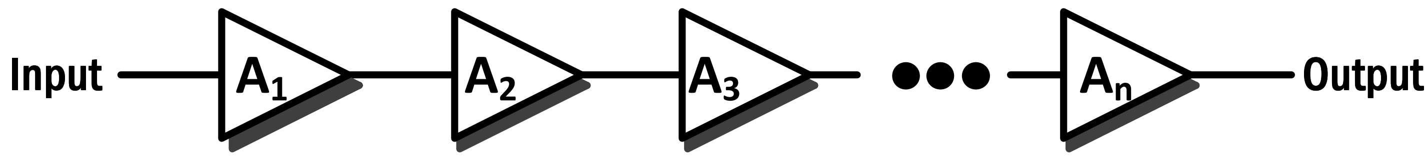
ECE 5220 RFIC Technology & Design (Noise)

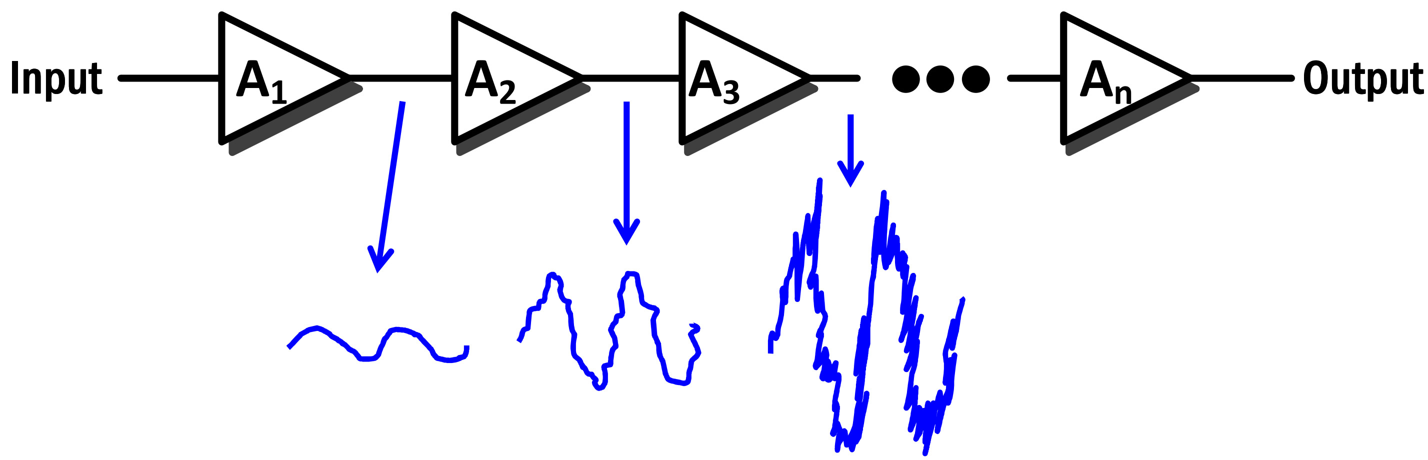
Noise (introduction)

- The broadest definition of noise is "everything except the desired signal".
- In practice, the term "noise" is used to refer to "random noise" having Gaussian distribution of its amplitude.
- In a strict sense "Interferer" is not random noise, but artificial man-made noise.
- Before the early 1900's, it was widely thought that the sensitivity of a system can be simply improved by cascading amplifiers with finite gain.



Noise (introduction)

- When it became possible to design amplifiers with enough gain using vacuum tubes in the early 1900's, it was observed that simply cascading amplifiers did not improve the sensitivity of the system.



- The degradation of the sensitivity of the system is observed as a continuous "hiss" in audio systems and a characteristic "snow" in video systems.

Noise (introduction)

- Nyquist, Johnson and Schottky were the first to publish a series of papers which carefully measuring and explaining the origin of noise.

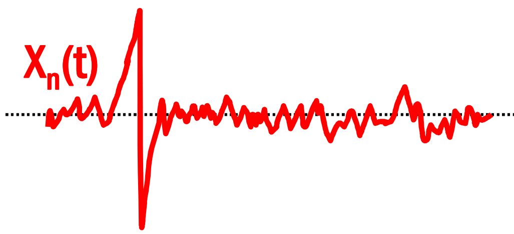
- The existence of noise in electronics is basically due to the fact that the movement of electrical carriers (electrons and holes) is not continuous, but discrete, and they have randomness in movement due to thermal energy and defects in semiconductor.

- Study of noise is important because it allows us to determine the lower limit to the size of signal that can be detected.

- Types of noise (covered in the lecture):
  - Thermal noise
  - Shot noise
  - Flicker noise (1/f noise)

Quantifying Noise Power

- Let's think about how to quantify random noise $X_n(t)$ which can be either random voltage or current signal.



- Random noise is unpredictable, but one thing well-known is that its time average is zero.

$$\langle X_n(t) \rangle = \lim_{T\to\infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} X_n(t)dt = 0$$

- So, we need a different way to quantify the noise. Let's mean square-value of $X_n(t)$.

This is called mean-square power (or just power) of $X_n(t)$ (with respect to 1Ω).

$$\langle (X_n(t))^2 \rangle = \lim_{T\to\infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} X_n(t)X_n(t)^* \, dt$$

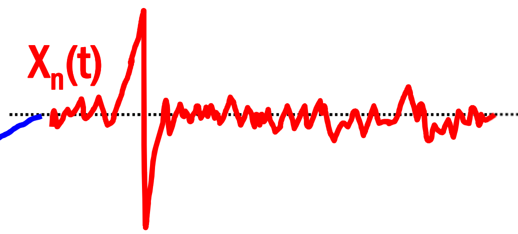
Because most noise is real quantity.

$$= \lim_{T\to\infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} X_n{}^2(t) \, dt \neq 0$$

$$= \overline{X_n{}^2(t)}$$

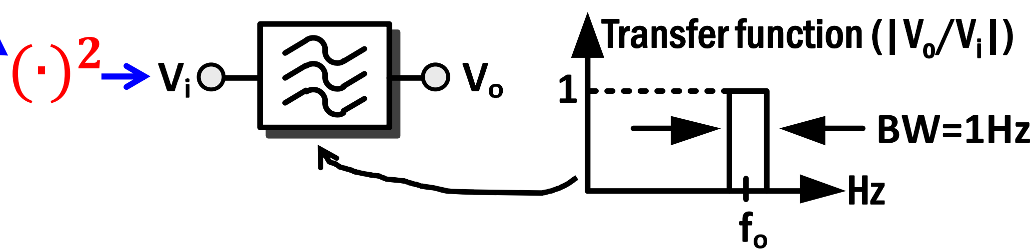
Quantifying Noise Power

- Apparently, mean-square power of $X_n(t)$ is not zero, but it's still difficult to calculate because of infinite time integral.



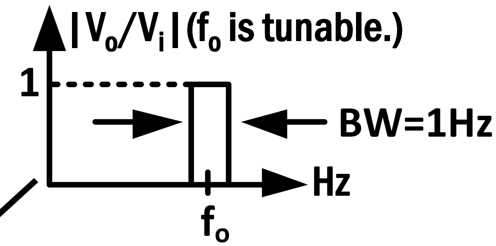
$$\overline{X_n^{\,2}(t)} = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} X_n^{\,2}(t)\,dt$$

- Any easy way ? Well, let's try it in an experimental way. First, imagine a tunable bandpass filter where center frequency f_o is tunable and passband BW is 1Hz having brick-wall type ideal unity gain response (mental model).

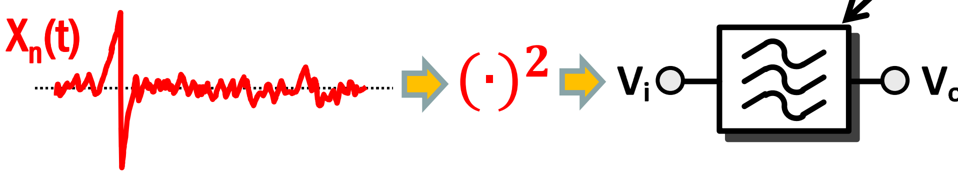


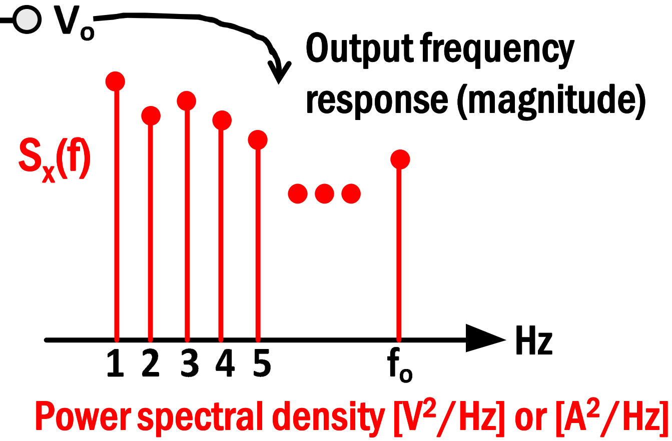
- Then apply the square of noise $X_n(t)$ to the input of the filter.

Quantifying Noise Power



- Now, you can measure frequency response of the $X^2_n(t)$ for every frequency per 1Hz step.





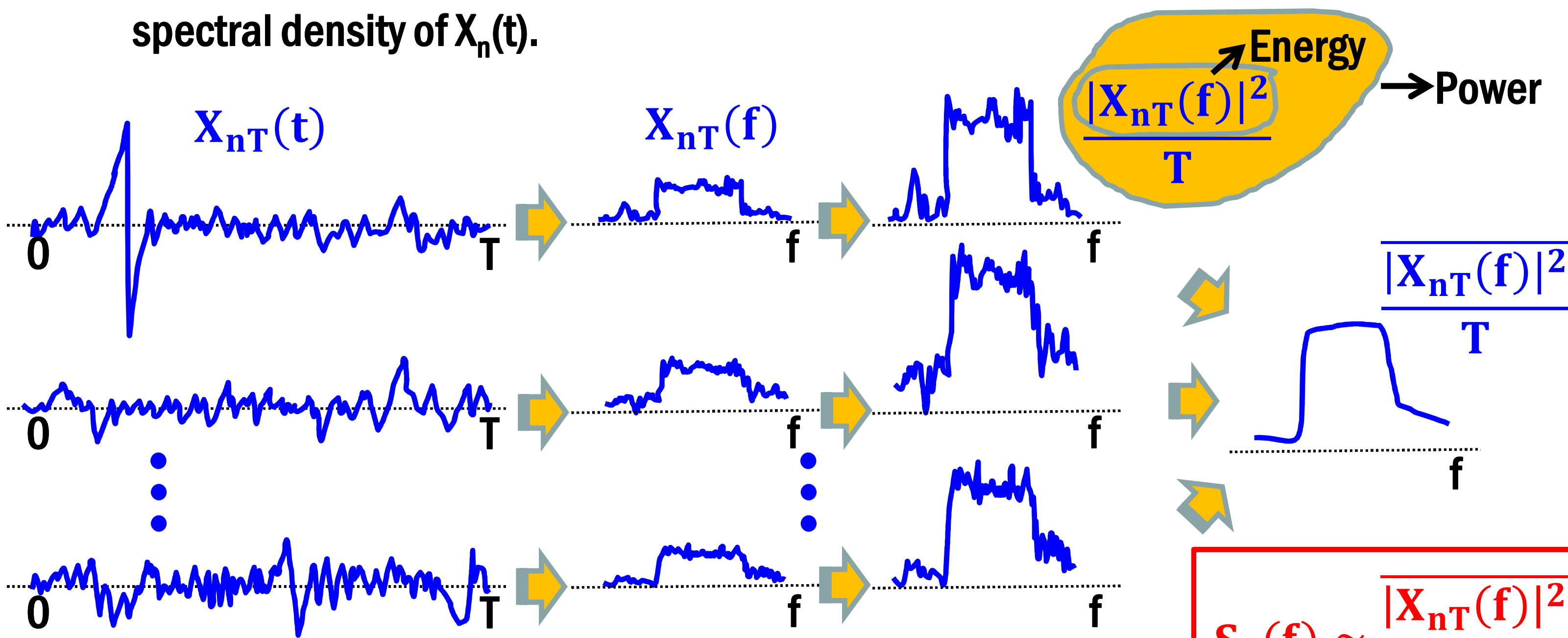
- This is basic concept of "power spectral density or power spectrum $S_x(f)$" of $X_n(t)$.
- Once $S_x(f)$ is known, power can be calculated by integrating $S_x(f)$ over the frequency band of interest (f_L-f_U).

$$\overline{X_n{}^2(t)} = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} X_n{}^2(t)\,dt$$

$$= \boxed{\int_{f_L}^{f_U} S_x(f)\,df = \overline{X_n{}^2}}$$

Quantifying Noise Power

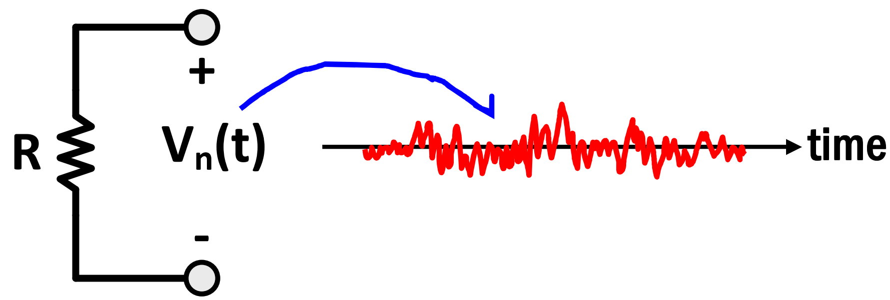
- In reality, it's impossible to have 1Hz BW tunable bandpass filter.
- Alternatively, we take sufficient number of $X_n(t)$ for a sufficient amount of time period T. Then take Fourier transform of each sample and calculate power of each sample in frequency domain. Finally average all the power. This gives approximate of real power spectral density of $X_n(t)$.



$$S_X(f) \approx \frac{\overline{|X_{nT}(f)|^2}}{T}$$

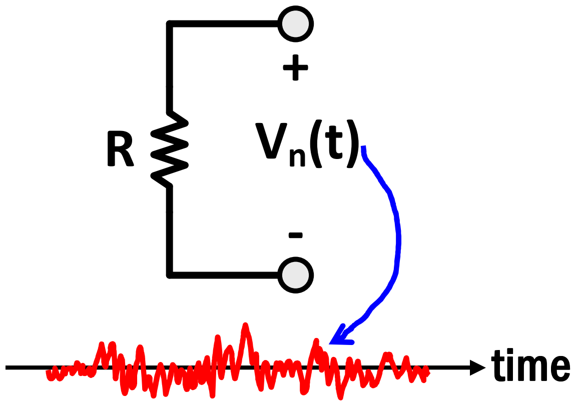
Resistor Thermal Noise

- If ambient temperature is not absolutely zero, then there is thermal energy everywhere ("Blackbody radiation"). In semiconductors including resistors, electrons (and holes) will be agitated by this thermal energy, which causes random movement of the carriers. This is the basic source of noise current (and therefore, noise voltage) in resistors.



- This fundamental physical phenomenon because discrete movement of electron charges. You will see later that this kind of noise also appears in antennas, which is basic noise source from the antennas.
- Some different names of thermal noise: Johnson noise, Nyquist noise

Resistor Thermal Noise



- It is know that the power spectral density of the resistor thermal noise, $V_n(t)$, is given as:

$$S_n(f) = \frac{\overline{v_n{}^2}}{\Delta f} = 2R\left(\frac{h|f|}{2} + \frac{h|f|}{exp\left(\frac{h|f|}{kT}\right) - 1}\right)$$

h: Plank's constant = 6.2×10^{-34} J·sec
k: Boltzman constant = 1.38×10^{-23} J/K
T: Absolute temperature = (273+°C) K
Δf: Bandwidth (Hz)

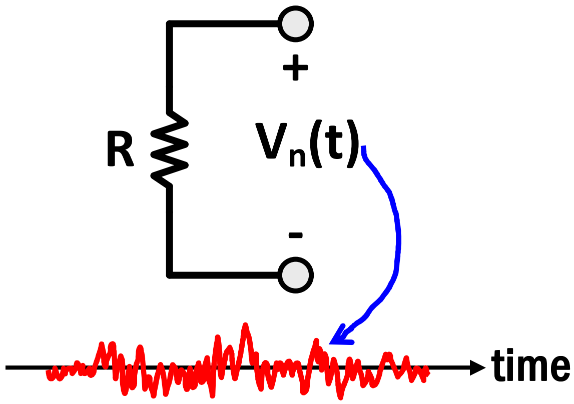
Apply this.

This assumption is valid for most of RF IC designs.

$$If, |f| < \frac{kT}{\cancel{2}h} = \frac{1.38 \times 300 \times 10^{-23}}{6.2 \times 10^{-34}} \approx 10^{13} = 10\,THz,$$

$$\text{then } exp\left(\frac{h|f|}{kT}\right) \approx 1 + \frac{h|f|}{kT} \quad (\because\ e^x \approx 1 + x).$$

Resistor Thermal Noise



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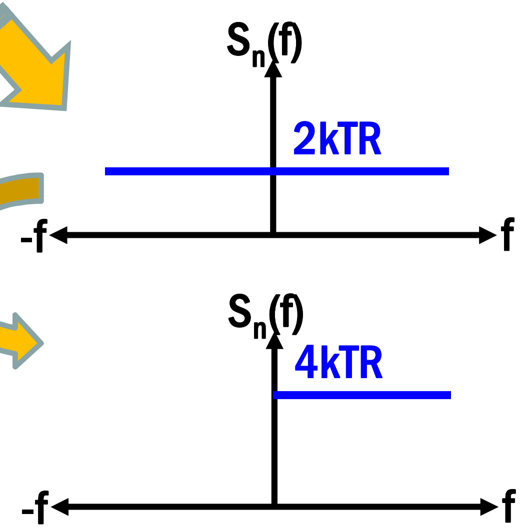
$$= 2\mathrm{kTR}$$

We call this as "double sided spectrum"



We call this as "single sided spectrum" (standard convention)

$$\boxed{S_n(f) = \frac{\overline{v_n^2}}{\Delta f} = 4\mathrm{kTR}}$$

Note: No frequency dependency (white noise)

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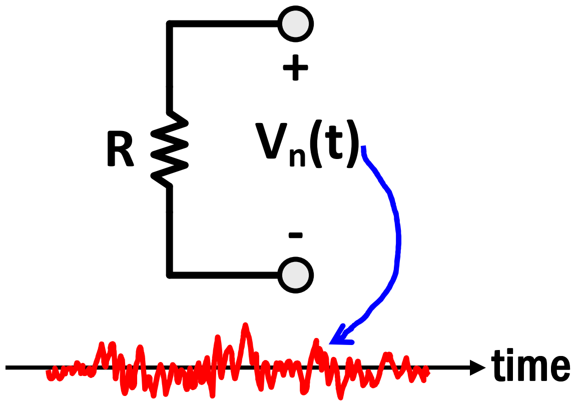
Resistor Thermal Noise - Modeling



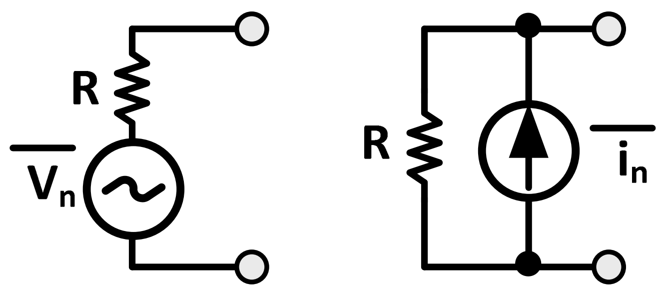
- ❑ Now, let's define noise power and effective noise voltage:

$$S_n(f) = \frac{\overline{v_n^{\,2}}}{\Delta f} = 4\mathrm{kTR}$$

Noise power = $\overline{v_n^{\,2}} = 4\mathrm{kTR}\Delta f$

Effective noise voltage = $\overline{v_n} = \sqrt{4\mathrm{kTR}\Delta f}$

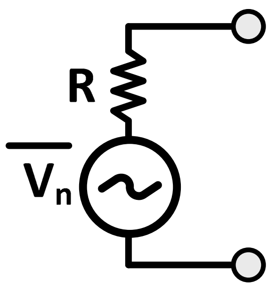
- ❑ Now, we can model resistor thermal noise as voltage source (Thevenin form) or current source (Norton form):

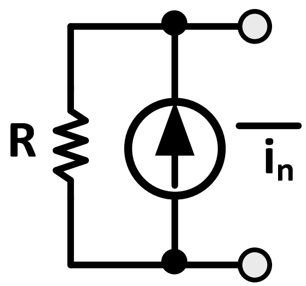


$$\frac{\overline{v_n}}{\sqrt{\Delta f}} = \sqrt{4\mathrm{kTR}} \quad \left(\frac{V}{\sqrt{Hz}}\right)$$

$$\frac{\overline{i}}{\sqrt{\Delta f}} = \frac{\overline{v_n}}{\sqrt{\Delta f}R} = \sqrt{\frac{4\mathrm{kT}}{\mathrm{R}}} \left(\frac{A}{\sqrt{Hz}}\right)$$

Resistor Thermal Noise - Modeling





$$\frac{\overline{v_n}}{\sqrt{\Delta f}} = \sqrt{4\mathrm{kTR}} \quad \left(\frac{V}{\sqrt{Hz}}\right)$$

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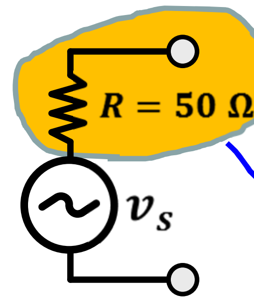
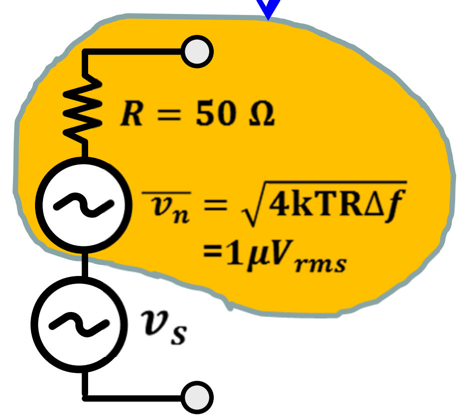




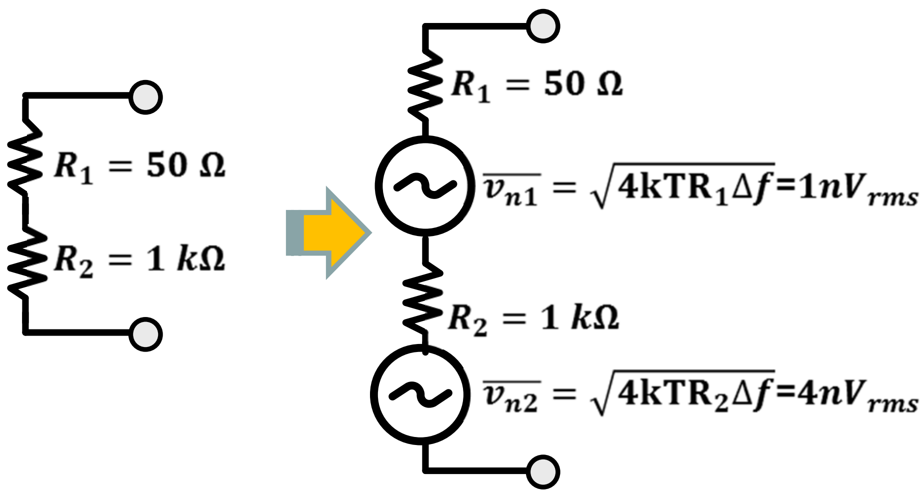
- Rule of thumb:
  - 1 kΩ, T=300 K, Δf=1 Hz, $\overline{V_n}$=4 nV_{rms}
  - 50 Ω, T=300 K, Δf=1 Hz, $\overline{V_n}$=1 nV_{rms}

- Example:

Consider a 50-Ω wireless system with 1 MHz signal bandwidth. The noise voltage from a 1 MHz BW for a 50-Ω resistor is $1nV_{rms}$ x 1000=$1uV_{rms}$. Therefore, the signal voltage with a BW of 1MHz needs to be more than 1uV to be detectable.

Noise Algebra

- Now, let's add two resistors in series. Then, how much is the overall noise voltage (Δf=1Hz) ?



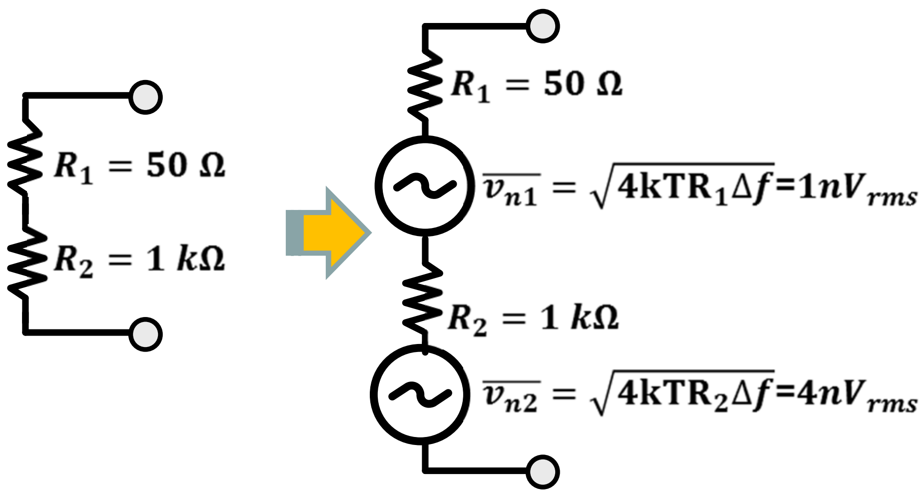
Don't calculate like this:

$$\overline{v_{n,total}} = \overline{v_{n1}} + \overline{v_{n2}} = 5\ nV_{rms}$$

Why ?

Noise Algebra

- ❑ Now, let's add two resistors in series. Then, how much is the overall noise voltage (Δf=1Hz) ?



When adding two noises, we need to consider correlation between the noises.

$$\overline{v_{n,total}}^2 = (\overline{v_{n1}} + \overline{v_{n2}})(\overline{v_{n1}} + \overline{v_{n2}})^*$$

$$= \overline{v_{n1}} \times \overline{v_{n1}}^* + \overline{v_{n2}} \times \overline{v_{n2}}^*$$

$$+ \overline{v_{n2}} \times \overline{v_{n1}}^* + \overline{v_{n1}} \times \overline{v_{n2}}^*$$

These terms will be zero, because noises from R1 and R2 are not correlated; i.e., time average of multiplication of two uncorrelated noise samples will be zero.

$$\overline{v_{n,total}}^2 = \overline{v_{n1}} \times \overline{v_{n1}}^* + \overline{v_{n2}} \times \overline{v_{n2}}^*$$

$$= \overline{v_{n1}}^2 + \overline{v_{n2}}^2$$

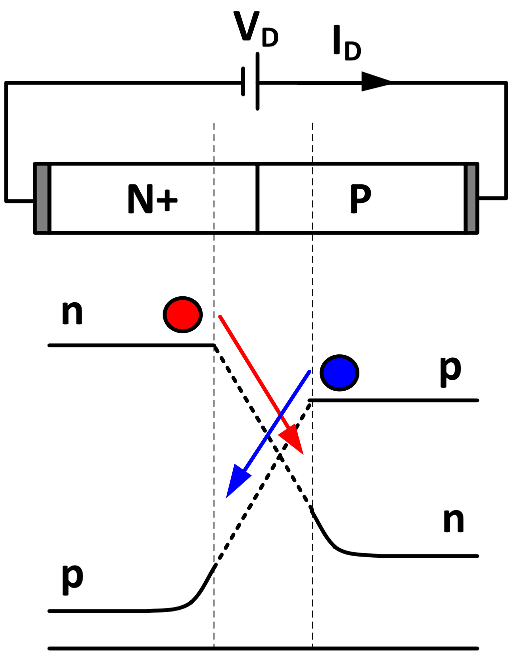
Note: we have to add noise in power domain or resistance, not voltage.

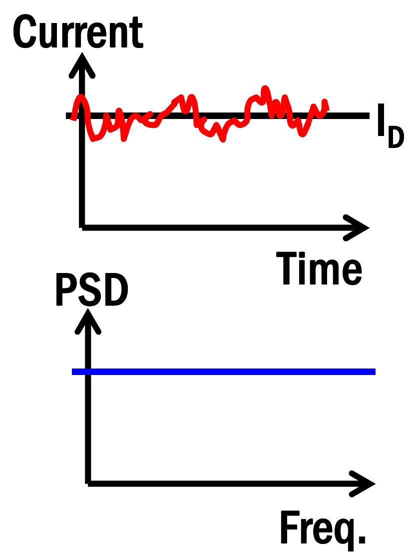
$$\therefore\ \overline{v_{n,total}} = \sqrt{\overline{v_{n1}}^2 + \overline{v_{n2}}^2} = \sqrt{4kT(R_1 + R_2)\Delta f}$$

$$= \sqrt{17}\ nV_{rms} \approx 4.12\ nV_{rms}$$

Shot Noise

- Noise source: any electron (e-) and hole (h+) has slightly different random energy level (not uniform energy). These different energy level causes randomness in the # of carrier hopping the energy barrier in the P-N junction under forward biasing.





These randomness of # of carriers (e-, h+) arriving at the contact generates random noise pulse of current.

$$\frac{\overline{i_n{}^2}}{\Delta f} = 2qI_D,$$
$$where\ q = 1.6 \times 10^{-19}\ C.$$

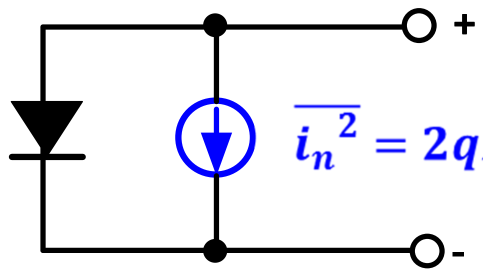
Note: this is white noise and has Gaussian distribution.

Shot Noise

- Condition for shot noise:

1. There must be DC current flow.
2. There must potential barrier (P-N junction) over which a charge carrier hops.

➡ Any diode and P-N junction device exhibits shot noise current. Noise of diode can be represented by a noise current source in parallel with the diode as shown in figure below.



$$\overline{i_n^{\,2}} = 2qI_D\Delta f$$

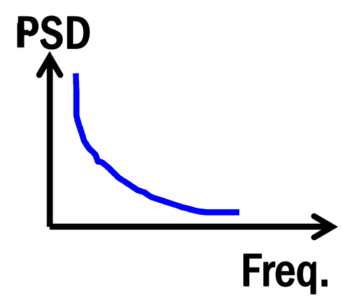
Ex) Rule of Thumb:

- 1mA of DC current produces 18 pA/√Hz of rms noise current density.
- 1mA of DC current produces 18 nA of rms noise current of a BW=1MHz.

Flicker Noise

- ❑ Condition for flicker noise: Flow of DC current
- ❑ Flicker noise is caused by the randomness of being captured (recombination) and released (injection) of charge in a crystal defect states (traps).
- \- Ex) In diode/BJT, P-N junction depletion region has some contamination and crystal defects, which play as trap states.
- \- Ex) In MOSFETs, the interface between the gate oxide and the silicon substrate contains many dangling bonds acting as trap sites.
- ❑ The traps capture and release carriers in a random fashion, and the time constants associated with the process give rise to the 1/f nature of the noise power spectral density (mostly low-frequency dominant noise).

$$\frac{\overline{i_n^{\,2}}}{\Delta f} = K_f \frac{I_{DC}^{\;af}}{f^{ef}}$$



- ❑ K_f, af, ef will be depending on fabrication process (but mostly $ef \sim 1$).
- ❑ It is observed that in typical IC process, BJT exhibits less 1/f-noise than CMOS.

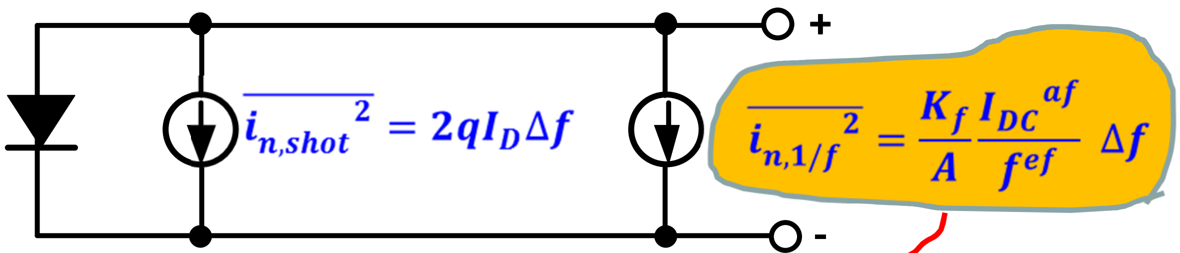
Flicker Noise in Junctions

- Flicker noise in a forward biased junction can be expresses as:

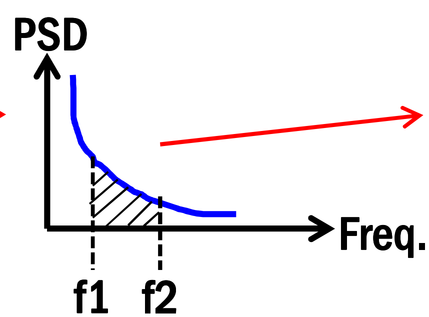
$$\overline{i_n^{\,2}} = \frac{K_f}{A}\frac{I_{DC}^{\;af}}{f^{ef}}\Delta f, where\ A = diode\ junction\ area,$$

$$K_f \approx 10^{-25}\ A\cdot m^2, 0.5 \le af \le 2, ef \approx 1.$$

- A more complete noise model of a forward biased diode is shown below:



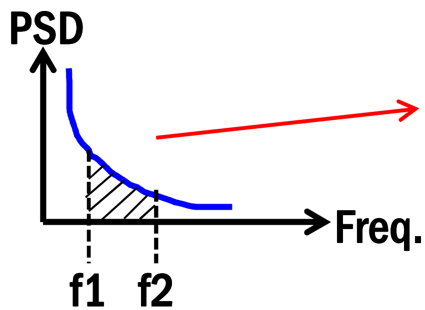
- Total flicker noise in a bandwidth from f1 and f2 is (assume af=ef=1):



$$\int_{f1}^{f2}\overline{i_{n,1/f}^{\;2}}\,df = \int_{f1}^{f2}\frac{K_f}{A}\frac{I_{DC}}{f}\,df = \frac{K_f I_{DC}}{A}\ln\left(\frac{f2}{f1}\right)$$

Flicker Noise

- Total flicker noise in a bandwidth from f1 and f2 is (assume af=ef=1):



$$\int_{f1}^{f2} \overline{i_{n,1/f}}^{2} df = \int_{f1}^{f2} \frac{K_f}{A} \frac{I_{DC}}{f} df = \boxed{\frac{K_f I_{DC}}{A} ln\left(\frac{f2}{f1}\right)}$$

Same ratio of frequencies gives same 1/f-noise. For example, 1/f-noise in a bandwidth from 30 Hz to 300 Hz is the same as the noise in a bandwidth from 300 Hz to 3000 Hz. More specifically, the 1/f-noise is constant for decade of frequency.

Ex) diode area A=1um x 1um, I_{DC}=1mA, BW=10-100 Hz, then

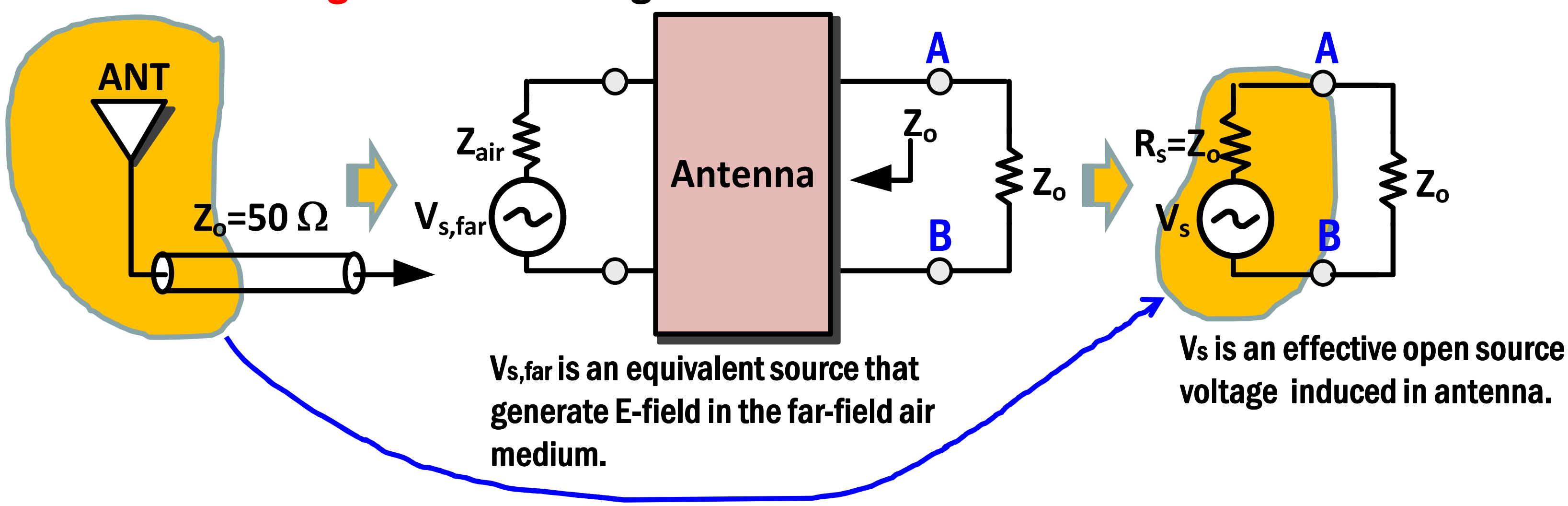
$$\int_{f1}^{f2} \overline{i_{n,1/f}}^{2} df = \frac{K_f I_{DC}}{A} ln\left(\frac{f2}{f1}\right)$$

$$= \frac{10^{-25} \times 10^{-3}}{10^{-12}} ln\left(\frac{100}{10}\right) = 2.3 \times 10^{-16} A^2$$

$$\therefore \overline{i_{n,1/f}}(one - decade) = \sqrt{2.3 \times 10^{-16}} = 15\ nA.$$

Antenna Noise

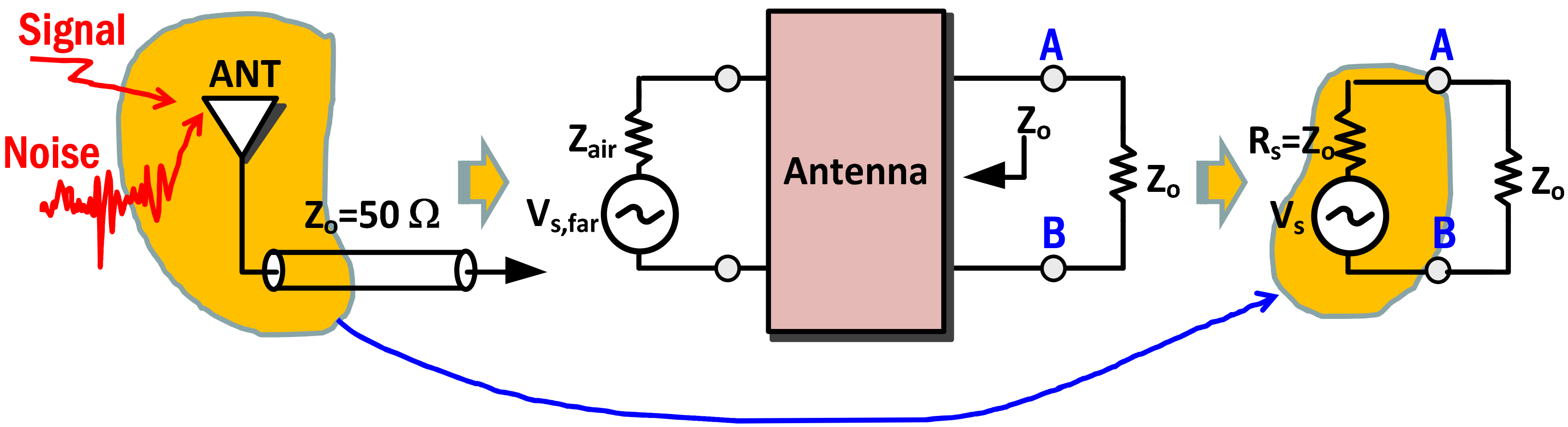
- Previous discussion allows this simple modeling of antenna in terms of signal for RF IC designs.



- R_s is not real resistance, but called as "radiation resistance".
- For 50-Ω system, R_s=Z_o=50 Ω
- For 75-Ω system, R_s=Z_o=75 Ω

Antenna Noise

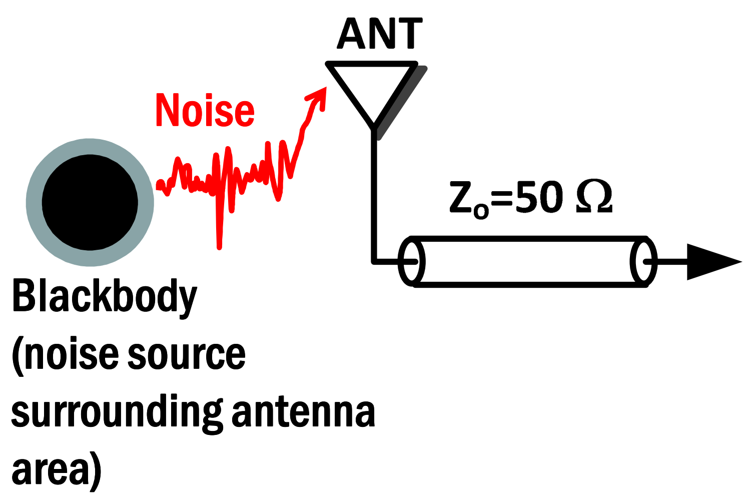
- We need to consider one more thing in this modeling.



- In real world,
  - Antenna receives not only signal energy but also noise energy.
  - Recall that every object above absolute temperature emits thermal energy which will be noise energy if it's not coming from the signal ("Blackbody radiation").

Q) How can we model the antenna noise ?

Antenna Noise



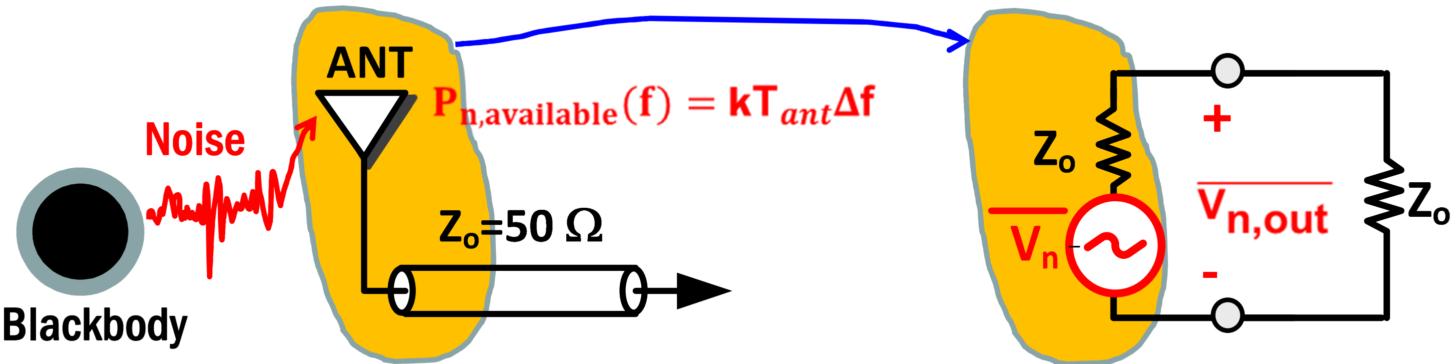
- Let's assume that the antenna is facing a blackbody with a temperature T_{ant}.
- The carriers in the antenna are thermally agitated by the radiation from the blackbody source.
- It is known that the available noise power depends on the temperature of the blackbody and BW of the system under Z-matched condition.

Available Noise Power ($P_{n,available}$),

$$P_{n,available}(f) = kT_{ant}\Delta f$$

k: Boltzman constant = 1.38 x 10^{-23} J/K
T: Absolute temperature = (273+°C) K
Δf: Bandwidth (Hz)

Antenna Noise



Blackbody
(noise source
surrounding antenna
area)

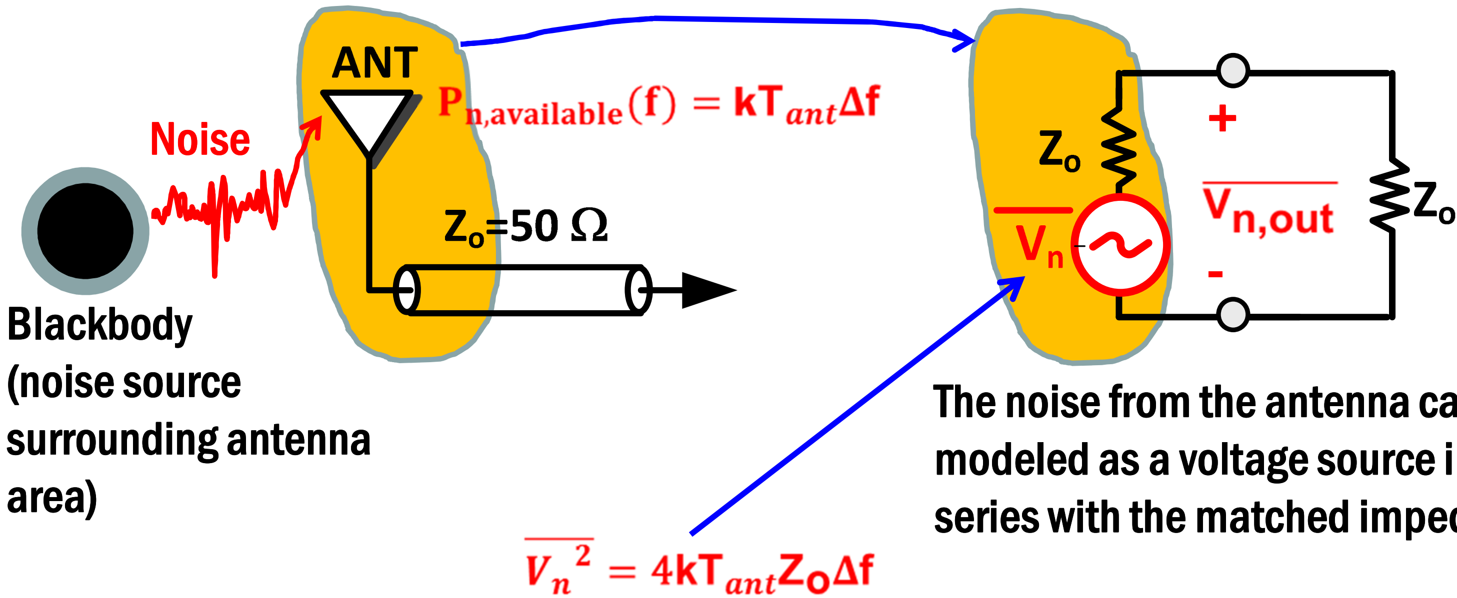
The noise from the antenna can be modeled as a voltage source in series with the matched impedance.

The equivalent noise voltage can be determined by this equation.

$$P_{n,available}(f) = kT_{ant}\Delta f = \frac{\overline{V_{n,out}}^2}{Z_o} = \frac{\overline{V_n}^2}{4Z_o}$$

$$\overline{V_n}^2 = 4kT_{ant}Z_o\Delta f$$

Antenna Noise



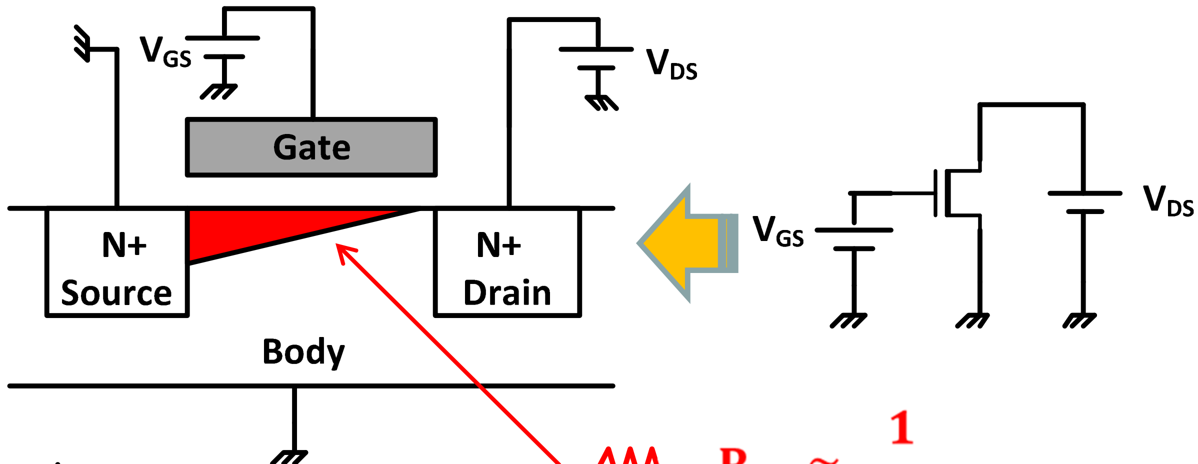
The noise from the antenna can be modeled as a voltage source in series with the matched impedance.

Q) How do we determine T_{ant} ?
Ans) Normal convention is to take ambient temperature, i.e., T_{ant}=300 K. But in reality, it could be a bit higher due to cosmic rays from outer space.

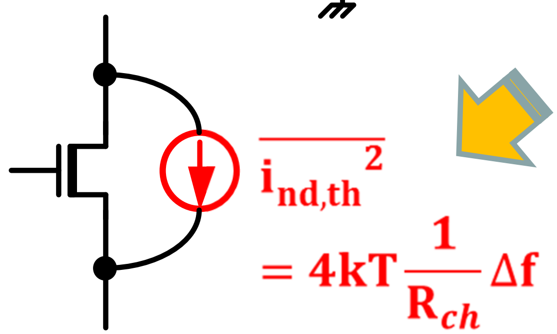
- Note that the equivalent noise voltage is the same form as in that of noisy resistor, although Z_o is not physical resistor.

MOSFET Noise Model

- The channel resistance of MOSFET exhibits thermal noise just like a normal resistor, and causes drain thermal noise current.



$$R_{ch} \approx \frac{1}{\gamma g_m}$$

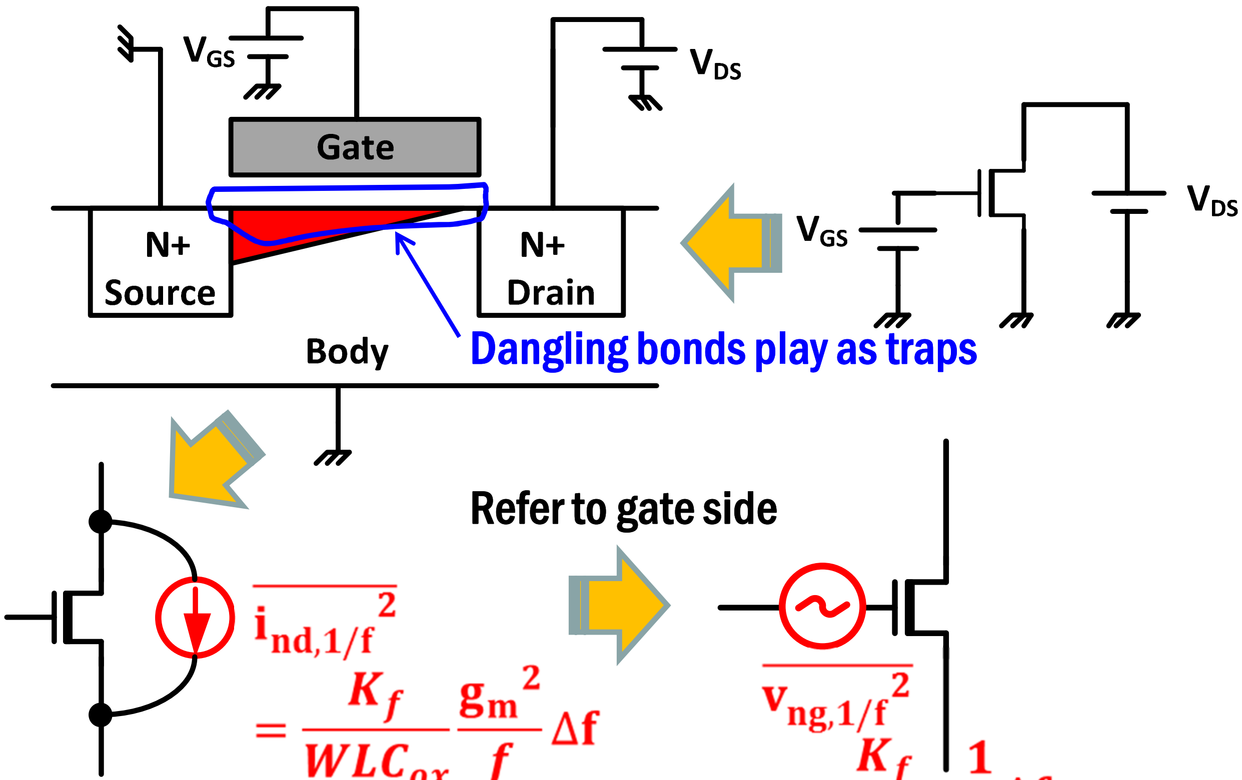


$$\overline{i_{nd,th}^{\,2}} = 4kT\frac{1}{R_{ch}}\Delta f = 4kT(\gamma g_m)\Delta f$$

g_m: transconductance,
γ: drain thermal noise coefficient
(~2/3 for long channel,
> 3-4 for short channel device)

MOSFET Noise Model

- Atomic bonds at the interface of gate oxide and silicon substrate is not perfect, but there are many dangling bonds which plays trap states. These traps give rise to flicker noise.



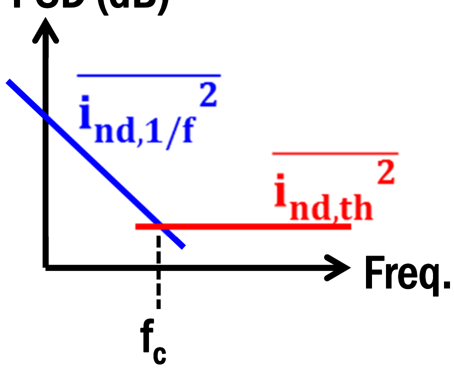
$$\overline{i_{nd,1/f}^{2}} = \frac{K_f}{WLC_{ox}}\frac{g_m^{2}}{f}\Delta f$$

$$\overline{v_{ng,1/f}^{2}} = \frac{K_f}{WLC_{ox}}\frac{1}{f}\Delta f$$

MOSFET Noise Model

- Drain noise current has two components; channel thermal noise and 1/f – noise.

$$\overline{i_{nd}^{\,2}} = \overline{i_{nd,th}^{\,2}} + \overline{i_{nd,1/f}^{\,2}}$$
$$= 4kT(\gamma g_m)\Delta f + \frac{K_f}{WLC_{ox}}\frac{g_m^{\,2}}{f}\Delta f$$



At f=fc, $\overline{i_{nd,th}^{\,2}} = \overline{i_{nd,1/f}^{\,2}}$

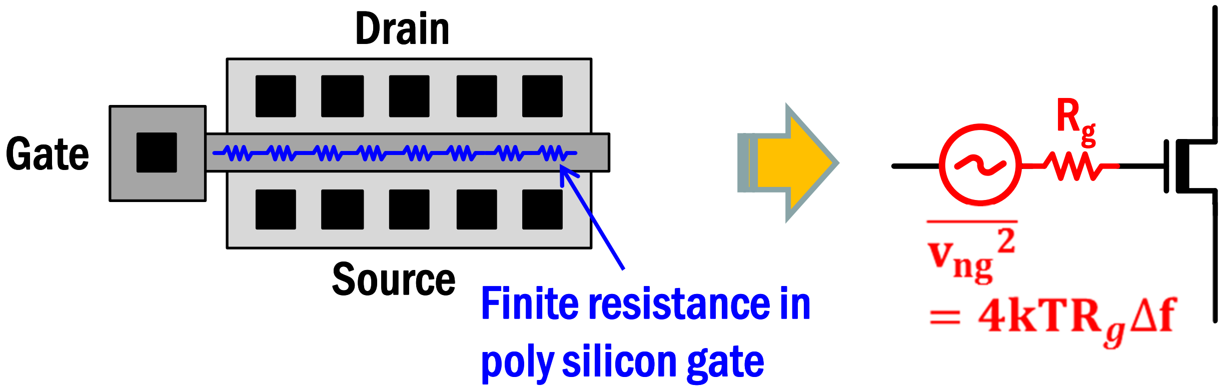
$$\rightarrow 4kT(\gamma g_m)\Delta f = \frac{K_f}{WLC_{ox}}\frac{g_m^{\,2}}{f_c}\Delta f$$

$$\rightarrow f_c = \frac{K_f}{WLC_{ox}}\frac{g_m}{4kT\gamma}$$

Ex) K_f=3x10^{-24} $V^2 \cdot F$ (for 0.18 um CMOS)
Cox=1 uF/cm^2 (for 0.18 um CMOS)
W= 60 um, L=0.18 um
gm=20 mS
→ f_c ~ 10MHz

MOSFET Noise Model

- In actual layout of MOSFET, the gate is made of poly silicon which has a finite resistance. The parasitic resistance generates its own thermal noise.



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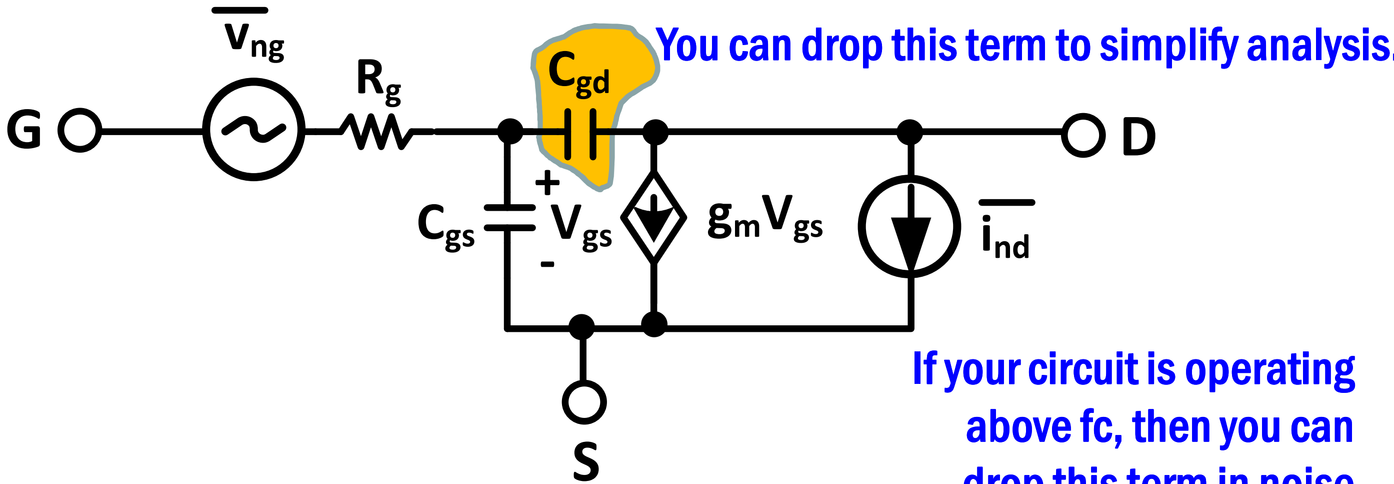
Small Signal MOSFET Model (with noises)

- Now, we can generate first-order small signal model for noise analysis of MOSFET (for high frequency applications).



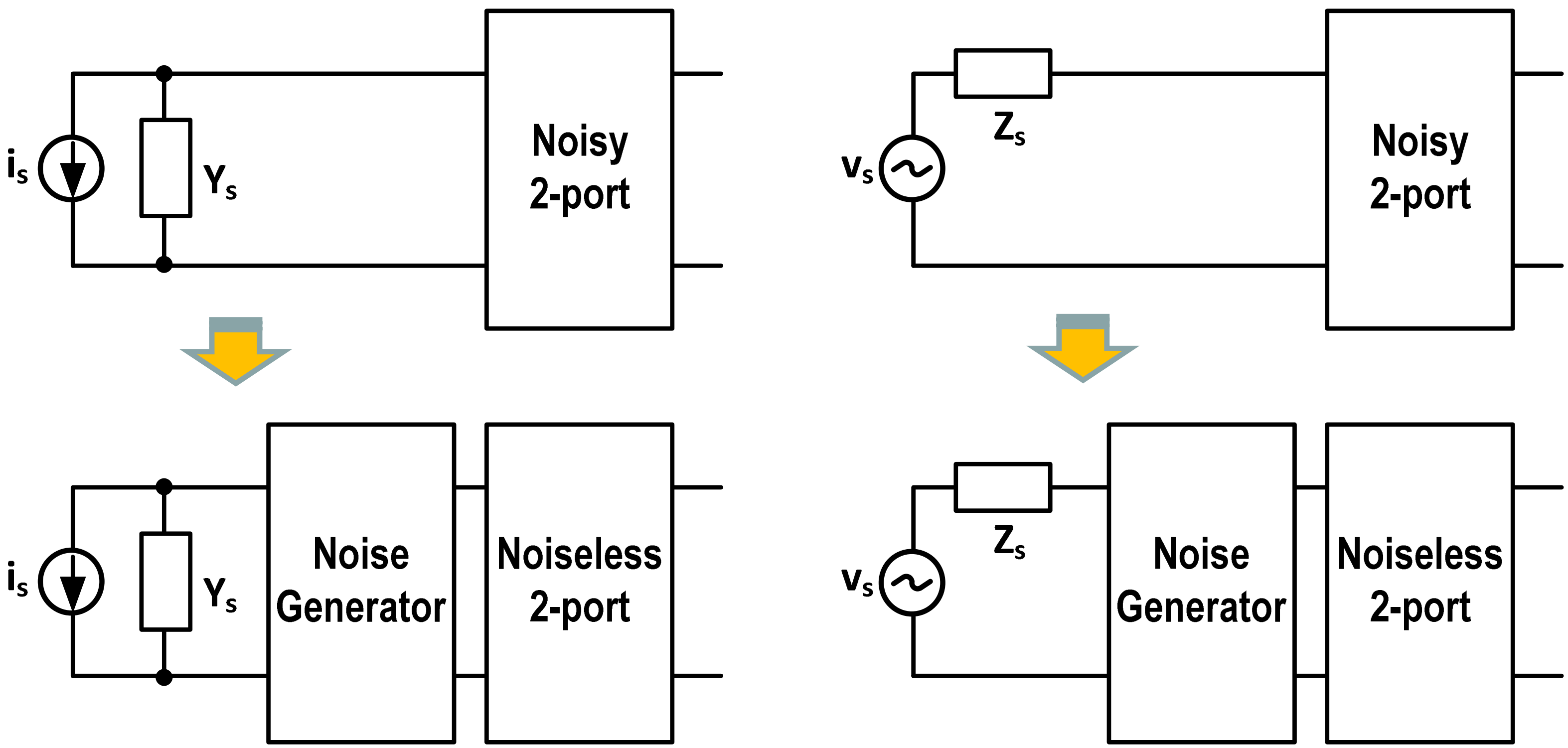
If your circuit is operating above fc, then you can drop this term in noise analysis.

$$\overline{v_{ng}^{2}} = 4kTR_g\Delta f$$

$$\overline{i_{nd}^{2}} = 4kT(\gamma g_m)\Delta f + \frac{K_f}{WLC_{ox}}\frac{g_m^{2}}{f}\Delta f$$

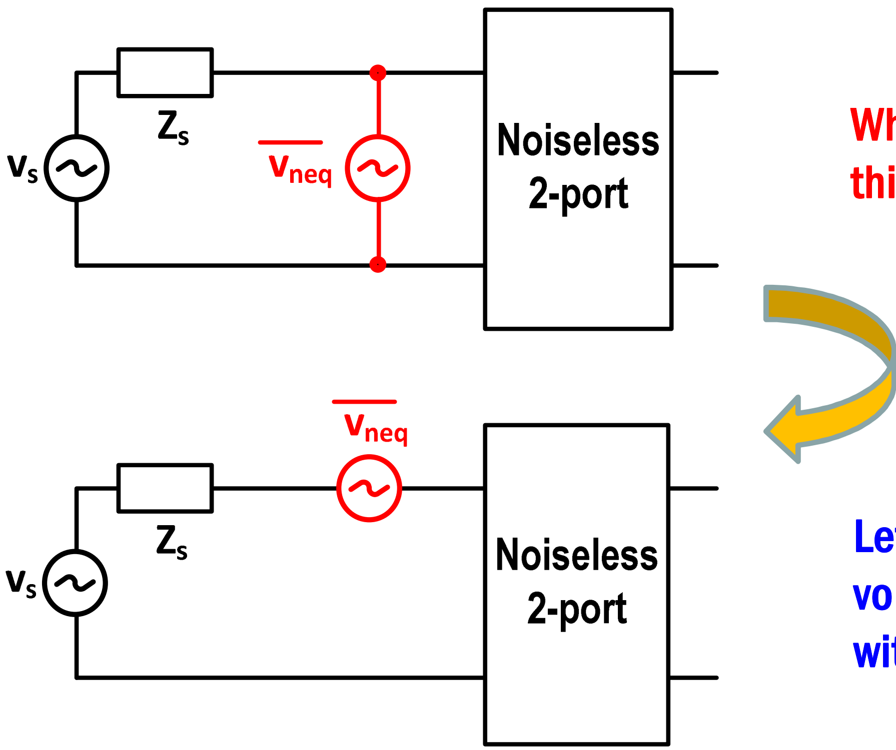
Equivalent Input Noise Generators

- We want to represent a noisy 2-port by a noiseless 2-port with equivalent noise source(s) outside the 2-port network, as shown below.



Equivalent Input Noise Generators

- Let's start with introducing an equivalent noise voltage source. Where can we put such a noise source? First let's try to put the noise voltage source in parallel with the 2-port network.

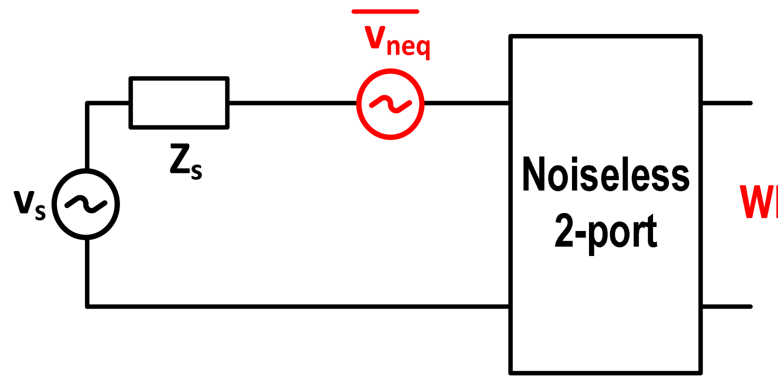


What's the problem in this configuration?

Let us put the noise voltage source in series with the 2-port network.

Equivalent Input Noise Generators

- The equivalent noise source should be able to generate the equivalent noise power at the output port regardless of **the input drive source** and **the output termination conditions**.

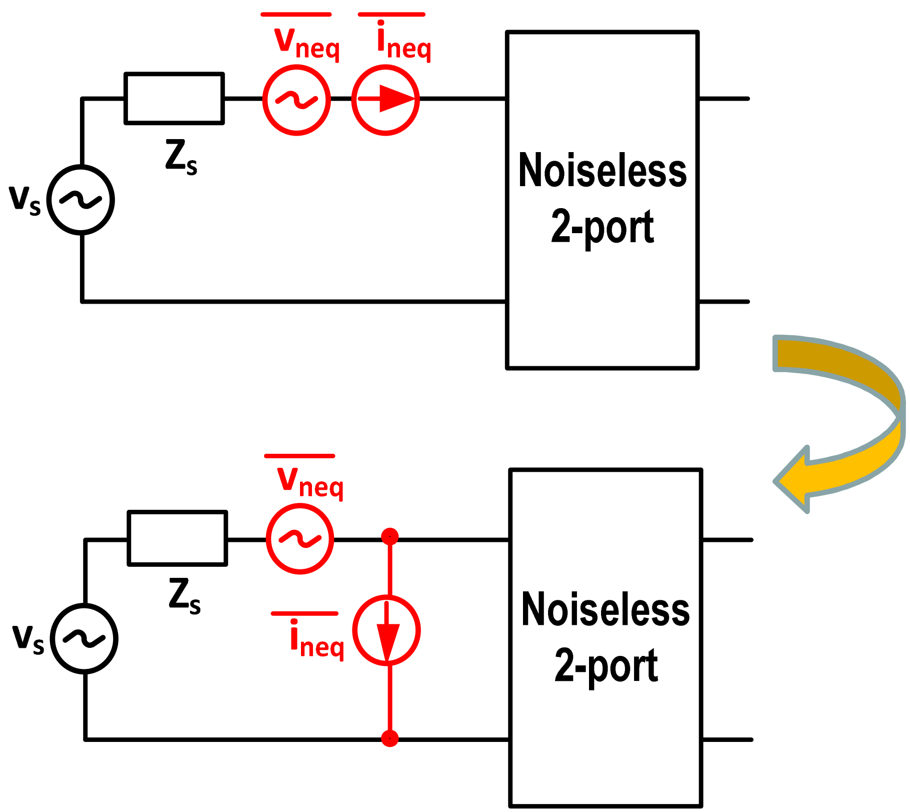


What if $Z_s = \infty$?

- If $Z_s = \infty$, then the noise voltage can play nothing, and output noise power will be zero. Clearly this not the case for all noisy 2-port networks. For instance, in CMOS, even though the input is open, there would be output noise current as long as it is biased. Therefore, using $\overline{V_{neq}}$ as the only noise generator is inadequate.

Equivalent Input Noise Generators

- Let's introduce a noise current source in series with noise voltage source.

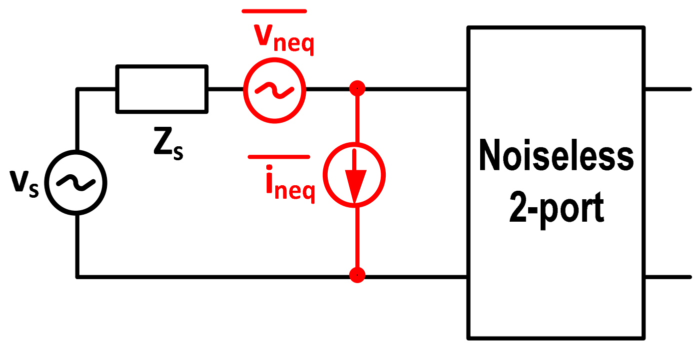


Then, the noise current source will block any signal current from source, not to mention the noise voltage source (not acceptable).

Let us put the noise current source in parallel with the 2-port network.

Equivalent Input Noise Generators

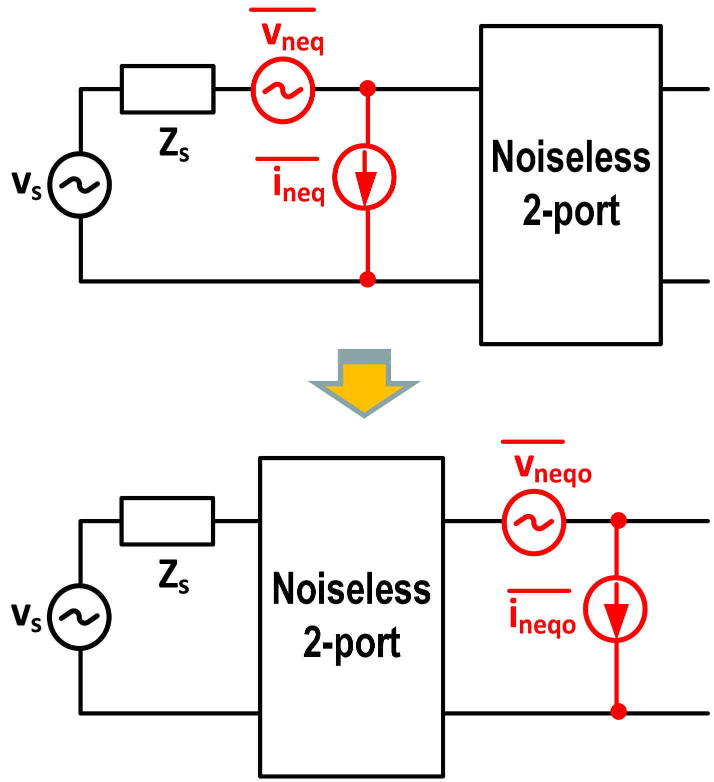
- Now, let's check the model.
  - All the signal current will go to the 2-port network.
  - If $Z_s=0$, $\overline{V_{neq}}$ will generate output noise ($\overline{i_{neq}}$ will do nothing).
  - If $Z_s=\infty$, $\overline{I_{neq}}$ will generate output noise ($\overline{V_{neq}}$ will do nothing).
  - If $0<Z_s<\infty$, both $\overline{V_{neq}}$ and $\overline{I_{neq}}$ will participate to generate output noise.



- The model is working fine, but we missed just one thing here, which is correlation between $\overline{V_{neq}}$ and $\overline{I_{neq}}$. We will see later how we should handle the correlation between the two noise amounts.

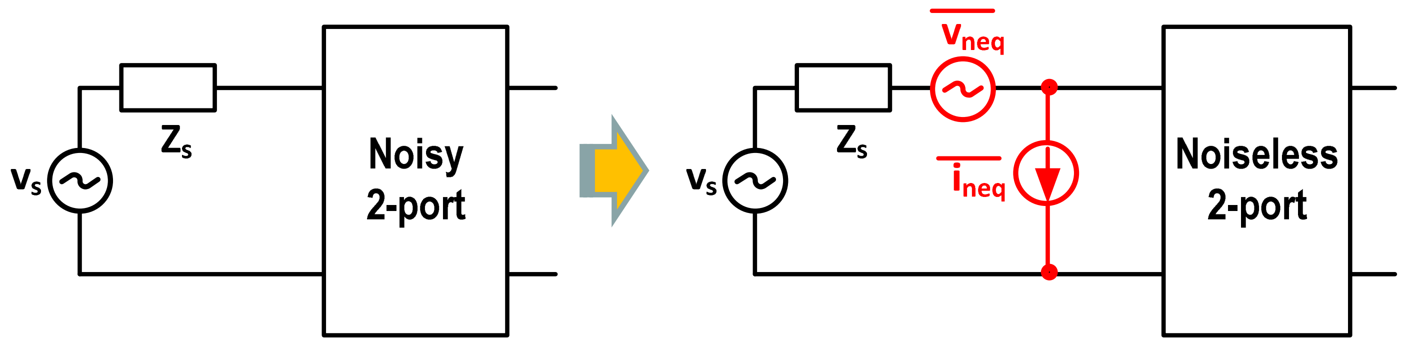
Equivalent Output Noise Generators

- We can represent the equivalent noises at the output side also.



Computing Equivalent Input Noise Sources

- ❑ How would we compute the equivalent noise voltage and current for a given noisy 2-port network ?

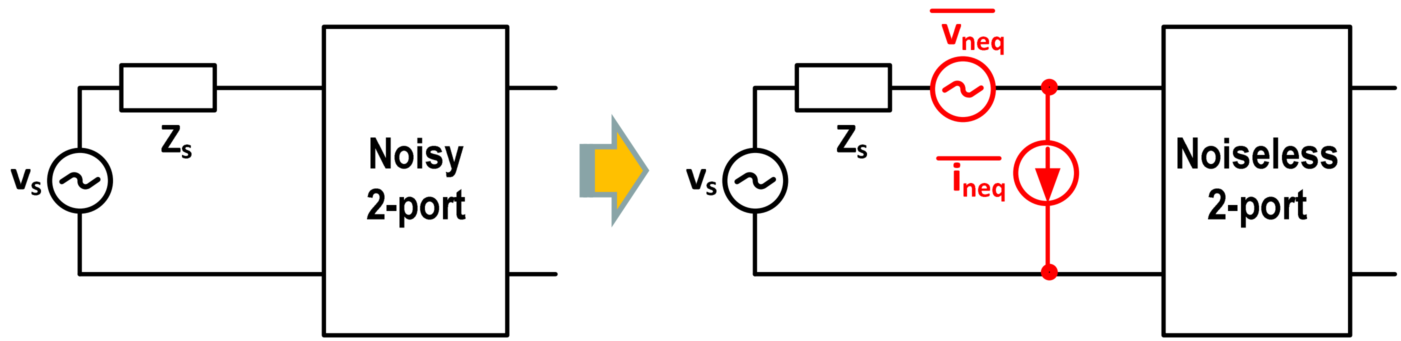


- ❑ Computing $\overline{V_{neq}}$:

- Set Zs=0 and Vs=0 for both networks. Under this condition, $\overline{i_{neq}}$ plays no role.
- Calculate output noise in the original network
- Calculate output noise in the equivalent network due to $\overline{V_{neq}}$.
- By equating these two noise amount, $\overline{V_{neq}}$ can be determined.

Computing Equivalent Input Noise Sources

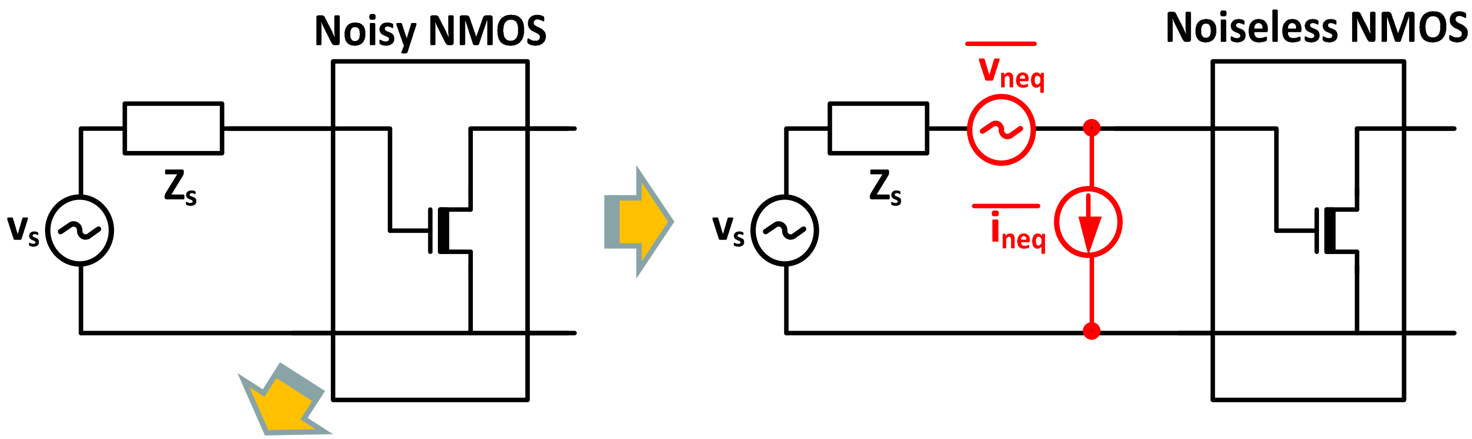
- How would we compute the equivalent noise voltage and current for a given noisy 2-port network ?

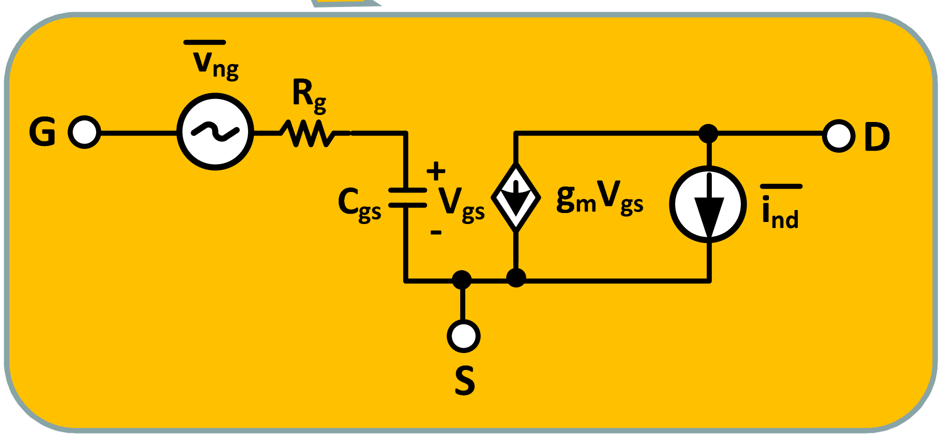


- Computing $\overline{i_{neq}}$:
- Set Zs=∞ for both networks. Under this condition, $\overline{V_{neq}}$ plays no role.
- Calculate output noise in the original network
- Calculate output noise in the equivalent network due to $\overline{i_{neq}}$.
- By equating these two noise amount, $\overline{i_{neq}}$ can be determined.

MOSFET Equivalent Input Noise Generators

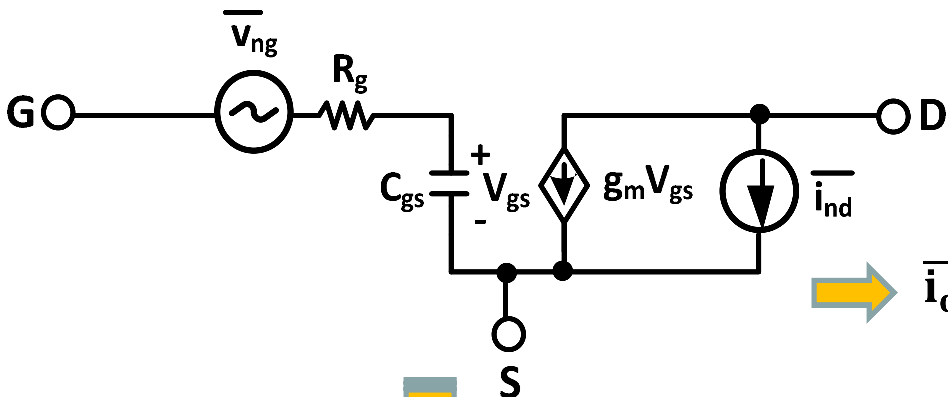
- As an example, let's compute input equivalent noises in NMOS.



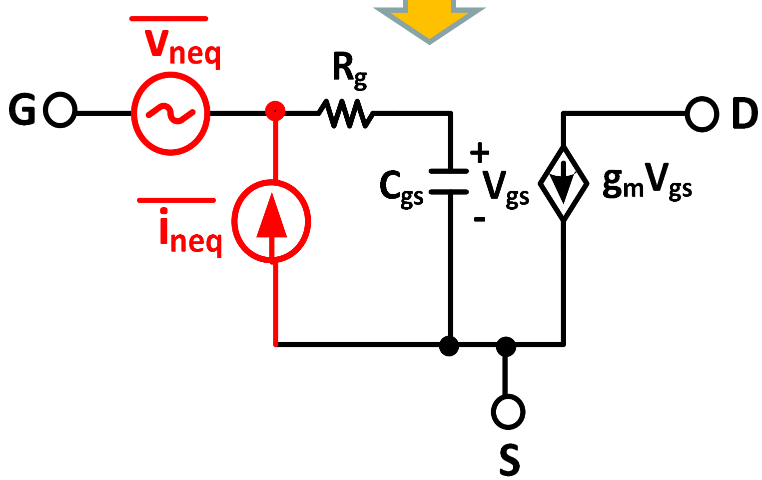


MOSFET Equivalent Input Noise Generators

- First let's find $\overline{i_{neq}}$. In order to do that, open Gate and Source for both cases.



$$\overline{i_{out}} = \overline{i_{nd}} \approx \sqrt{4kT\gamma g_m \Delta f} \quad (1)$$

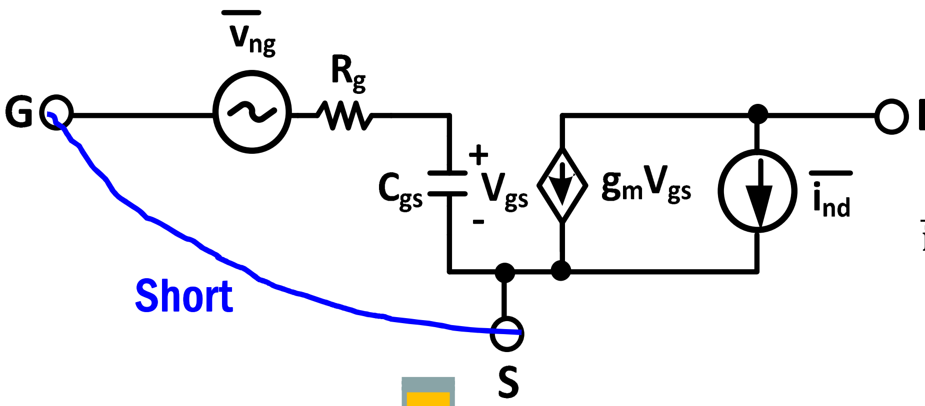


$$\overline{i_{out}} = g_m V_{gs} = g_m \frac{\overline{i_{neq}}}{j\omega C_{gs}} \quad (2)$$

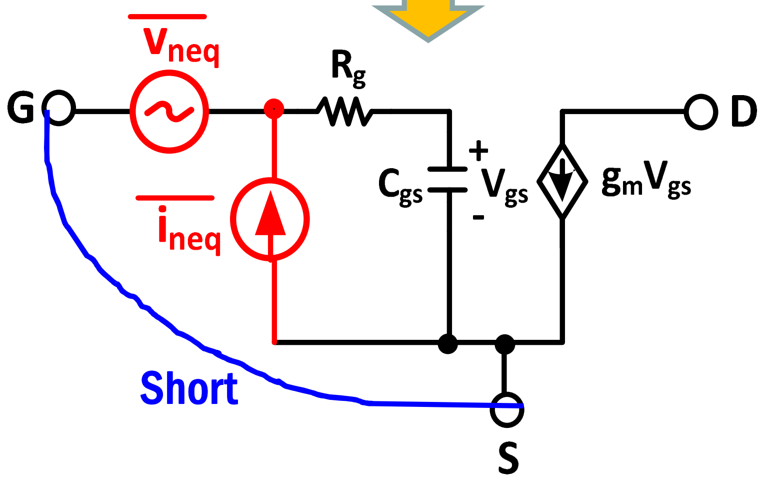
(1)=(2), $\boxed{\therefore \overline{i_{neq}} = \overline{i_{nd}} \frac{j\omega C_{gs}}{g_m}}$

MOSFET Equivalent Input Noise Generators

- Next let's find $\overline{v_{neq}}$. In order to do that, short Gate and Source for both cases.



$$\overline{i_{out}} = \overline{i_{nd}} + g_m \left(\frac{\frac{1}{j\omega C_{gs}}}{R_g + \frac{1}{j\omega C_{gs}}} \right) \overline{v_{ng}} \quad (1)$$



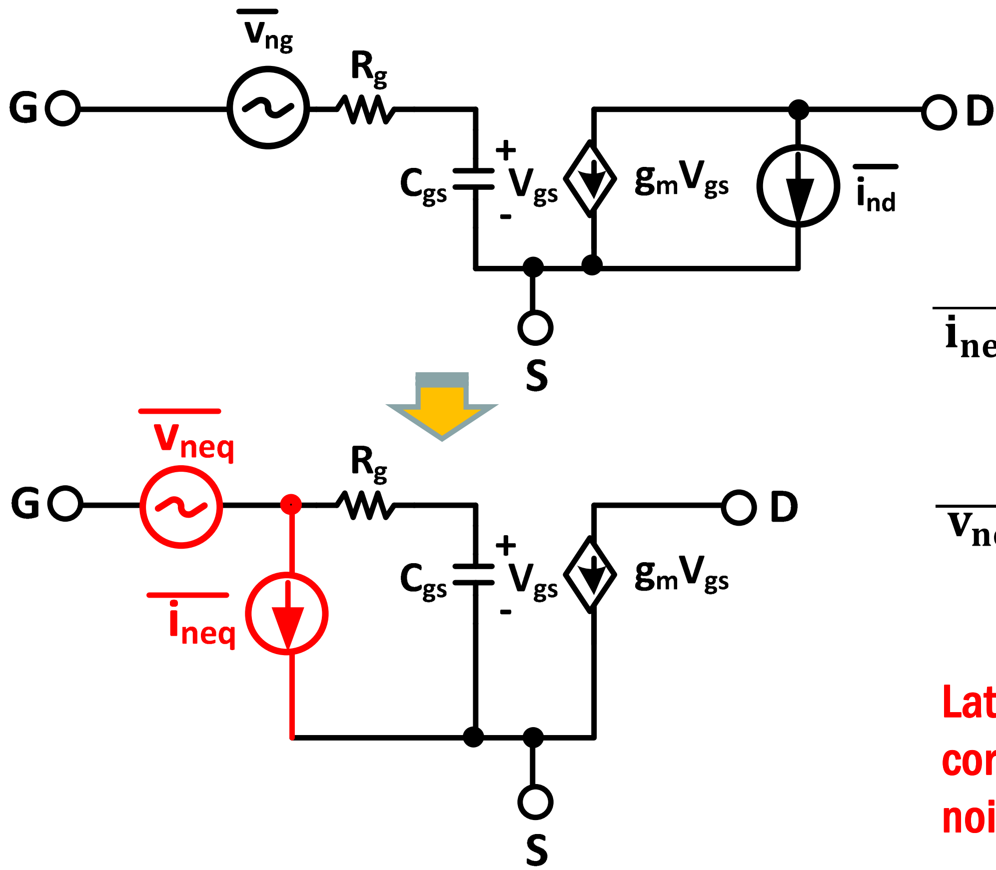
$$\overline{i_{out}} = g_m \left(\frac{\frac{1}{j\omega C_{gs}}}{R_g + \frac{1}{j\omega C_{gs}}} \right) \overline{v_{neq}} \quad (2)$$

$$(1)=(2), \therefore \overline{v_{neq}} = \overline{v_{ng}} + \frac{\overline{i_{nd}}}{g_m \left(\frac{\frac{1}{j\omega C_{gs}}}{R_g + \frac{1}{j\omega C_{gs}}} \right)}$$

$$= \boxed{\overline{v_{ng}} + \overline{i_{nd}} \frac{1 + j\omega C_{gs} R_g}{g_m}}$$

MOSFET Equivalent Input Noise Generators

- Now we calculate equivalent noise voltage and current sources in NMOS. But the noise sources have correlation part with each other.



Correlation parts

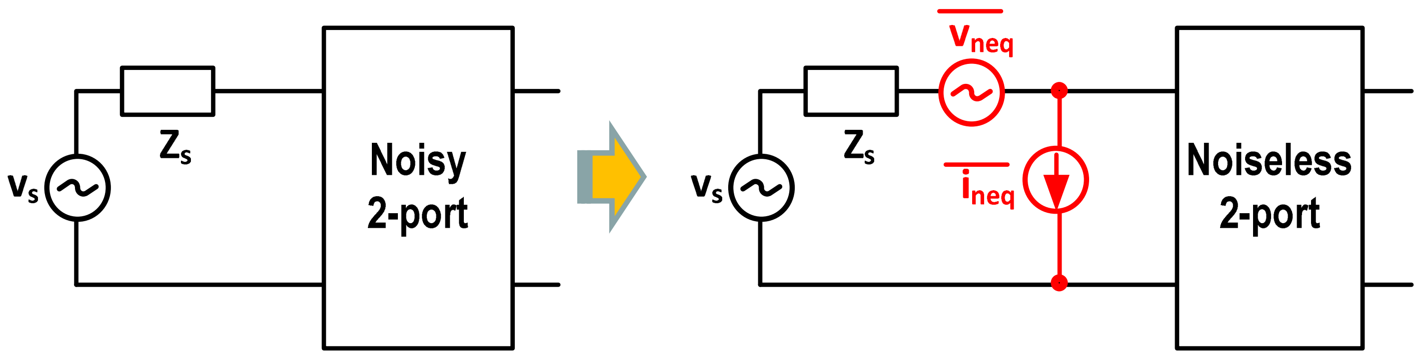
$$\overline{i_{neq}} = \overline{i_{nd}}\,\frac{j\omega C_{gs}}{g_m}$$

$$\overline{v_{neq}} = \overline{v_{ng}} + \overline{i_{nd}}\,\frac{1 + j\omega C_{gs}R_g}{g_m}$$

Later, we will come back to this correlation issue to minimize noise of a system.

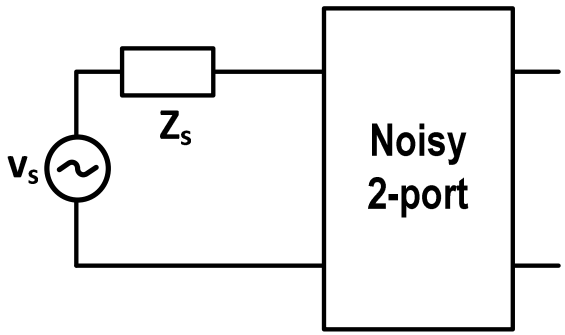
Input Referred Noise

- This is general representation of input equivalent noises, which will be valid at any input source impedance.

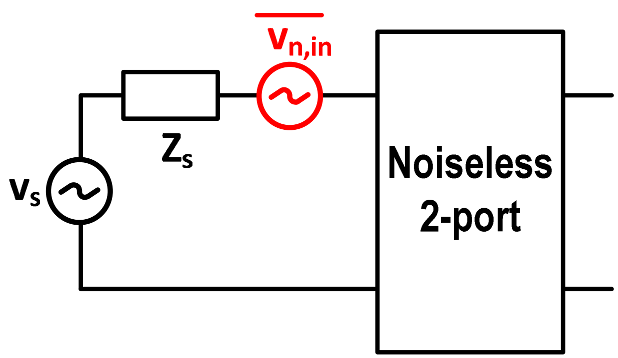


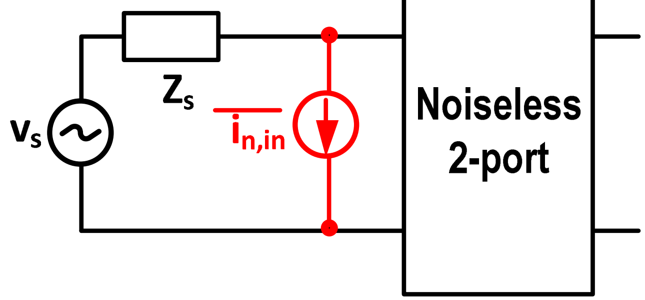
Input Referred Noise

- However, if input source impedance is clearly known, then the noisy 2-port network can be represented as:
  - Equivalent noise voltage in series with noiseless 2-port, or
  - Equivalent noise current in parallel with noiseless 2-port.



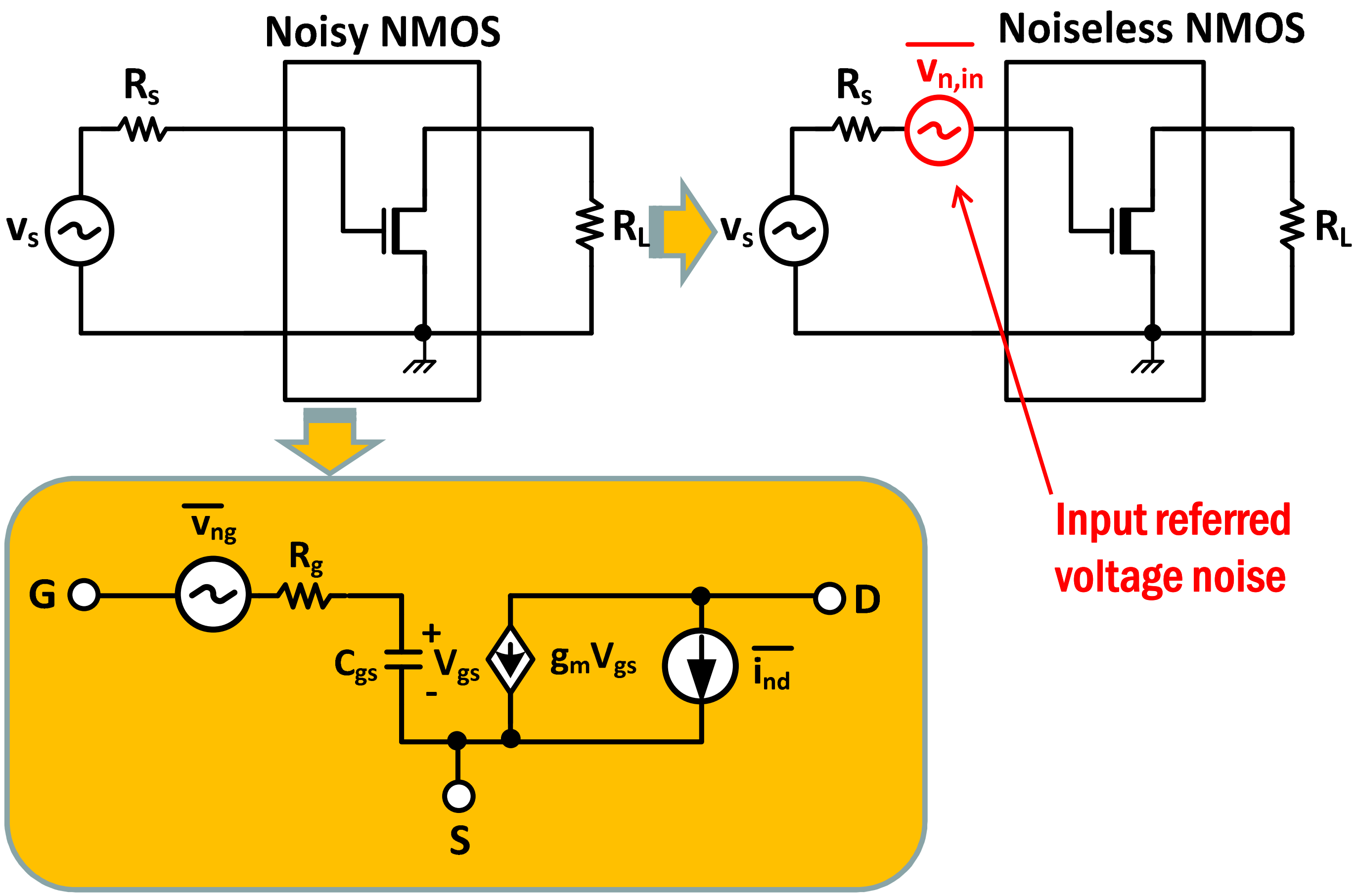
This is called input referred noise. And for many case of RFIC designs, we just input referred noise for analysis.





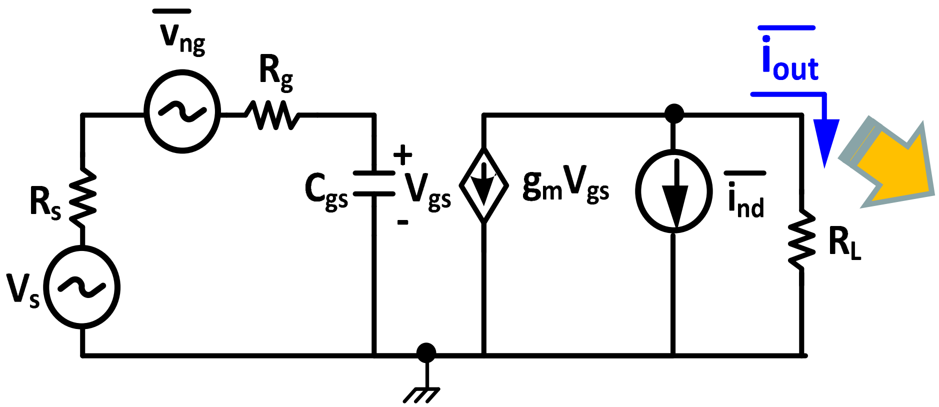
MOSFET Input Referred Noise

- Most case of RFIC designs, input source and output load impedances will be defined clearly.

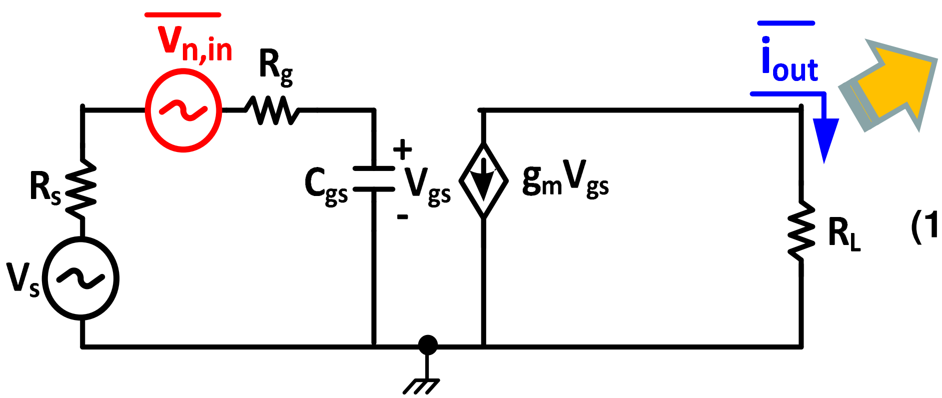


MOSFET Input Referred Noise

- Assume, V_s=0, R_L=noiseless.



$$\overline{i_{out}} = \overline{i_{nd}} + g_m \left(\frac{\frac{1}{j\omega C_{gs}}}{R_s + R_g + \frac{1}{j\omega C_{gs}}} \right) \overline{v_{ng}} \quad (1)$$



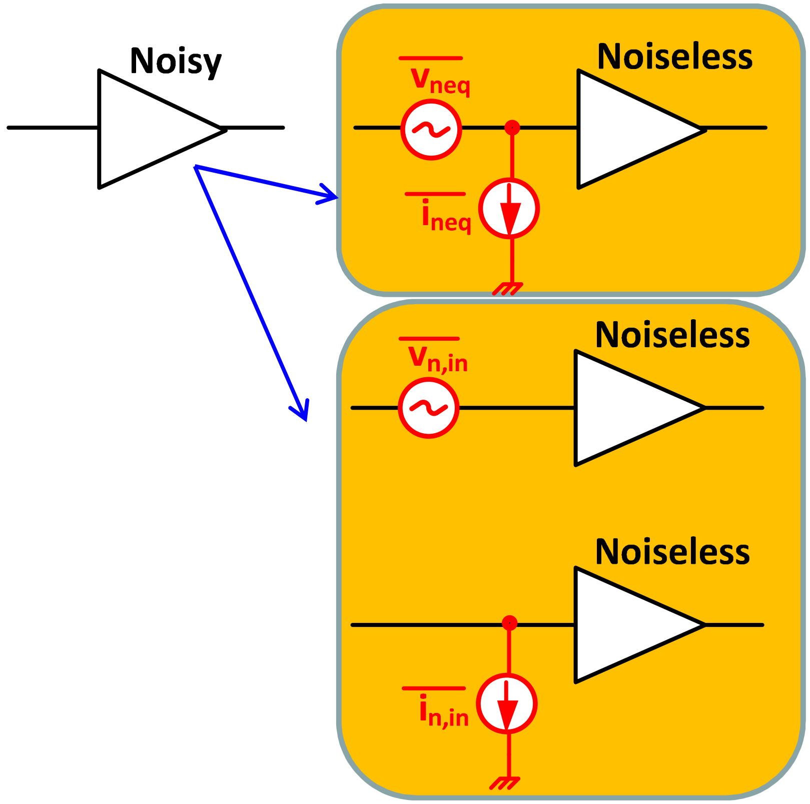
$$\overline{i_{out}} = g_m \left(\frac{\frac{1}{j\omega C_{gs}}}{R_s + R_g + \frac{1}{j\omega C_{gs}}} \right) \overline{v_{n,in}} \quad (2)$$

(1)=(2), $\therefore \overline{v_{n,in}} = \overline{v_{ng}} + \dfrac{\overline{i_{nd}}}{g_m \left(\frac{\frac{1}{j\omega C_{gs}}}{R_s + R_g + \frac{1}{j\omega C_{gs}}} \right)}$

$$= \boxed{\overline{v_{ng}} + \overline{i_{nd}} \frac{1 + j\omega C_{gs}(R_s + R_g)}{g_m}}$$

Noise Factor

- What we have been investigating is to model the noisy 2-port network with input equivalent noise sources or input referred noise sources.



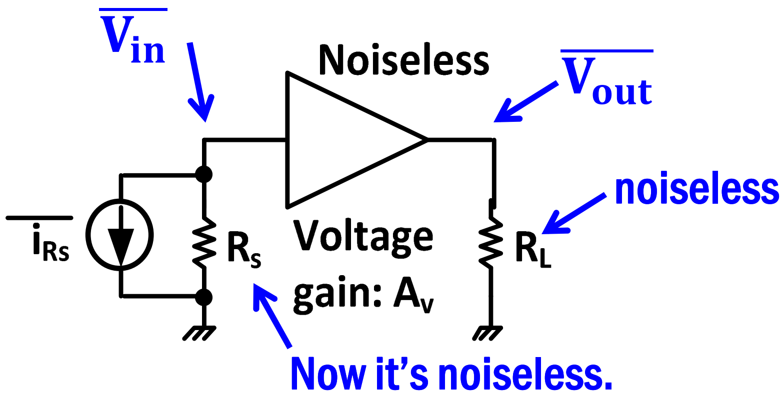
input equivalent noise sources : when input source impedance is not clearly defined.

input referred noise sources : when input source impedance is clearly defined.

- Next topic is to characterize the noise of the 2-port network using single number.

Noise Factor

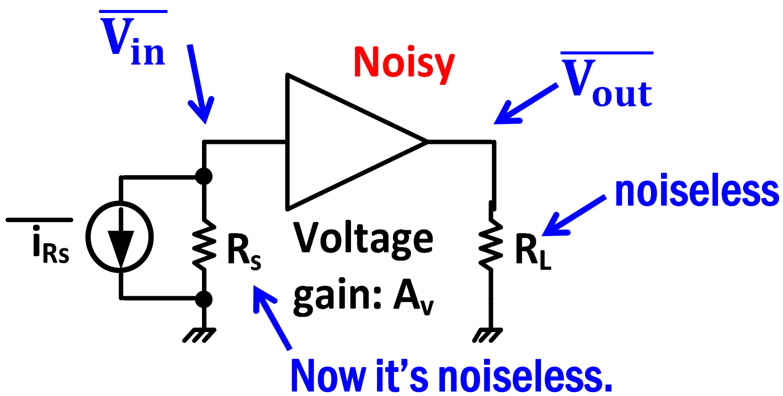
- Let's start with an example (assume R_L is noiseless, but R_s is noisy).



$$\overline{V_{in}} = \overline{i_{Rs}} \times R_s = \sqrt{4kTR_s\Delta f}$$

$$\overline{V_{out}} = \overline{V_{in}} \times A_V \qquad (1)$$

Apparently there is no additional noise from amplifier.



$$\overline{V_{in}} = \overline{i_{Rs}} \times R_s = \sqrt{4kTR_s\Delta f}$$

$$\overline{V_{out}} = \overline{V_{in}} \times A_V + \overline{V_{n,amp}} \qquad (2)$$

Output noise due to input noise source only

Output noise by the noisy amplifier

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Noise Factor

- Now, let's take output noise power ratio of (1) and (2) in previous slide.

$$\frac{\text{Power of (2)}}{\text{power of (1)}} = \frac{\left(\overline{V_{in}} \times A_V + \overline{V_{n,amp}}\right)\left(\overline{V_{in}} \times A_V + \overline{V_{n,amp}}\right)^*}{\left(\overline{V_{in}} \times A_V\right)\left(\overline{V_{in}} \times A_V\right)^*}$$

$$= \frac{\overline{V_{in}^{\ 2}}A_V^{\ 2} + \overline{V_{n,amp}^{\ 2}}}{\overline{V_{in}^{\ 2}}A_V^{\ 2}} = \boxed{1 + \frac{\overline{V_{n,amp}^{\ 2}}}{\overline{V_{in}^{\ 2}}A_V^{\ 2}}}$$

This is basically noise factor of the amplifier.

- Noise factor (F) definition:

$$F = \frac{\text{Total Noise Power delivered to load due to all noise sources}}{\text{Total Noise Power delivered to load due to source noise only}}$$

- Meaning of noise factor (F) : Noise factor is a measure of relative amount of noise added by a 2-port noisy network, compared with the noise added by the source.

Noise Figure

- Noise Figure (NF) is just dB-scale of noise factor (F).

$$\mathrm{NF} = 10 \times \log(\mathrm{F}) \quad \textbf{(dB)}$$

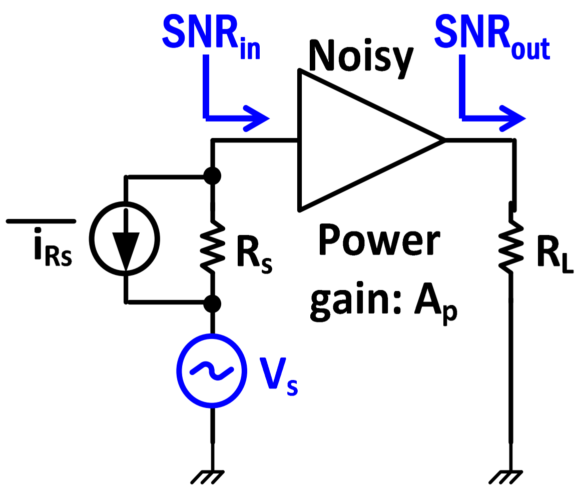
- Rule of thumb

Noise Factor (F)	Noise Figure (NF)	Meaning
1	0 dB	No noise in the 2-port
1.5	1.76 dB	The 2-port will add 50% of the noise added by the input noise source
2	3 dB	The 2-port will add the same noise amount as input noise source

- Typical LNA NF performances
- CMOS, SiGe : 2~3.5 dB (with gain > 20 dB)
- GaAs, InP: 1 ~ 2 dB (very good designs <1 dB, with gain > 20 dB)

Another Definition of Noise Factor

- Another way of defining noise factor (F): it's just different way of same meaning.



S_{in}: input signal power
S_{out}: output signal power
N_{in}: input noise power
N_{out}: output noise power
A_p: power gain of 2-port network

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{S_{in}/N_{in}}{S_{out}/N_{out}} = \frac{N_{out}}{N_{in}}\frac{S_{in}}{S_{out}} = \frac{N_{out}}{N_{in}A_p}$$

$$= \frac{\text{Total Noise Power delivered to load due to all noise sources}}{\text{Total Noise Power delivered to load due to source noise only}}$$

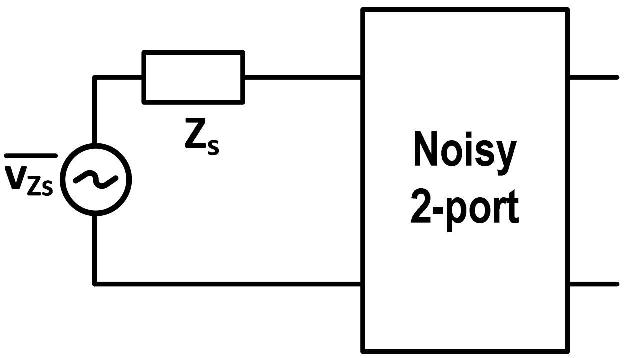
$$F = \frac{N_{out}}{N_{in}A_p} = \frac{\left(\frac{N_{out}}{A_p}\right)}{N_{in}}$$

$$= \frac{\text{Input referred noise power due to all noise sources}}{\text{Input referred noise power due to source noise only}}$$

This gives short-cut to calculate noise factor (see next page).

Short Cuts to Noise Factor (from Input Referred Noise)

- Recall that if source impedance is well defined, the noisy 2-port network can be represented with input referred noise voltage or current.

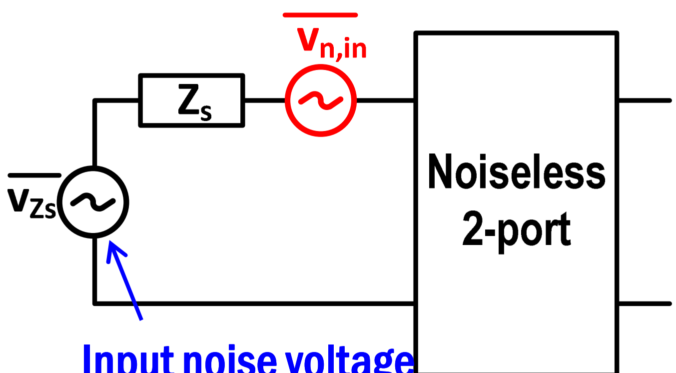


$$F = \frac{\left(\overline{v_s^2}+\overline{v_{n,in}^2}\right)/Z_s}{\overline{v_{Zs}^2}/Z_s} = \boxed{1+\frac{\overline{v_{n,in}^2}}{\overline{v_{Zs}^2}}}$$

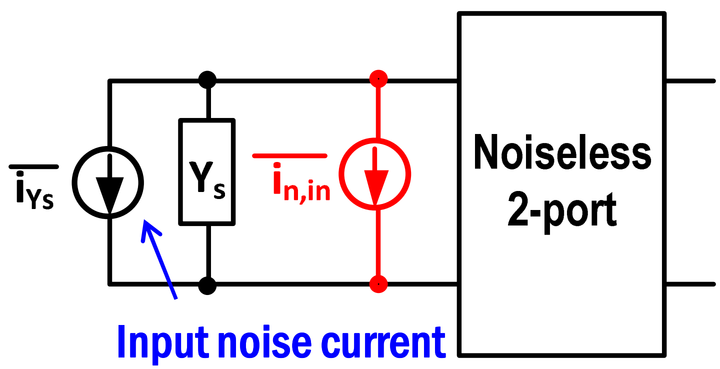
$$F = \frac{\left(\overline{i_s^2}+\overline{i_{n,in}^2}\right)\times Y_s}{\overline{i_{Ys}^2}\times Y_s} = \boxed{1+\frac{\overline{i_{n,in}^2}}{\overline{i_{Ys}^2}}}$$

Thevenin form

Norton form



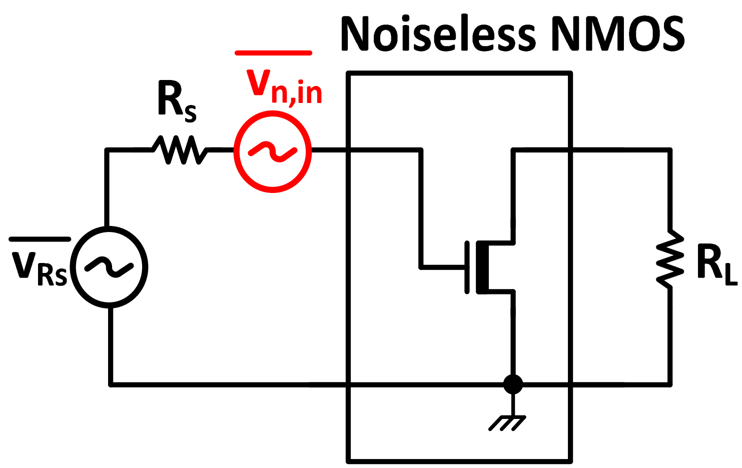
Input noise voltage due to Z_s



Input noise current due to Y_s

MOSFET Noise Factor

- Previously we calculate input referred noise voltage of NMOS.



$$\overline{v_{n,in}} = \overline{v_{ng}} + \overline{i_{nd}}\,\frac{1 + j\omega C_{gs}(R_s + R_g)}{g_m}$$

$$F = 1 + \frac{\overline{V_{n,in}}^2}{\overline{V_{Rs}}^2}$$

$$= 1 + \frac{\overline{V_{ng}}^2}{\overline{V_{Rs}}^2} + \frac{\overline{i_{nd}}^2}{\overline{V_{Rs}}^2}\left(\frac{1+\omega^2 C_{gs}{}^2 (R_s+R_g)^2}{g_m{}^2}\right)$$

apply

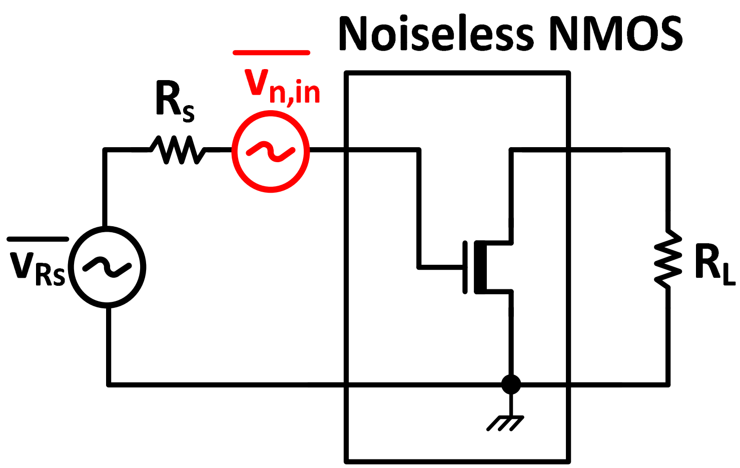
$$\overline{V_{ng}}^2 = 4kTR_g\Delta f$$
$$\overline{V_{Rs}}^2 = 4kTR_s\Delta f$$
$$\overline{i_{nd}}^2 = 4kT\gamma g_m\Delta f$$

$$F = 1 + \frac{R_g}{R_s} + \frac{\gamma}{g_m}\left(\frac{1+\omega^2 C_{gs}{}^2 (R_s+R_g)^2}{R_s}\right)$$

Ex) If you apply, R_s=50Ω, R_g=3Ω, γ=3.5, ω=2πx2.5 GHz, W=180um, L=0.18um, C_{ox}=1uF/cm^2 (for 0.18um CMOS), C_{gs}=2/3 C_{ox}WL=324 fF, g_m=60mS,
then, F=2.31, NF=10log(F)=3.64 dB.

MOSFET Noise Factor

- Let us think about how we can minimize F.



$$F = 1 + \frac{R_g}{R_s} + \frac{\gamma}{g_m}\left(\frac{1+\omega^2 C_{gs}{}^2 (R_s+R_g)^2}{R_s}\right)$$

- From the noise factor equation, we can observe that in order to minimize F,
  - Increase g_m (more current → more power consumption).
  - There is optimum R_s where F can be minimized, which can be found by this equation:

$$\frac{dF}{dR_s} = -\frac{R_g}{R_s{}^2} + \frac{\gamma}{g_m}\left(\frac{-1}{R_s{}^2} + \omega^2 C_{gs}{}^2 - \frac{\omega^2 C_{gs}{}^2 R_g{}^2}{R_s{}^2}\right) = 0$$

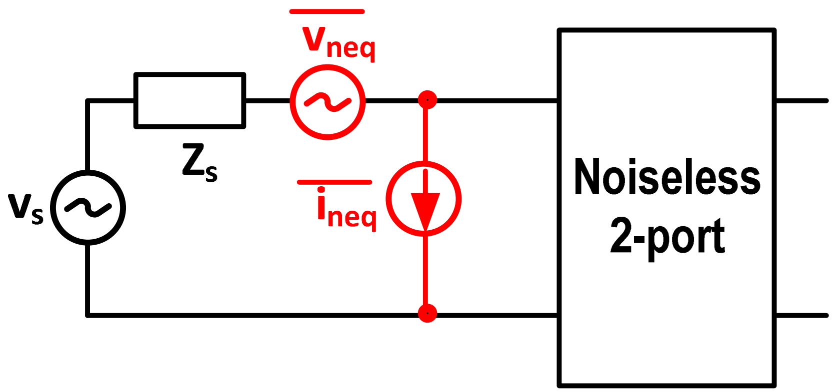
$$R_{s,opt} = \sqrt{\frac{g_m R_g + \gamma + \gamma\omega^2 C_{gs}{}^2 R_g{}^2}{\gamma\omega^2 C_{gs}{}^2}}$$

Ex) If you apply, R_g=3Ω, γ=3.5, ω=2πx2.5 GHz, W=180um, L=0.18um, C_{ox}=1uF/cm² (for 0.18um CMOS), C_{gs}=2/3 C_{ox}WL=324 fF, g_m=60mS, then, $R_{s,opt}$=201.5Ω, F=1.62, NF_{opt}=10log(F)=2.1 dB.

This result is a lot better than previous one. Can we do better?

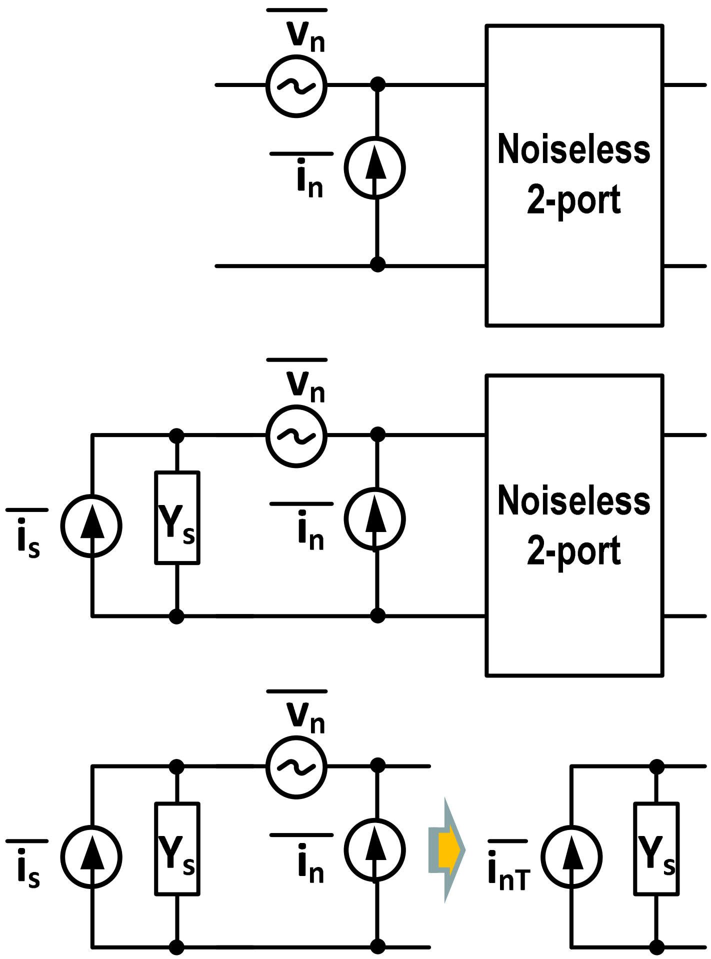
Classical 2-Port Noise Theory (intro)

- As we have seen in the previous example, there is optimum source impedance, Zs, where noise factor will be minimized for a given noisy 2-port network.



- How can we find the optimum Zs? To answer this question, we need to develop a classical 2-port noise theory.

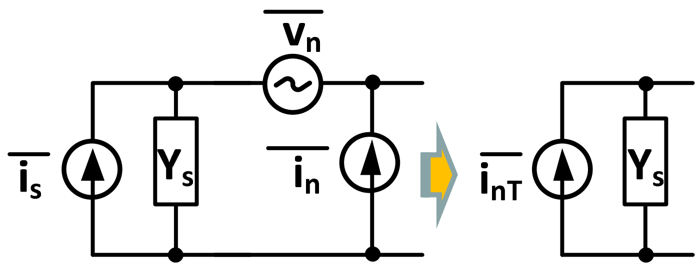
Classical 2-Port Noise Theory (1)



- First, let's draw noisy 2-port network with equivalent input noise sources.
- Next, apply input source with Norton equivalent form (Y_s: source admittance, $\overline{i_s}$: noise current from the source admittance) .
- Then let's change whole input part in Norton form.

$$\overline{i_{nT}} = \overline{i_s} + \overline{i_n} + \overline{v_n} Y_s$$

Classical 2-Port Noise Theory (2)



- Then let's change whole input part in Norton form.

$$\overline{i_{nT}} = \overline{i_s} + \overline{i_n} + \overline{v_n} Y_s$$

- Let's calculate noise factor of the system.

$$F = \frac{\text{Total noise current power}}{\text{Noise current power due to source only}} = \frac{\overline{i_{nT}}^2}{\overline{i_s}^2} = \frac{(\overline{i_s} + \overline{i_n} + \overline{v_n} Y_s)(\overline{i_s}^* + \overline{i_n}^* + \overline{v_n}^* Y_s^*)}{\overline{i_s}^2}$$

We need to develop some math to handle this correlation.

In this equation,
- $\overline{i_s}$ and $\overline{i_n}$ are not correlated.
- $\overline{i_s}$ and $\overline{v_n}$ are not correlated.
- $\overline{v_n}$ and $\overline{i_n}$ are correlated.

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Classical 2-Port Noise Theory (3)

- Let's decompose $\overline{i_n}$ into two components.

$$\boxed{\overline{i_n} = \overline{i_{nc}} + \overline{i_{nu}}}$$

$\overline{i_{nc}}$: noise component correlated to $\overline{v_n}$
$\overline{i_{nu}}$: noise component uncorrelated to $\overline{v_n}$

- Now let's define correlation admittance as:

$$Y_c = \frac{\overline{i_{nc}}}{\overline{v_n}} \rightarrow \boxed{\overline{i_{nc}} = Y_c \overline{v_n},}$$

$$\therefore \boxed{\overline{i_{nu}} = \overline{i_n} - Y_c \overline{v_n}}$$

→How can we calculate Y_c?

$$\begin{aligned}\overline{v_n}^* \overline{i_n} &= \overline{v_n}^* (\overline{i_{nc}} + \overline{i_{nu}}) \\ &= \overline{v_n}^* (Y_c \overline{v_n} + \overline{i_{nu}}) \\ &= \overline{v_n}^2 Y_c + \cancel{\overline{v_n}^* \overline{i_{nu}}} \\ &= \overline{v_n}^2 Y_c\end{aligned}$$

This term will go zero because of no correlation.

$$\boxed{\therefore Y_c = \frac{\overline{v_n}^* \overline{i_n}}{\overline{v_n}^2}}$$

Classical 2-Port Noise Theory (4)

- Let's substitute the equations in the previous page into noise factor equation.

$$F = \frac{(\overline{i_s} + \overline{i_n} + \overline{v_n}Y_s)(\overline{i_s}^* + \overline{i_n}^* + \overline{v_n}^*Y_s^*)}{\overline{i_s^2}}$$

$$= \boxed{1 + \frac{\overline{i_{nu}^2} + |Y_s + Y_c|^2\overline{v_n^2}}{\overline{i_s^2}}}$$

Apply these variables into noise factor equation (the result is in next slide).

- Now let's define these variables.

$$R_n = \frac{\overline{v_n^2}}{4kT\Delta f}$$

$$G_u = \frac{\overline{i_{nu}^2}}{4kT\Delta f}$$

$$G_s = \frac{\overline{i_s^2}}{4kT\Delta f}$$

These are just change of variables.

$$Y_s = G_s + jB_s$$

$$Y_c = G_c + jB_c$$

These are decomposition of source admittance and correlation admittance.

Classical 2-Port Noise Theory (5)

- When new variables applied into the noise factor equation,

$$\mathbf{F = 1 + \frac{\overline{i_{nu}^{\,2}} + |Y_s + Y_c|^2\overline{v_n^{\,2}}}{\overline{i_s^{\,2}}} = 1 + \frac{G_u + |Y_s + Y_c|^2 R_n}{G_s}}$$

$$= \boxed{\mathbf{1 + \frac{G_u + \left((G_s + G_c)^2 + (B_s + B_c)^2\right) R_n}{G_s}}}$$

- Remind that all the objective of these mathematics is to find optimum $Y_s=G_s+jB_s$ to minimize the noise factor.
  Clearly to minimize F,
  1. $B_s=-B_c$ (possible)
  2. $G_s=-G_c$ (not possible, why?).

Classical 2-Port Noise Theory (6)

- The condition on Gs for optimum noise factor can be found by differentiating the F with respect to Gs and setting it to zero.

$$\frac{dF}{dG_s} = -\frac{G_u}{{G_s}^2} - \frac{{G_c}^2}{{G_s}^2} R_n + R_n = 0$$

$$\therefore G_{s,opt} = \sqrt{\frac{G_u}{R_n} + {G_c}^2}$$

- The condition for source impedance for minimum noise factor can be summarized as:

$$Y_{s,opt} = G_{s,opt} + jB_{s,opt}$$

$$\boxed{= \sqrt{\frac{G_u}{R_n} + {G_c}^2} - jB_c}$$

Classical 2-Port Noise Theory (7)

- By substituting the $Y_{s,opt}$ into the noise factor equation, we can get minimum noise factor of a system.

$$F_{min} = 1 + \frac{G_u + \left(G_{s,opt} + G_c\right)^2 R_n}{G_{s,opt}}$$

$$\boxed{= 1 + 2R_n\left(\sqrt{\frac{G_u}{R_n} + {G_c}^2} + G_c\right)}$$

- A general equation of noise factor can be expressed using the F_{min} as:.

$$\boxed{F = F_{min} + \frac{R_n}{G_s}\left|Y_s - Y_{s,opt}\right|^2}\text{, where } Y_{s,opt} = \sqrt{\frac{G_u}{R_n} + {G_c}^2} - jB_c$$

Classical 2-Port Noise Theory (summary)

1. Find input equivalent voltage and noise current sources.

$$\overline{V_n} \quad \overline{i_n}$$

2. Find correlation admittance.

$$Y_c = \frac{\overline{V_n^* i_n}}{\overline{V_n^2}}$$

3. Find uncorrelated noise current.

$$\overline{i_{nu}} = \overline{i_n} - Y_c \overline{V_n}$$

4. Find R_n & G_u.

$$\overline{V_n} \rightarrow R_n$$

$$\overline{i_{nu}} \rightarrow G_u$$

5. Compute $G_{s,opt}$ & $B_{s,opt}$

$$G_{s,opt} = \sqrt{\frac{G_u}{R_n} + G_c^2}$$

$$B_{s,opt} = -B_c$$

6. Calculate F_{min} & F

$$F_{min} = 1 + 2R_n\left(\sqrt{\frac{G_u}{R_n} + G_c^2} + G_c\right)$$

$$F = F_{min} + \frac{R_n}{G_s}\left|Y_s - Y_{s,opt}\right|^2$$

MOSFET Minimum Noise Factor (step-1)

Find input equivalent voltage and noise current sources.

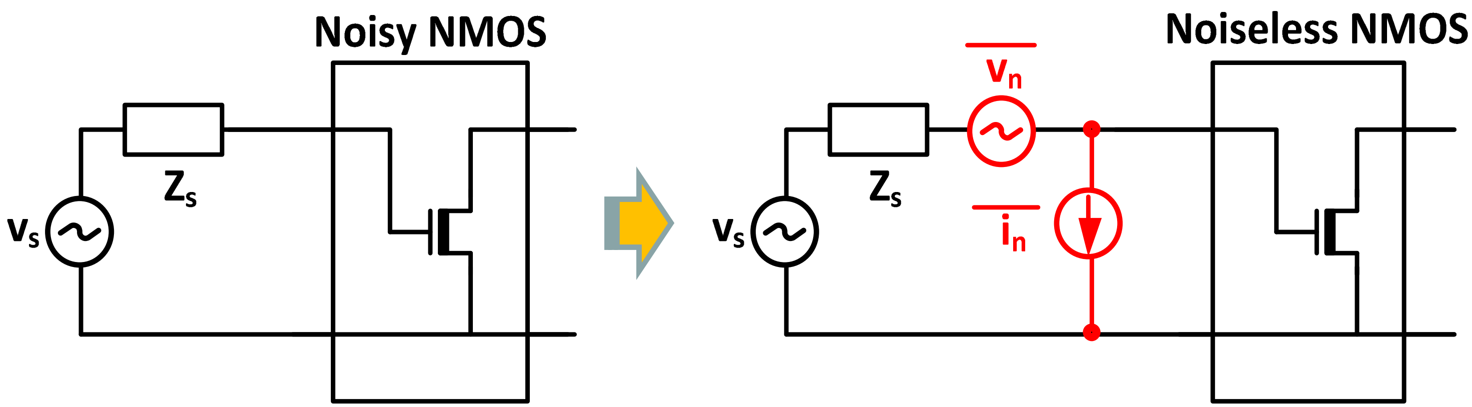
$\overline{V_n}$ $\overline{i_n}$

- We already computed these noise amounts.

$$\overline{v_n} = \overline{v_{ng}} + \overline{i_{nd}}\frac{1 + j\omega C_{gs}R_g}{g_m} \approx \overline{v_{ng}} + \frac{\overline{i_{nd}}}{g_m}$$

$$\overline{i_n} = \overline{i_{nd}}\frac{j\omega C_{gs}}{g_m}$$

Assume $R_g << 1/\omega C_{gs}$



$$\overline{i_{nd}^{\,2}} \approx 4kT\gamma g_m \Delta f$$

$$\overline{v_{ng}^{\,2}} = 4kTR_g \Delta f$$

MOSFET Minimum Noise Factor (step-2 & 3)

Find correlation admittance.

$$Y_c = \frac{\overline{V_n^* i_n}}{\overline{V_n^2}}$$

$$Y_c = \frac{\overline{V_n^* i_n}}{\overline{V_n^2}} = \frac{\overline{\left(v_{ng}^* + \frac{i_{nd}^*}{g_m}\right) i_{nd}}\frac{j\omega C_{gs}}{g_m}}{\overline{v_{ng}^2} + \frac{\overline{i_{nd}^2}}{g_m^2}} = \frac{\frac{j\omega C_{gs}}{g_m^2}\overline{i_{nd}^2}}{\overline{v_{ng}^2} + \frac{\overline{i_{nd}^2}}{g_m^2}}$$

$$= \frac{\frac{j\omega C_{gs}}{g_m^2}}{\frac{\overline{v_{ng}^2}}{\overline{i_{nd}^2}} + \frac{1}{g_m^2}}$$

Assume $g_m R_g \ll 1$

$$= \frac{j\omega C_{gs}}{1 + \frac{g_m R_g}{\gamma}} \approx j\omega C_{gs}$$

Find uncorrelated noise current.

$$\overline{i_{nu}} = \overline{i_n} - Y_c \overline{V_n}$$

$$\overline{i_{nu}} = \overline{i_n} - Y_c \overline{V_n}$$

$$= \overline{i_{nd}}\frac{j\omega C_{gs}}{g_m} - j\omega C_{gs}\left(\overline{v_{ng}} + \frac{\overline{i_{nd}}}{g_m}\right)$$

$$= -j\omega C_{gs}\overline{v_{ng}}$$

MOSFET Minimum Noise Factor (step-4 & 5)

Find R_n & G_u.

$$\overline{V_n} \rightarrow R_n$$
$$\overline{i_{nu}} \rightarrow G_u$$

$$R_n = \frac{\overline{v_n}^2}{4kT\Delta f} = \frac{\overline{v_{ng}}^2 + \frac{\overline{i_{nd}}^2}{g_m{}^2}}{4kT\Delta f} = \frac{4kTR_g\Delta f + \frac{4kT\gamma g_m \Delta f}{g_m{}^2}}{4kT\Delta f}$$

$$= R_g + \frac{\gamma}{g_m} \approx \frac{\gamma}{g_m}$$

$$G_u = \frac{\overline{i_{nu}}^2}{4kT\Delta f} = \frac{\omega^2 C_{gs}{}^2 \overline{v_{ng}}^2}{4kT\Delta f} = \omega^2 C_{gs}{}^2 R_g$$

Compute $G_{s,opt}$ & $B_{s,opt}$

$$G_{s,opt} = \sqrt{\frac{G_u}{R_n} + G_c{}^2}$$
$$B_{s,opt} = -B_c$$

$$G_{s,opt} = \sqrt{\frac{G_u}{R_n} + G_c{}^2} = \sqrt{\frac{\omega^2 C_{gs}{}^2 R_g}{\frac{\gamma}{g_m}}} = \omega C_{gs}\sqrt{\frac{g_m R_g}{\gamma}}$$

$$B_{s,opt} = -B_c = -\omega C_{gs}$$

MOSFET Minimum Noise Factor (step-6)

Calculate F_{min}

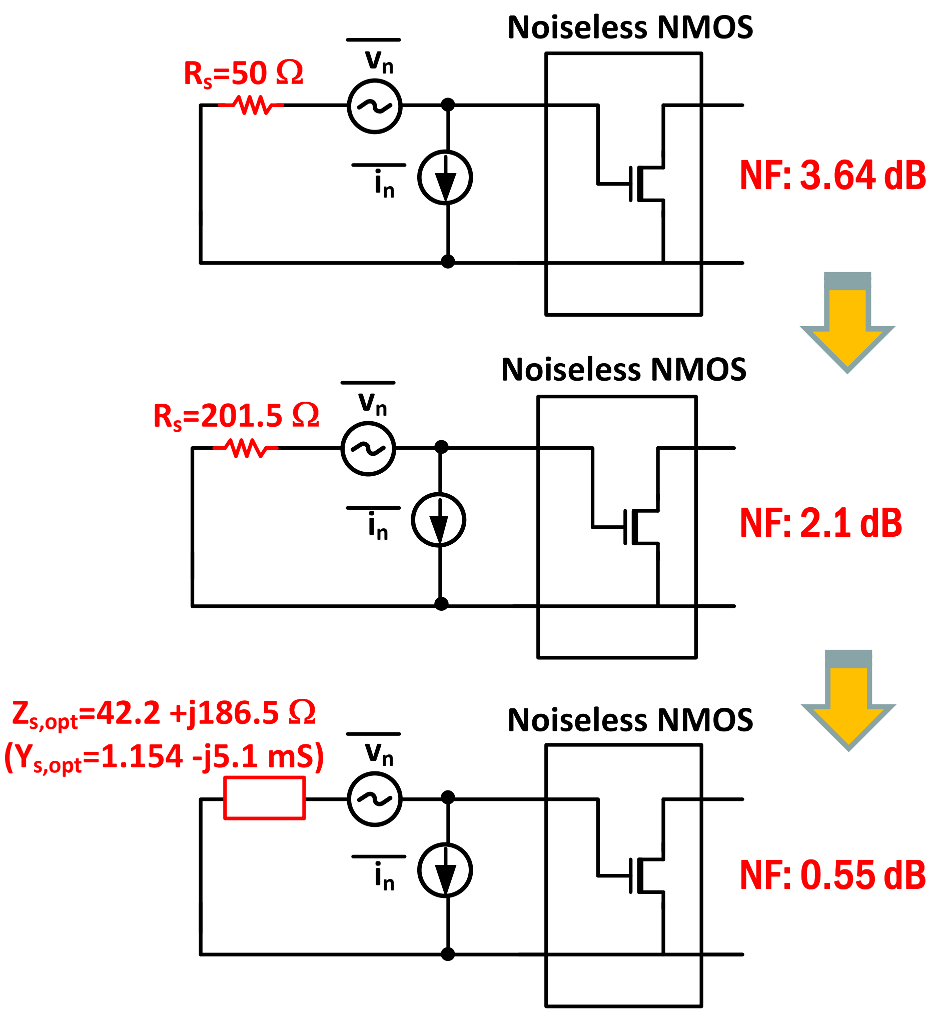
$$F_{min} = 1 + 2R_n\left(\sqrt{\frac{G_u}{R_n} + {G_c}^2} + G_c\right)$$

$$= 1 + 2\frac{\gamma}{g_m}\,\omega C_{gs}\sqrt{\frac{g_m R_g}{\gamma}}$$

$$= 1 + 2\omega C_{gs}\sqrt{\gamma\frac{R_g}{g_m}}$$

Ex) If you apply, R_g=3Ω, γ=3.5, ω=2πx2.5 GHz, W=180um, L=0.18um, C_{ox}=1uF/cm^2 (for 0.18um CMOS), C_{gs}=2/3 C_{ox}WL=324 fF, g_m=60mS, then, $G_{s,opt}$=1.154 mS, $B_{s,opt}$=-5.1 mS, F_{min}=1.136, NF_{min}=10log(F_{min})=0.55 dB.

MOSFET Noise Matching

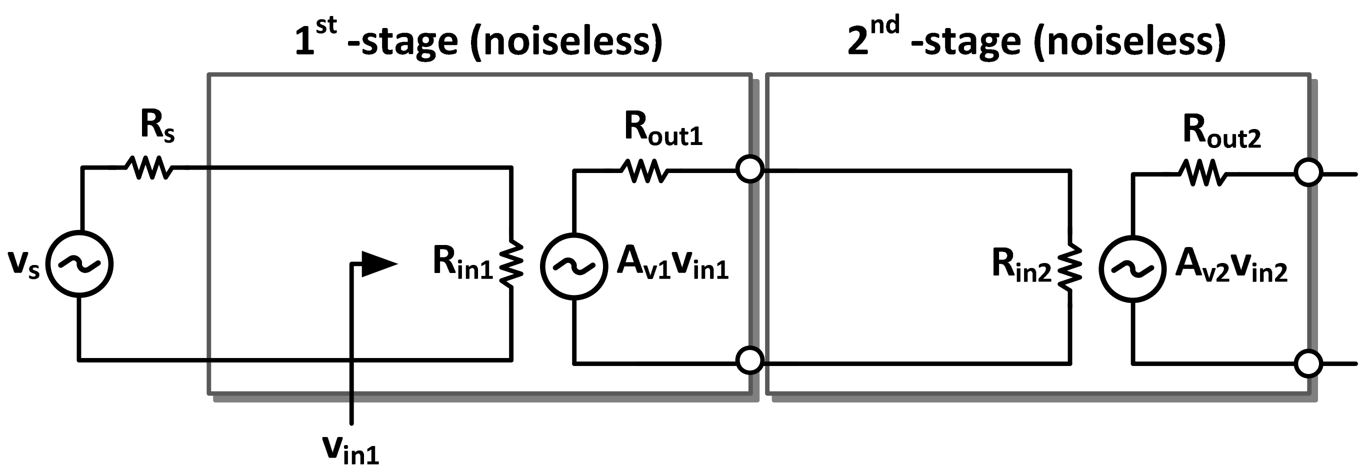


R_g=3Ω, γ=3.5, ω=2πx2.5 GHz,
W=180um, L=0.18um,
C_{ox}=1uF/cm^2 (for 0.18um CMOS),
C_{gs}=2/3 C_{ox}WL=324 fF,
g_m=60mS

- For a given bias condition and device parameters, there is optimum source impedance, $Z_{s,opt}$ ($Y_{s,opt}$) where F (NF) is minimum.
- Matching source impedance to $Z_{s,opt}$ is called "noise matching".
- Usually it's difficult to match source impedance to optimum noise point and maximum power transfer at the same time.
- We will study how to handle this issue in LNA design section.

Cascaded Noise Factor (General Case)

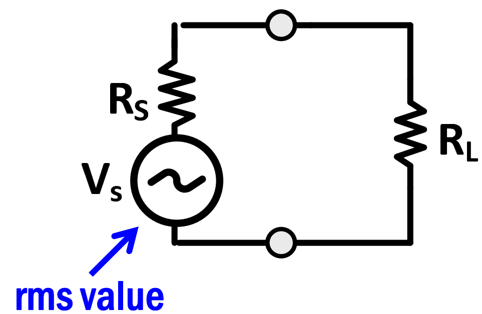
- Until now, we have focused on characterization of single-stage 2-port network. Let's continue computing NF of multiple cascaded system.
- Let's think about noiseless 2-port network with applying signal source.



- Simply, we can model the noiseless 2-port network using parameters, input impedance (R_{in}), output impedance (R_{out}) and voltage gain (A_v).

Available Power vs. Transducer Power (review)

- Before continuing further, let's review available power gain (P_{av}) and transducer power gain (P_L)



❖ <u>Available power</u> (P_{av})

- The possible maximum power deliverable to a load
- Happens when $R_s = R_L$ (matched condition).

$$P_{av} = \frac{V_s^{\,2}}{4R_s}$$

- Depends on source impedance only.

❖ <u>Transducer power</u> (P_L)

- Actual power delivered to a load

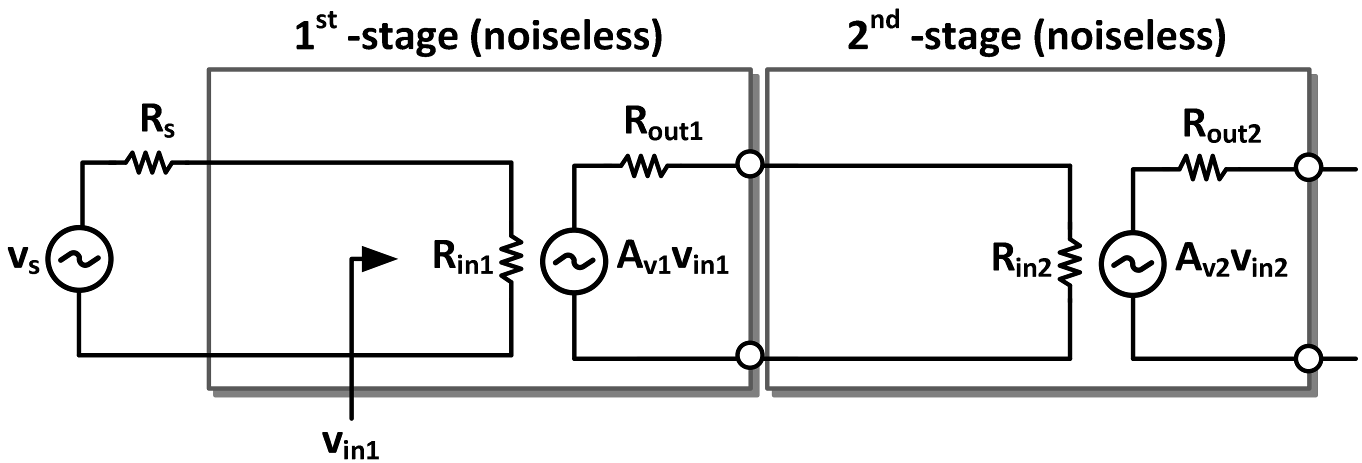
$$P_L = \left(\frac{R_L}{R_s + R_L} V_s\right)^2 \frac{1}{R_L}$$

- Depends on both source and load impedances.

$$\boxed{P_{L,max} = P_{av}}$$

Available Power Gain (G_{av})

- Let's calculate available power gain (G_{av}) in the first stage.



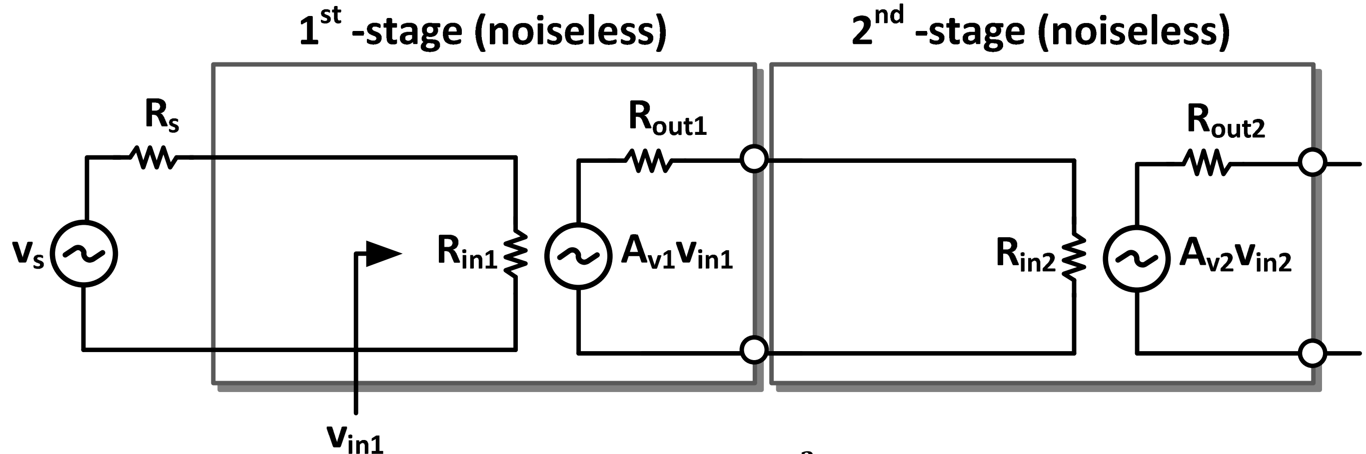
$$P_{av,in} = \frac{V_s^2}{4R_s} \qquad P_{av,out} = \frac{(A_{v1}V_{in1})^2}{4R_{out1}} = A_{v1}^2 \left(\frac{R_{in1}}{R_s + R_{in1}}\right)^2 \frac{V_{in1}^2}{4R_{out1}}$$

$$\boxed{G_{av} = \frac{P_{av,out}}{P_{av,in}} = A_{v1}^2 \left(\frac{R_{in1}}{R_s + R_{in1}}\right)^2 \frac{R_s}{R_{out1}}}$$

- Note that available power is only dependent of source impedance, since it is assumed matched condition.

Transducer Power Gain (G_T)

- Let's calculate available power gain (G_{av}) in the first stage.



$$P_{av,in}=\frac{V_s^2}{4R_s}\qquad P_{L,out}=\frac{\left(A_{v1}V_{in1}\frac{R_{in2}}{R_{out1}+R_{in2}}\right)^2}{R_{in2}}=A_{v1}^{\;2}\left(\frac{R_{in1}}{R_s+R_{in1}}\right)^2\left(\frac{R_{in2}}{R_{out1}+R_{in2}}\right)^2\frac{V_s^{\;2}}{R_{in2}}$$

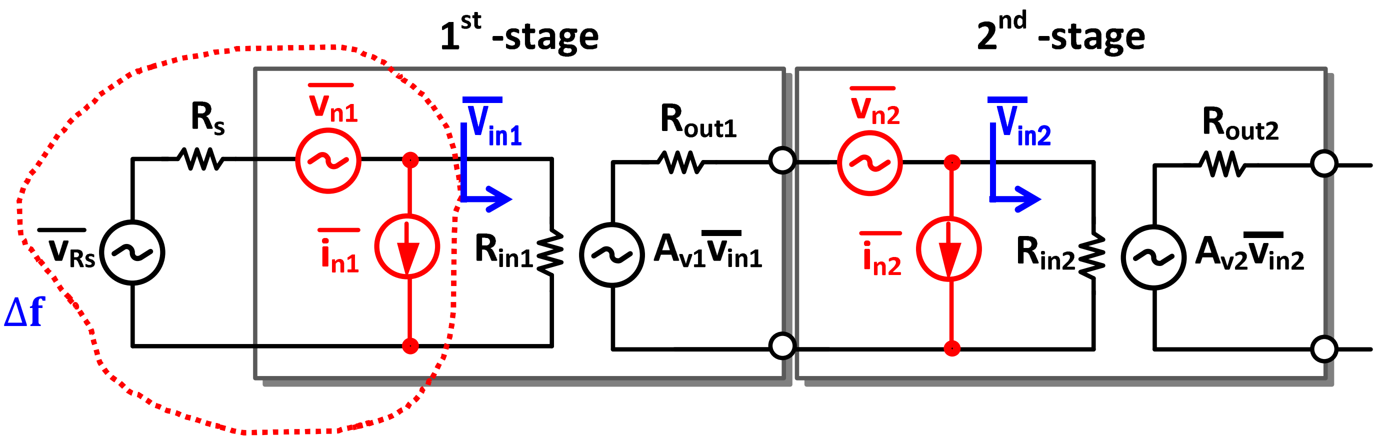
$$\boxed{G_T=\frac{P_{L,out}}{P_{av,in}}=A_{v1}^{\;2}\left(\frac{R_{in1}}{R_s+R_{in1}}\right)^2\left(\frac{R_{in2}}{R_{out1}+R_{in2}}\right)^2\frac{4R_s}{R_{in2}}}$$

$$\boxed{G_{av}=G_T\frac{(R_{out1}+R_{in2})^2}{4R_{out1}R_{in2}}}$$

Apparently, if $R_{out1}=R_{in2}$, then, $G_{av}=G_T$.

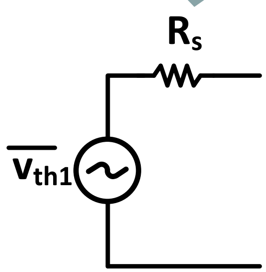
Cascaded Noise Factor (General Case)

- Let's consider noisy cascaded 2-port network.



$\overline{v_{Rs}^2} = 4kTR_s\Delta f$

Thevenin equivalence

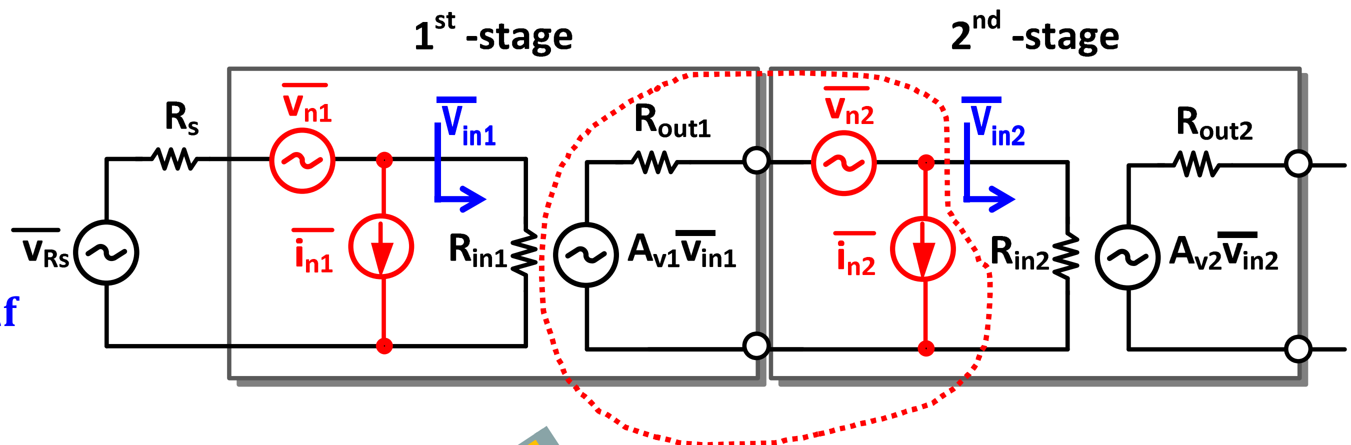


$$\overline{v_{th1}} = \overline{v_{Rs}} + \overline{v_{n1}} + \overline{i_{n1}}R_s$$

$$\therefore \ \overline{v_{in1}} = \frac{R_{in1}}{R_s + R_{in1}}\overline{v_{th1}} = \frac{R_{in1}}{R_s + R_{in1}}(\overline{v_{Rs}} + \overline{v_{n1}} + \overline{i_{n1}}R_s)$$

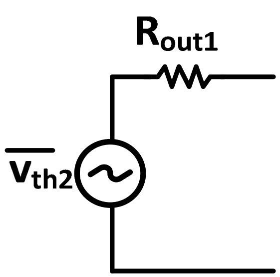
Cascaded Noise Factor (General Case)

- Now let's calculate $\overline{V_{in2}}$.



$$\overline{v_{Rs}}^2 = 4kTR_s\Delta f$$

Thevenin equivalence



$$\overline{v_{th2}} = A_{v1}\overline{v_{in1}} + \overline{v_{n2}} + \overline{i_{n2}}R_{out}$$

$$\therefore\ \overline{v_{in2}} = \frac{R_{in2}}{R_{out1} + R_{in2}}\overline{v_{th2}} = \frac{R_{in2}}{R_{out1} + R_{in2}}\left(A_{v1}\overline{v_{in1}} + \overline{v_{n2}} + \overline{i_{n2}}R_{out1}\right)$$

Cascaded Noise Factor (General Case)

- From previous results,

Noise voltage : $\overline{v_{in2}} = \frac{R_{in2}}{R_{out1}+R_{in2}}\overline{v_{th2}} = \frac{R_{in2}}{R_{out1}+R_{in2}}\left(A_{v1}\overline{v_{in1}} + \overline{v_{n2}} + \overline{i_{n2}}R_{out1}\right)$

$$= \frac{R_{in2}}{R_{out1}+R_{in2}}\left\{A_{v1}\left(\frac{R_{in1}}{R_s+R_{in1}}\left(\overline{v_{Rs}} + \overline{v_{n1}} + \overline{i_{n1}}R_s\right)\right) + \overline{v_{n2}} + \overline{i_{n2}}R_{out1}\right\}$$

Noise power :

$$\overline{v_{in2}{}^2} = \left(\frac{R_{in2}}{R_{out1}+R_{in2}}\right)^2\left\{A_{v1}{}^2\left(\left(\frac{R_{in1}}{R_s+R_{in1}}\right)^2\left(\overline{v_{Rs}{}^2} + \overline{(v_{n1} + i_{n1}R_s)^2}\right)\right) + \overline{(v_{n2} + i_{n2}R_{out1})^2}\right\}$$

Cascaded Noise Factor (F) : $F = \frac{\overline{v_{in2}{}^2}\textit{,due to all noise sources}}{\overline{v_{in2}{}^2}\textit{,due to source noise only}}$

$$= \frac{\left(\frac{R_{in2}}{R_{out1}+R_{in2}}\right)^2\left\{A_{v1}{}^2\left(\left(\frac{R_{in1}}{R_s+R_{in1}}\right)^2\left(\overline{v_{Rs}{}^2} + \overline{(v_{n1} + i_{n1}R_s)^2}\right)\right) + \overline{(v_{n2} + i_{n2}R_{out1})^2}\right\}}{\left(\frac{R_{in2}}{R_{out1}+R_{in2}}\right)^2 A_{v1}{}^2\left(\frac{R_{in1}}{R_s+R_{in1}}\right)^2\overline{v_{Rs}{}^2}}$$

Cascaded Noise Factor (General Case)

$$F=\frac{\left(\frac{R_{in2}}{R_{out1}+R_{in2}}\right)^2\left\{A_{v1}^2\left(\left(\frac{R_{in1}}{R_s+R_{in1}}\right)^2\left(\overline{v_{Rs}^2}+\overline{(v_{n1}+i_{n1}R_s)^2}\right)\right)+\overline{(v_{n2}+i_{n2}R_{out1})^2}\right\}}{\left(\frac{R_{in2}}{R_{out1}+R_{in2}}\right)^2 A_{v1}^2\left(\frac{R_{in1}}{R_s+R_{in1}}\right)^2\overline{v_{Rs}^2}}$$

1st-stage F

$$=\frac{\overline{v_{Rs}^2}+\overline{(v_{n1}+i_{n1}R_s)^2}}{\overline{v_{Rs}^2}}+\frac{\overline{(v_{n2}+i_{n2}R_s)^2}}{A_{v1}^2\left(\frac{R_{in1}}{R_s+R_{in1}}\right)^2\overline{v_{Rs}^2}}=\boxed{F_1+\frac{\overline{(v_{n2}+i_{n2}R_s)^2}}{A_{v1}^2\left(\frac{R_{in1}}{R_s+R_{in1}}\right)^2\overline{v_{Rs}^2}}}$$

Change this in terms of available power gain of 1-st stage, G_{av1}.

$$=F_1+\frac{\overline{(v_{n2}+i_{n2}R_s)^2}}{G_{av1}\frac{R_{out}}{R_s}\overline{v_{Rs}^2}}$$

apply, $\overline{v_{Rs}^2}=4kTR_s\Delta f$

$$=F_1+\frac{\frac{\overline{(v_{n2}+i_{n2}R_s)^2}}{4kTR_{out}\Delta f}}{G_{av1}}$$

$$=\boxed{F_1+\frac{F_2-1}{G_{av1}}}$$

Note that 2nd –stage noise is divided by available power gain of the 1-st stage.

Cascaded Noise Factor (General Case)

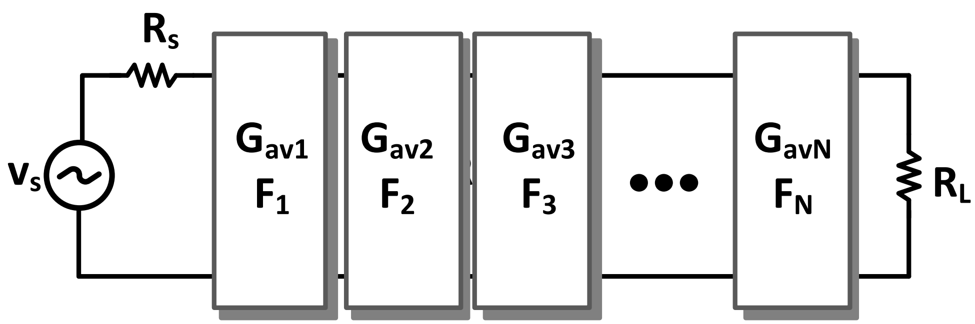
- Another form of noise factor of cascaded system

$$F = F_1 + \frac{F_2 - 1}{G_{av1}}$$

Change this in terms of transducer power gain of first stage.

$$= F_1 + \frac{F_2 - 1}{G_{T1}} \left\{ \frac{4R_{out1}R_{in2}}{(R_{out1} + R_{in2})^2} \right\}$$

- Using the 2-stage cascaded noise factor expression, you can verify this Frii's Formula.



$$F_{overall} = F_1 + \frac{F_2 - 1}{G_{av1}} + \frac{F_3 - 1}{G_{av1}G_{av2}} + \cdots + \frac{F_N - 1}{G_{av1}G_{av2}\cdots G_{av(N-1)}}$$

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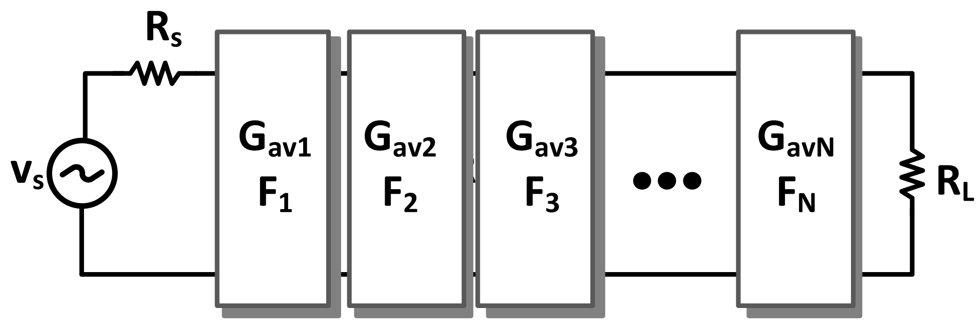
Some Noise Considerations in Cascaded System



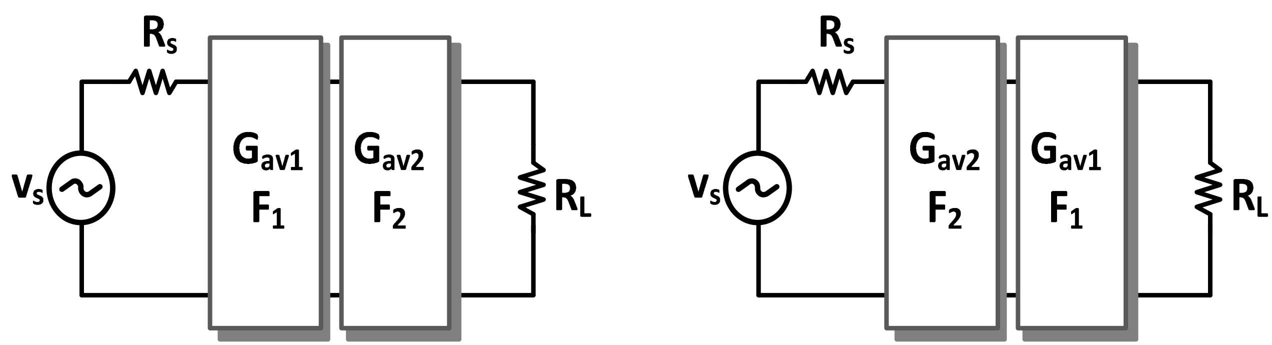
$$F_{overall} = F_1 + \frac{F_2 - 1}{G_{av1}} + \frac{F_3 - 1}{G_{av1} G_{av2}} + \cdots + \frac{F_N - 1}{G_{av1} G_{av2} \cdots G_{av(N-1)}}$$

- Each stage in a cascaded system adds noise to the system. As a consequence of this SNR gets worse toward the end of the cascaded chain. And therefore, if SNR drops below acceptable range in intermediate stage, you should stop the design, and rethink the design.

- In typical RF systems, the first few stages set overall noise level of the system. This is particularly the reason why the noise performance of LNA and mixer is very crucial in determining the NF of the system.

Cascaded Systems and Noise Margin

- In cascading two systems, the choice of optimum arrangement for low-noise is simple.

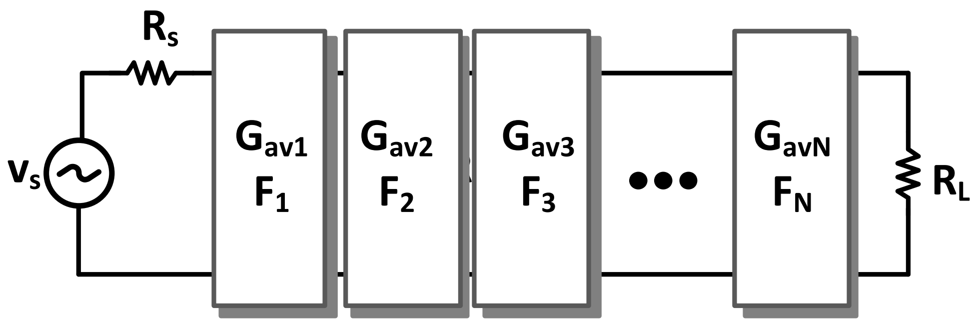


$$F_{overall} = F_1 + \frac{F_2 - 1}{G_{av1}} \quad \Leftrightarrow \quad F_{overall} = F_2 + \frac{F_1 - 1}{G_{av2}}$$

Compare and pick the arrangement giving less noise factor.

Cascaded Systems and Noise Margin

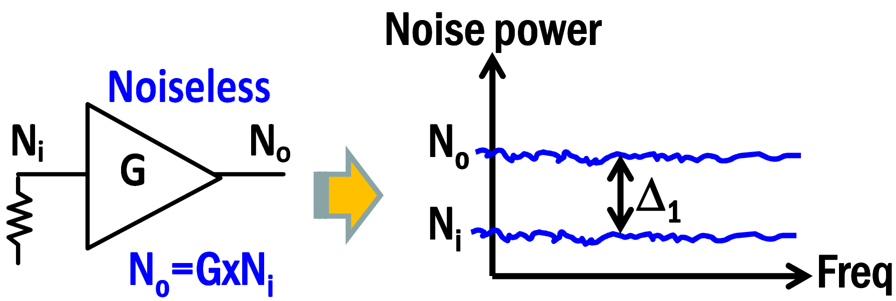
- But when cascading many blocks, we need to consider many different combination of the arrangement of each block, and the choice is time consuming work.



- A generalized technique to resolve this issue is to consider "**noise margin**" of each building block and **place the block having lower noise margin first** in cascading the systems.

Cascaded Systems and Noise Margin

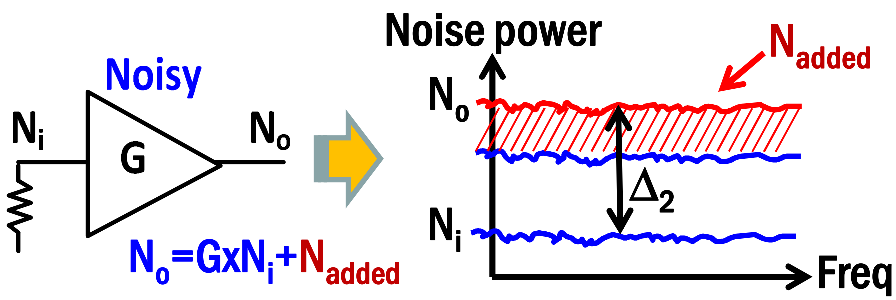
- What is noise margin (NM) ?



$$NM = \frac{\Delta_2}{\Delta_1} = \frac{GN_i - N_i + N_{added}}{GN_i - N_i}$$

$$= 1 + \frac{N_{added}}{GN_i - N_i} = 1 + \frac{N_{added}}{GN_i\left(1 - \frac{1}{G}\right)}$$

$$\boxed{= 1 + \frac{F - 1}{\left(1 - \frac{1}{G}\right)}} \approx F, \text{if } G \gg 1.$$

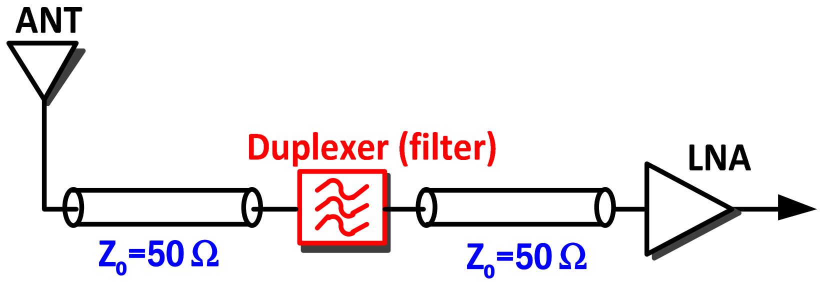


Once we know F & G of each block, we can figure out NM of the block.

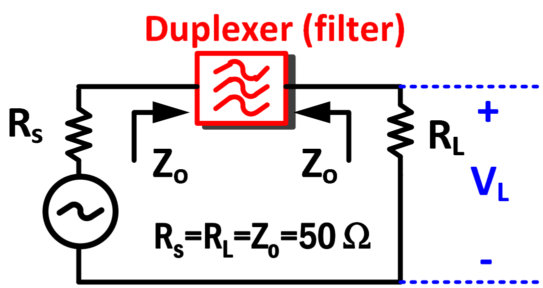
To achieve minimum F, place the block with lower NM first (verify this claim !).

Loss of Filters and T-lines

- In typical RF systems, a filter will be placed before LNA to filter wanted signal band, and it typically has a loss, L (< 2-3 dB).



- How to define loss L in passive filter ?



$$L=\frac{P_{in}}{P_{out}}=\frac{\frac{V_S^2}{4Z_O}}{\frac{V_L^2}{R_L}}=\boxed{\left(\frac{\frac{1}{2}V_S}{V_L}\right)^2}$$

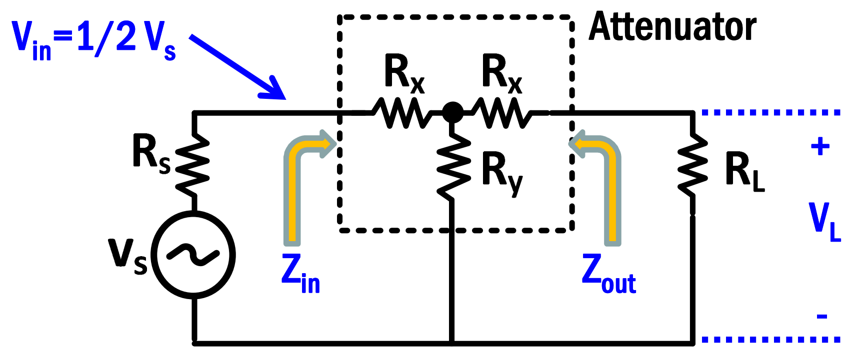
This is V_{in} of the duplexer.

Note, $G=\frac{P_{out}}{P_{in}}=\frac{1}{L}$

- Loss of transmission lines can be defined using the same way.

Loss of Attenuator

- Attenuator is a resistor network whose input and output impedance are matched to Z_o (=50 Ω), and signal power is attenuated by a specific number.



$R_s=R_L=Z_o=50\ \Omega$

$$Z_{in}=Z_{out}=Z_o=R_x+\left(R_y \parallel (R_x+R_L)\right)=\boxed{R_x\sqrt{1+2\frac{R_y}{R_x}}}$$

$$V_L=\frac{1}{2}V_s\frac{Z_o-R_x}{Z_o}\frac{R_L}{R_L+R_x}=\frac{1}{2}V_s\frac{Z_o-R_x}{Z_o}\frac{Z_o}{Z_o+R_x}=\frac{1}{2}V_s\frac{Z_o-R_x}{Z_o+R_x}$$

$$\therefore\ L=\frac{P_{in}}{P_{out}}=\left(\frac{\frac{1}{2}V_s}{V_L}\right)^2=\boxed{\left(\frac{Z_o+R_x}{Z_o-R_x}\right)^2}$$

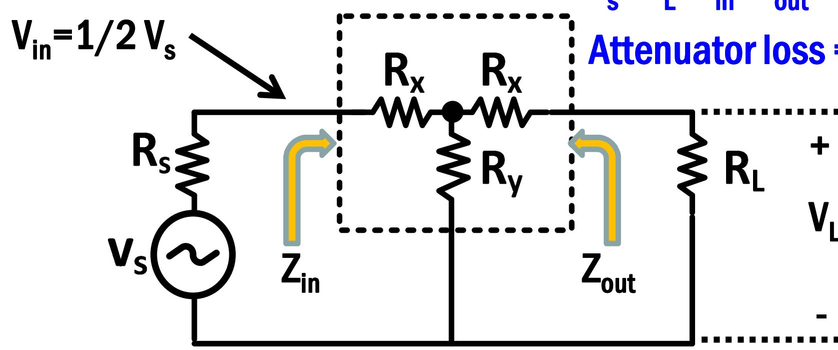
Ex) R_x=8.55 Ω, R_y=141.9 Ω, will give Z_{in}=Z_{out}=50 Ω, and L=3 dB.

Noise Factor of Attenuator

- Let's think about noise factor of attenuator.

R_s=R_L=Z_{in}=Z_{out}=Z_o=50 Ω

Attenuator loss = P_{in}/P_{out}=L



$$P_{in} = \left(\frac{1}{2}V_s\right)^2 \frac{1}{Z_o}$$

$$P_{out} = \frac{1}{L}\left(\frac{1}{2}V_s\right)^2 \frac{1}{Z_o}$$

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{in}/N_{in}}{P_{out}/N_{out}} = \boxed{L}$$

Loss=Noise Factor

No change !

$$N_{in} = \left(\frac{1}{4}\overline{V_{n,Rs}}^2\right)\frac{1}{Z_{in}} = kT\Delta f,\ \text{where}\ \overline{V_{n,Rs}}^2 = 4kTR_s\Delta f.$$

$$N_{out} = \left(\frac{1}{4}\overline{V_{n,Zout}}^2\right)\frac{1}{R_L} = kT\Delta f,\ \text{where}\ \overline{V_{n,Zout}}^2 = 4kTZ_{out}\Delta f.$$

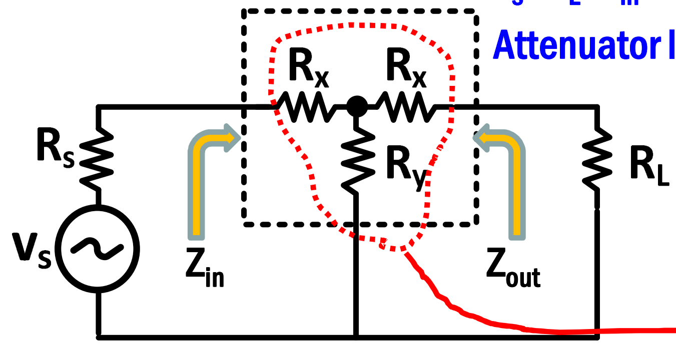
Q: How should we interpret this ?

Noise Factor of Attenuator

- Noise behaves just like signal and the input noise due to R_s will be attenuated by the attenuator.

$R_s = R_L = Z_{in} = Z_{out} = Z_o = 50\ \Omega$

Attenuator loss = $P_{in}/P_{out} = L$



Noise behaves just like signal.

$$N_{in} = \left(\frac{1}{4}\overline{V_{n,Rs}^{\,2}}\right)\frac{1}{Z_{in}} = kT\Delta f, \text{ where } \overline{V_{n,Rs}^{\,2}} = 4kTR_s\Delta f.$$

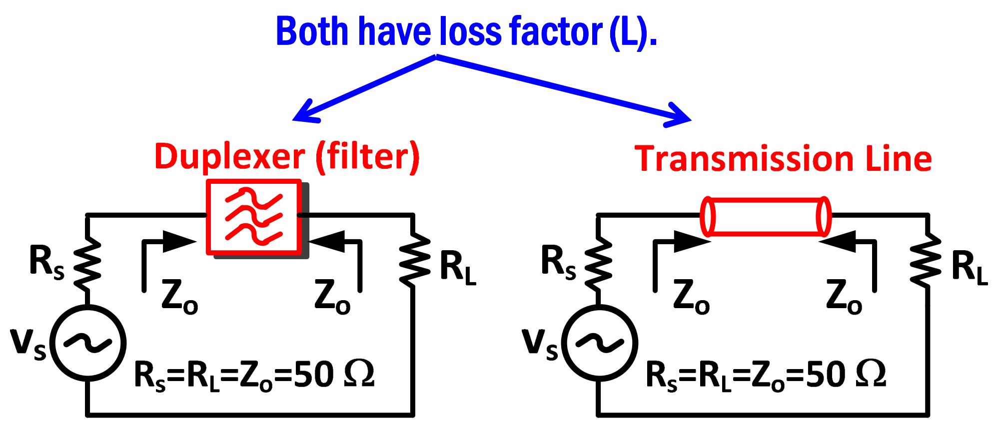
$$N_{out,\text{due to Rs}} = \frac{N_{in}}{L} = \frac{1}{L}kT\Delta f$$

- But, the resistors in the attenuator will generate the exactly same amount of noise power lost in the attenuator. In overall there is no noise power loss in the network (Proof of this is HW).

$$N_{out,\text{due to Attenuator}} = \left(1 - \frac{1}{L}\right)kT\Delta f$$

Noise Factor of Passives

- In general under matched condition, duplexer and T-line having a loss can be modeled as attenuator within passband frequency range.

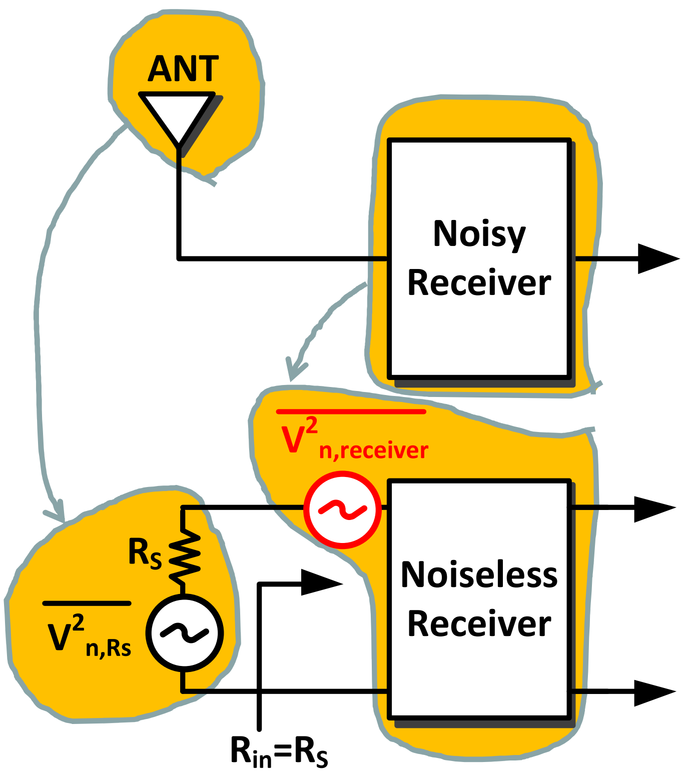


- And we can apply the same theory developed in previous slides, and confirm that the loss factor L is the noise figure of the passive devices.

- Generally speaking, under matched condition noise power delivered to a load from a pure passive device will be $KT\Delta f$.

Minimum Noise Floor of Receiver

- The minimum noise floor of a receiver is total noise power which is noise power from antenna plus equivalent input noise power of a receiver.



$$\overline{V_{n,Rs}}^2 = 4kTR_s\Delta f$$

$$\text{Minimum Noise Floor} = \frac{\overline{V_{n,Rs}}^2}{4R_s} + \frac{\overline{V_{n,receiver}}^2}{4R_s}$$

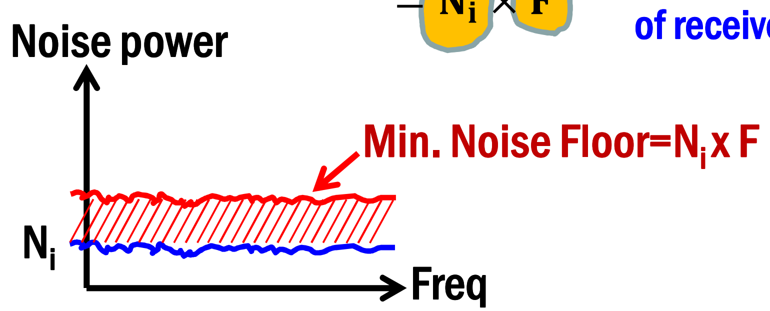
$$= \frac{\overline{V_{n,Rs}}^2}{4R_s}\left(1 + \frac{\overline{V_{n,receiver}}^2}{\overline{V_{n,Rs}}^2}\right)$$

$$= kT\Delta f\left(1 + \frac{\overline{V_{n,receiver}}^2}{\overline{V_{n,Rs}}^2}\right)$$

$$= N_i \times F$$

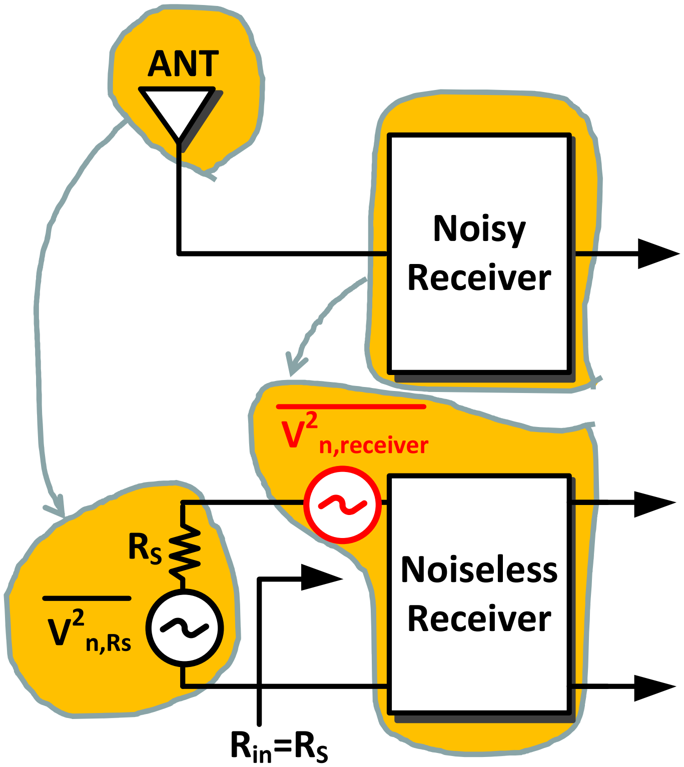
Noise power from antenna

Noise factor of receiver



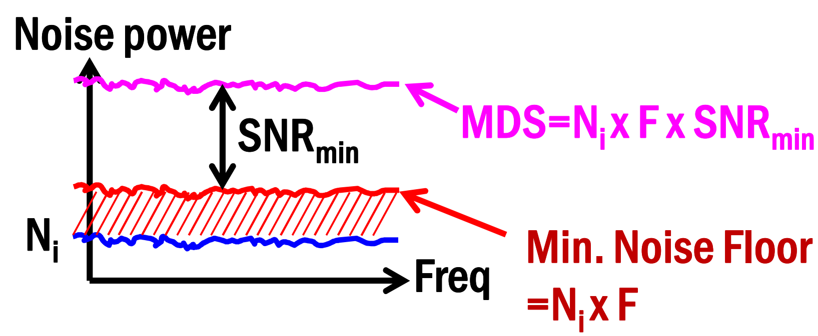
Minimum Detectable Signal (MDS)

- In typical wireless comm., we need certain level of signal to noise ratio for an acceptable bit-error-rate required by a system. We call this SNR_{min} (typ. 10-20 dB).



$$\overline{V_{n,Rs}}^2 = 4kTR_s\Delta f$$

- This means the minimum detectable signal (MDS) for a given system is $MDS= N_i \times F \times SNR_{min}$.



Ex) Δf=2 MHz, T=300 K, F=2 (3dB), SNR_{min}=10 dB
→N_i=KTΔf x F= -174 dBm+10log(2MHz) + 3 dB=-108 dBm
→MDS=N_i x SNR_{min}=-108 dB + 10 dB = -98 dBm