)$ and I_0 are given by (5) and (6), respectively. The small-signal capacitances per unit width are defined as

$$\begin{aligned} C_{gg} &= \frac{\partial Q}{\partial V_g} = \frac{\partial Q}{\partial y}\frac{\partial y}{\partial V_g} + \frac{\partial Q}{\partial z}\frac{\partial z}{\partial V_g} \\ &= \frac{1}{W_{00}}\left(\frac{\partial Q}{\partial y} + \frac{\partial Q}{\partial z}\right) \qquad (54) \end{aligned}$$

$$C_{gd} = -\frac{\partial Q}{\partial V_d} = -\frac{\partial Q}{\partial y}\frac{\partial y}{\partial V_d} = \frac{1}{W_{00}}\frac{\partial Q}{\partial y} \qquad (55)$$

and hence

$$C_{gs} = C_{gg} - C_{gd} = \frac{1}{W_{00}}\frac{\partial Q}{\partial z}. \qquad (56)$$

Carrying out the differentiation yields

$$C_{gs} = \frac{2\rho_0 a L}{W_{00}} f_2(y, z); \qquad C_{gd} = \frac{2\rho_0 a L}{W_{00}} f_3(y, z) \qquad (57)$$

where

For zero drain bias $(y = z)$ one finds from de l'Hopital's rule

$$f_2(z, z) = f_3(z, z) = \frac{1}{4z^{1/2}} \qquad (58)$$

which goes to infinity for zero gate bias $(z = 0)$, as expected, and equals $\frac{1}{4}$ for complete cutoff $(z = 1)$.

For saturation $(y = 1)$,

$$f_2(1, z) = \frac{3}{2}\frac{(1 + z^{1/2})}{(1 + 2z^{1/2})^2}; \qquad f_3(1, z) = 0 \qquad (59)$$

so that $f_2(1,0) = \frac{3}{2}$ and $f_2 = (1,1) = \frac{1}{3}$.

[1] The value of g_n corresponds to an equivalent saturated diode current of about 3 μa. The experimental values [4] for units of similar capacitance are smaller. This discrepancy is probably caused by series resistance effects in the channel.

References

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