Potential inside coaxial line

 (Book "Electromagnetics"
by John D. ~~Kar~~ Kraus)

Since any point at a radius of 400 mm has an absolute potential of 5 V and any point at a radius of 100 mm an absolute potential of 20 V, we can draw circles at these radii and label them as 5 and 20 V equipotential contours. Also drawing circles at other potentials and radial field lines, we obtain the contour map of Fig. 2-16*a*. We observe that ***field and potential lines contours always intersect at right angles.***

In Fig. 2-16*b* the absolute potential is plotted as a function of radius. In this presentation the slope of the potential versus radius is a measure of the electric field intensity: The steeper the slope of the potential "hill," the greater the field intensity. Thus, at $r = 100$ mm, V = 20 V and at $r = 200$ mm, V = 10 V so that the average field intensity for this distance interval is

$$|\mathbf{E}| = \frac{\Delta V}{\Delta r} = \frac{20 - 10}{0.2 - 0.1} = \frac{10}{0.1} = 100 \text{ V m}^{-1}$$

Reducing the distance interval to an infinitesimal value, we can obtain the magnitude of the field intensity at a point as

$$|\mathbf{E}| = \lim_{\Delta r \to 0} \frac{\Delta V}{\Delta r} = \frac{dV}{dr} \tag{6}$$

Thus, the ***magnitude of the field equals the slope or gradient of the potential*** which is the inverse of (5) in which we obtain the potential as the line integral of the field. We shall discuss gradient further in Sec. 2-14.

The work to move a test charge along an equipotential contour or surface is zero ($\theta = 90°$). The maximum amount of work per unit distance is performed by moving normal to an equipotential surface. This coincides with the direction of the field.

The work to transport a charge around any *closed path* in a static field is zero since the path starts and ends at the same point. Thus, the upper and lower limits of the integrals in (5) become the same, and the result is zero. A property of the *static electric field* is, then, that *the line integral of this field around any closed path is zero*, that is,

$$\oint \mathbf{E} \cdot d\mathbf{L} = 0 \tag{7}$$

The circle on the integral sign of (7) signifies a closed path. A field for which (7) holds is called a *conservative or lamellar field.* It follows that the potential difference between any two points of a conservative field is independent of the path.

2-10 ELECTRIC FIELD STREAMLINES AND EQUIPOTENTIAL CONTOURS; ORTHOGONALITY

A field line indicates the direction of the force on a positive test charge introduced into the field. If the test charge is released, it accelerates in the direction of a field line.

In a uniform field the **E** lines are parallel and the equipotential lines form a parallel orthogonal set of lines as in Fig. 2-17*a*. Actually the equipotentials are plane surfaces perpendicular to **E** and for a fixed voltage increment ΔV ($= 10$ V in the figure) are spaced uniformly.

In a nonuniform field the **E** lines diverge in going from a stronger to a weaker field region, as in Fig. 2-17*b*. Furthermore, for a fixed voltage increment, such as 10 V, the equipotential surfaces become more widely spaced in the weaker field region. The uniform and nonuniform fields are shown in three dimensions in Figs. 2-17*c* and *d*.

Note that a potential *rise* is always opposite to the direction of **E**.

As another example of a nonuniform field, let us consider the field in the vicinity of a long straight wire shown in Fig. 2-18. The wire perpendicular to the page carries a uniform positive charge.

From (2-7-8) the electric field at a distance r from a long (ideally infinite) wire is

$$\boxed{E_r = \frac{\rho_L}{2\pi\epsilon_0 r}} \quad (\mathrm{V\,m^{-1}}) \qquad (1)$$

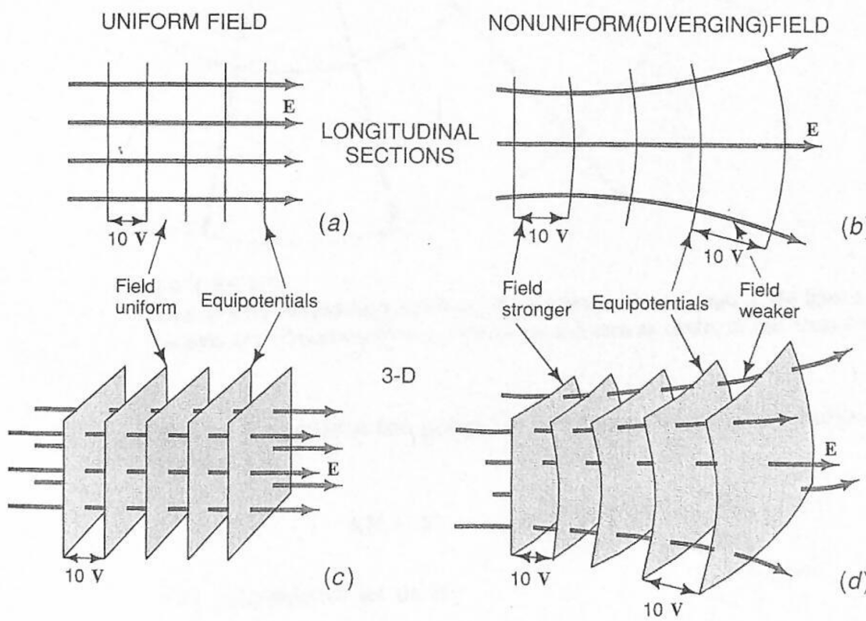

FIGURE 2-17
(*a*) Uniform electric field. (*b*) Nonuniform electric field. Equipotential surfaces are planar in (*a*) and curved in (*b*) as seen in three dimensions in (*c*) and (*d*). For a given potential difference or increment (10 V), the equipotential surfaces are equally spaced in a uniform field (*c*), but in the nonuniform field (*d*) the spacing increases as the field becomes weaker.

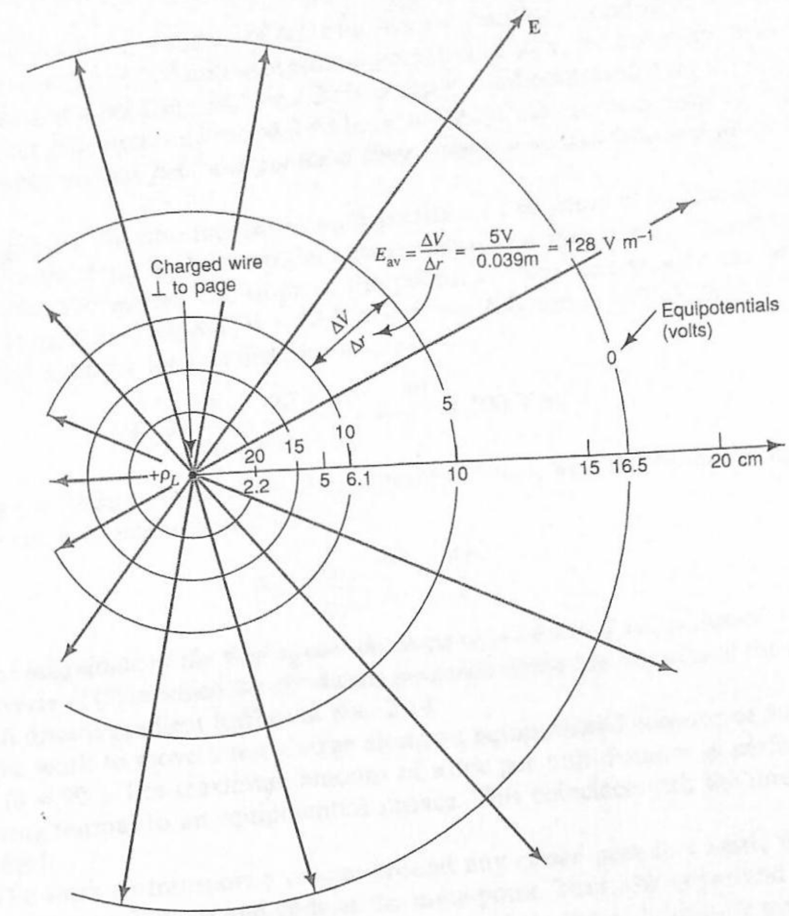

FIGURE 2-18
Electric field around long charged wire perpendicular to page. Field lines are radial. Equipotential surfaces are cylinders concentric with wire and seen as circles in this cross-section.

and by integration the potential difference between two radial distances r_1 and r_2 $(r_2 > r_1)$ is

$$\Delta V = V_1 - V_2 = \frac{\rho_L}{2\pi\epsilon_0}\int_{r_1}^{r_2}\frac{dr}{r} = \boxed{\frac{\rho_L}{2\pi\epsilon_0}\ln\frac{r_2}{r_1}} \quad \text{(V)} \tag{2}$$

For convenience let us set

$$\frac{\rho_L}{2\pi\epsilon_0} = 10\ \text{V} \tag{3}$$

which means that the charge per unit length

$$\rho_L = 20\pi\epsilon_0 = 20\pi 8.85 \times 10^{-12} = 556\ \text{pC m}^{-1}$$