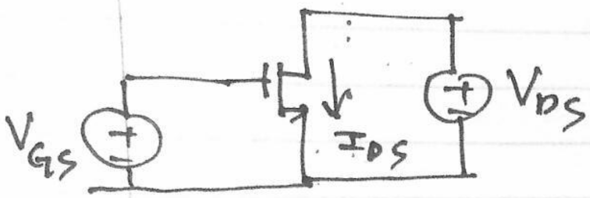
MOS I-V characteristics



1) long-channel NMOS ($L \geq 1\mu m$)

① Linear (triode) region

⇒ $V_{DS} < \underbrace{V_{GS} - V_{th}}_{\text{overdriving voltage } = \Delta} = V_{DS,sat}$.

⇒ $I_{DS} = \mu_n C_{ox} \frac{W}{L} \left\{ (V_{GS} - V_{th}) V_{DS} - \frac{1}{2} V_{DS}^2 \right\}$

⇒ Effective dynamic channel conductance, g_{d0} @ $V_{DS} = 0$

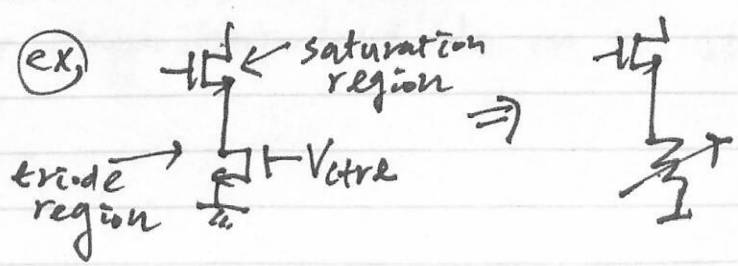
$$g_{d0} = \left(\frac{\partial I_{DS}}{\partial V_{DS}} \right)_{@ V_{DS}=0} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th}) = \mu_n C_{ox} \frac{W}{L} \cdot \Delta$$

⇒ Effective dynamic channel resistance

, $r_{ds} = \frac{1}{g_{d0}} = \frac{1}{\mu_n C_{ox} \frac{W}{L} \cdot \Delta}$ @ V_{DS} is very small.

⇒ Can be used as variable resistor

(ex,)



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② Saturation region

$V_{DS} \geq V_{GS} - V_{th} = V_{DS,sat} = \Delta$

$\Rightarrow I_{DS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})^2$

channel length modulation

$\Rightarrow I_{DS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})^2(1+\lambda V_{DS})$

⇒ Effective dynamic channel resistance

$r_o = \left(\frac{\partial V_{DS}}{\partial I_{DS}}\right) = \frac{1}{\lambda \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})^2} \approx \frac{1}{\lambda I_D}$

⇒ Transconductance, g_m

$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})$

$= \sqrt{2\mu_n C_{ox}\frac{W}{L} I_{DS}}$

$= \frac{2 I_{DS}}{V_{GS}-V_{th}}$

⇒ For long channel CMOS, $g_m = g_{do}$.

⇒ g_{do} plays important role in noise analysis.

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2) Short-channel NMOS ($L < 1\mu m$)

① linear (triode) region

$$I_{DS} = \underbrace{\mu_n C_{ox} \frac{W}{L} \left\{ (V_{GS} - V_{th}) V_{DS} - \frac{1}{2} V_{DS}^2 \right\}}_{\text{same as in long-channel case}} \times \boxed{\frac{1}{1 + \frac{V_{DS}}{E_c \cdot L}}}$$

⇒ E_c: critical field where carrier velocity will be saturated to $v_{sat} = 8 \times 10^6$ cm/s

ex.
- for electron, $E_{cn} = 6 \times 10^4$ V/cm
- for hole, $E_{cp} = 24 \times 10^4$ V/cm
- $v_{sat} = 8 \times 10^6$ cm/s → same for electron and hole.

ex. NMOS

$L = 1\mu m \rightarrow E_c \cdot L = 6V$

$L = 0.2\mu m \rightarrow E_c \cdot L = 1.2V$

$L = 0.1\mu m \rightarrow E_c \cdot L = 0.6V$

$L = 50nm \rightarrow E_c \cdot L = 0.3V$

⇒ $g_{do} = \left(\frac{\partial I_{DS}}{\partial V_{DS}} \right)_{@ V_{DS} = 0} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th})$

⇒ g_{do} is same as in ~~short~~ long channel case.

$$\rightarrow \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})^2 \frac{1}{1+\frac{1}{E_c \cdot L}(V_{GS}-V_{th})} \quad (*)$$

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$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})^2 \frac{1}{1+\theta(V_{GS}-V_{th})}$$

② Saturation region

$$I_{DS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})^2 \frac{E_c \cdot L}{(V_{GS}-V_{th})+E_c \cdot L}(1+\lambda V_{DS})$$

$$\simeq \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})^2 \frac{E_c \cdot L}{(V_{GS}-V_{th})+E_c \cdot L}$$

$$= \underbrace{\frac{1}{2}\mu_n E_c}_{=v_{sat}} C_{ox} W (V_{GS}-V_{th})^2 \frac{1}{(V_{GS}-V_{th})+E_c \cdot L}$$

$$= v_{sat} \cdot W \cdot C_{ox}(V_{GS}-V_{th})^2 \frac{1}{(V_{GS}-V_{th})+E_c \cdot L}$$

↓ (if) $V_{GS}-V_{th} \gg E_c \cdot L$

$$\simeq v_{sat} \cdot W \cdot C_{ox}(V_{GS}-V_{th})$$

⇒ I_{DS} is linearly dependent on $(V_{GS}-V_{th})$.

⇒ For extreme case, $g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = v_{sat} \cdot W \cdot C_{ox}$

⇒ g_m is constant, not dependent on biasing.

Q) How to determine $V_{DS,sat}$ at short channel case?

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From ① & ② in previous pages,

$I_{DS,\,linear\text{-}region} = I_{DS,\,sat\text{-}region}$, at $V_{DS} = V_{DS,sat}$

$$\mu_n C_{ox} \frac{W}{L} \left\{ (V_{GS}-V_{th}) V_{DS,sat} - \frac{1}{2} V_{DS,sat}^2 \right\} \frac{1}{1+\frac{V_{DS,sat}}{E_c \cdot L}}$$

$$= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS}-V_{th})^2 \frac{E_c \cdot L}{(V_{GS}-V_{th}) + E_c \cdot L}$$

$$\Rightarrow V_{DS,sat} = (V_{GS}-V_{th}) \boxed{\frac{E_c \cdot L}{(V_{GS}-V_{th}) + E_c \cdot L}}$$

→ This is always < 1

∴ $V_{DS,sat}$ of short channel is always smaller than that of long channel

⊛ Some remarks

For 0.13 ~ 0.18 μm CMOS, $E_c \cdot L = 0.5 \sim 1V$ range and overdriving voltage $(V_{GS}-V_{th})$ ~~usually smaller than 0.5V for small signal circuit designs such as LNA and mixer.~~ ~~V_{DS}~~ ~~But it could be comparable~~ Could be comparable to $E_c \cdot L$. ~~V_{DS} is~~ V_{DS} also could be reached to $E_c \cdot L$.

⇒ We need short channel DC-model for hand analysis.

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- g_m for short channel I-V characteristics

$$\Rightarrow g_m = \left(\frac{\partial V_{DS}}{\partial I_{DS}}\right)$$

$$= \mu_n C_{ox} \frac{W}{L} (V_{GS}-V_{th}) \frac{(V_{GS}-V_{th})E_c \cdot L}{(V_{GS}-V_{th})+E_c \cdot L} \; \frac{\frac{1}{2} + \frac{E_c \cdot L}{V_{GS}-V_{th}}}{(V_{GS}-V_{th})+E_c \cdot L}$$

$$= \underbrace{\mu_n C_{ox} \frac{W}{L} (V_{GS}-V_{th})}_{g_{d0}} \times \underbrace{\boxed{\frac{V_{DS,sat}}{V_{GS}-V_{th}}}}_{<1} \times \underbrace{\boxed{\frac{\frac{1}{2}(V_{GS}-V_{th}) + E_c \cdot L}{(V_{GS}-V_{th}) + E_c \cdot L}}}_{<1}$$

~~in short channel.~~

- Now Let's define the ratio of g_m/g_{d0} in short channel CMOS,

$$\boxed{\alpha = \frac{g_m}{g_{d0}} = \frac{V_{DS,sat}}{V_{GS}-V_{th}} \frac{\frac{1}{2}(V_{GS}-V_{th}) + E_c \cdot L}{(V_{GS}-V_{th}) + E_c \cdot L}} < 1$$

(ex) 0.18 μm NMOS → $E_c \cdot L \simeq 1.2V$
typical overdriving → $V_{GS}-V_{th} \simeq 0.2V$
⇒ $V_{DS,sat} \simeq 171.4$ mV
⇒ $\alpha \simeq 0.8$ (cf) for long channel, $\alpha = 1$)

Note) the constant α frequently appeared in noise expression in CMOS.