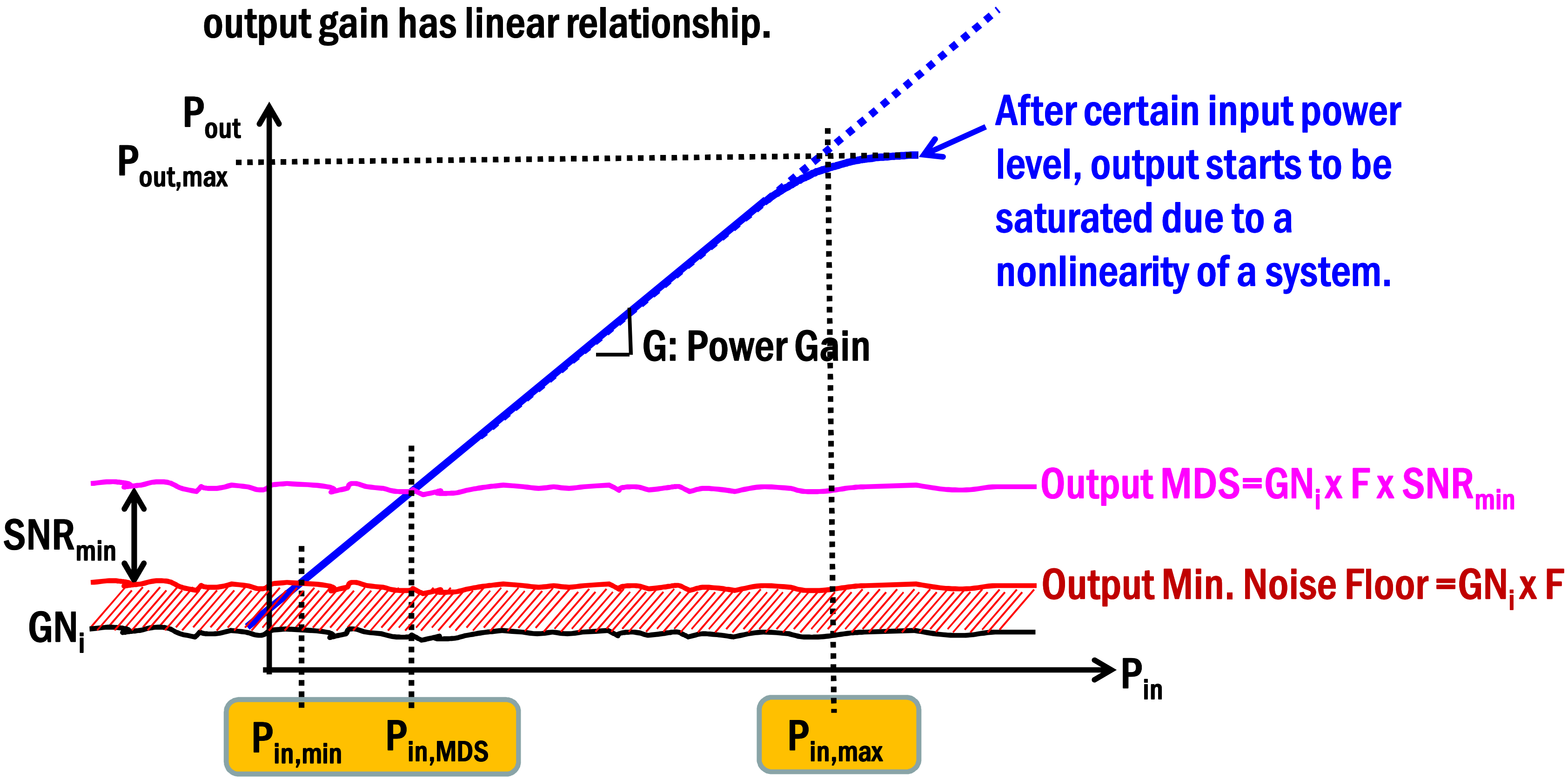
ECE 5220 RFIC Technology & Design (Linearity)

Linearity (introduction)

- Most RF and analog electronics have limited input range where input and output gain has linear relationship.



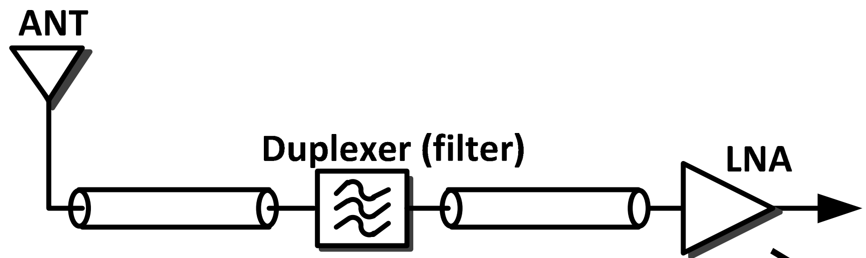
Determined by noise of a system.

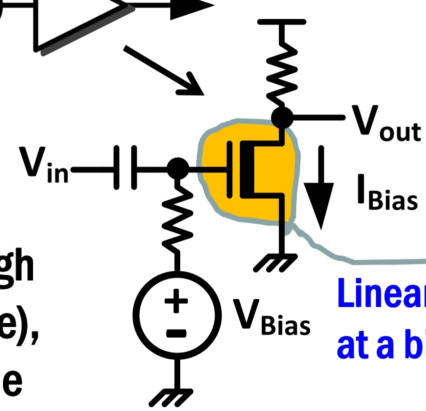
Determined by a nonlinearity of a system.

$P_{in,min}$=kTΔf x F, $P_{in,MDS}$=kTΔf x F x SNR_{min}

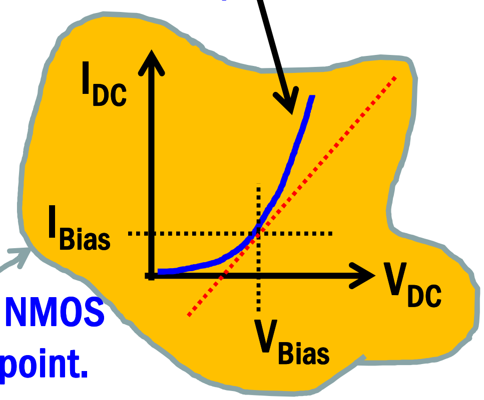
Nonlinearity Source (from DC point of view)

- Where does the nonlinearity come from? Let's think about LNA design.





NMOS DC characteristic is non-linear (typically square-law behavior with long channel NMOS).



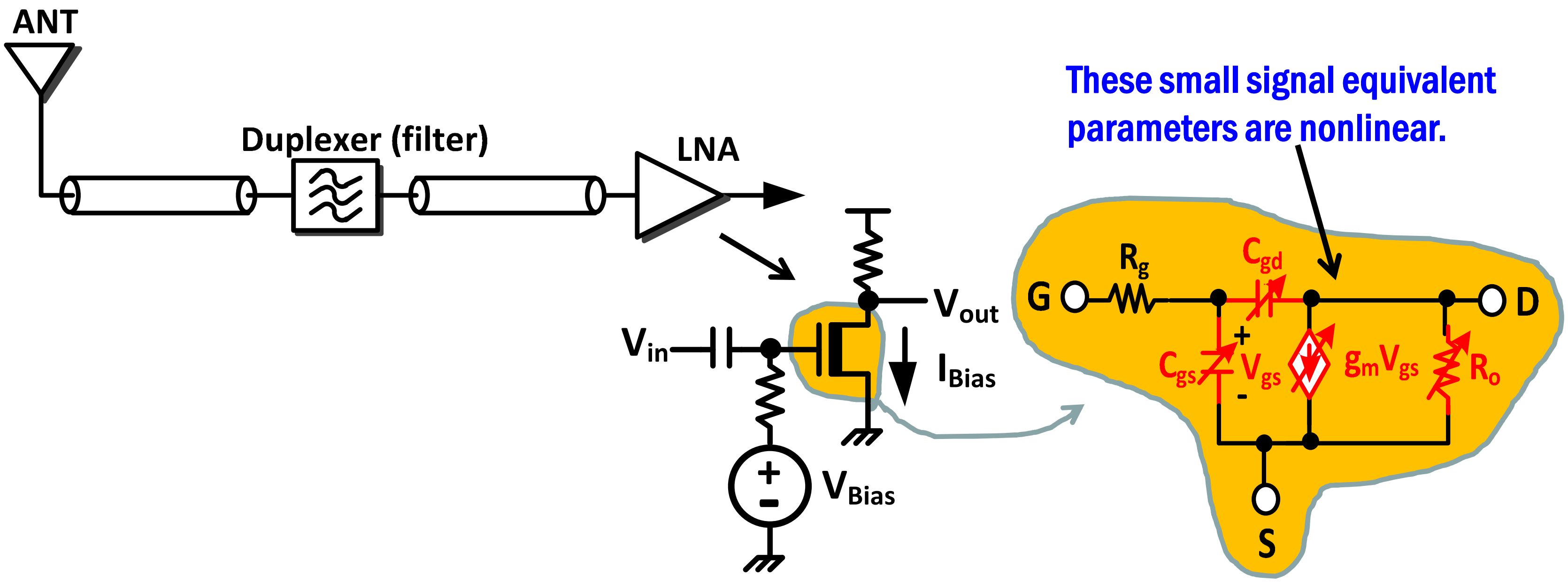
Linearize NMOS at a bias point.

$$\left.\frac{\partial I_{DC}}{\partial V_{DC}}\right|_{@V_{DC}=V_{Bias}} = g_m$$

- If applied V_{in} is small enough (for instance, V_{in} ~µV range), then the linearization will be valid.
- However, if the V_{in} is getting large (> ~ mV range), then the linearization no longer holds, and the LNA shows nonlinear behavior.

Nonlinearity Source (from AC point of view)

- Where does the nonlinearity come from? Let's think about LNA design.

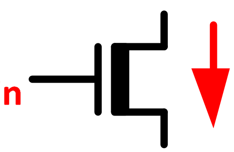


- The small signal equivalent parameters are not static, but depending on input and output signals, which causes a nonlinearity of the system.

Weak & Strong Nonlinearities

- ❑ **Weak nonlinearity**
  - If input and output signal levels are reasonable small (< ~-20 dBm, ~10's mV), the degree of nonlinearity will not be strong.
  - This is the most cases of RF and analog front-end receiver blocks; LNAs, mixers baseband amplifiers and filters.
  - Still, we can characterize the circuits approximately, using small signal model with modeling output current in terms of power series of input V_{in}.

V_{in}



$$I_{out}=a_1V_{in}+a_2V_{in}+a_3V_{in}+\ldots.$$

- ❑ **Strong nonlinearity**
  - If input and output signal levels are large (> 0 dBm, ~100's mV), the degree of nonlinearity will be quite strong.
  - This is the most cases of power amplifiers.
  - We need large signal model and large signal S-parameter to characterize the nonlinearity (we will be back to this issue in power amplifier section).

Gain Compression (Self-Jamming)

- Let's assume a weak nonlinear system. All weak nonlinear function can be described by a power series.

$V_{in} \rightarrow$ Nonlinear system $\rightarrow V_{out}$

$$V_{out} = a_1 V_{in} + a_2 {V_{in}}^2 + a_3 {V_{in}}^3 + \text{higher orders}$$

$$\approx a_1 V_{in} + a_2 {V_{in}}^2 + a_3 {V_{in}}^3$$

Apply $V_{in} = V_p \cos\omega_o t$

$$a_1 V_p \cos\omega_o t$$

fundamental

$$a_1 {V_p}^2 \cos^2\omega_o t$$

$$\frac{1}{2} a_2 {V_p}^2 (\cos 2\omega_o t + 1)$$

2nd –harmonic + DC

$$a_3 {V_p}^3 \cos^3\omega_o t$$

$$\frac{1}{4} a_3 {V_p}^3 (3\cos\omega_o t + \cos 3\omega_o t)$$

fundamental+ 3rd -harmonic

Gain Compression (Self-Jamming)

- Let's assume a weak nonlinear system. All weak nonlinear function can be described by a power series.

$$V_{out} = a_1 V_{in} + a_2 {V_{in}}^2 + a_3 {V_{in}}^3 + \text{higher orders}$$
$$\approx a_1 V_{in} + a_2 {V_{in}}^2 + a_3 {V_{in}}^3$$

Apply $V_{in} = V_p \cos\omega_o t$

$a_1 V_p \cos\omega_o t$
fundamental

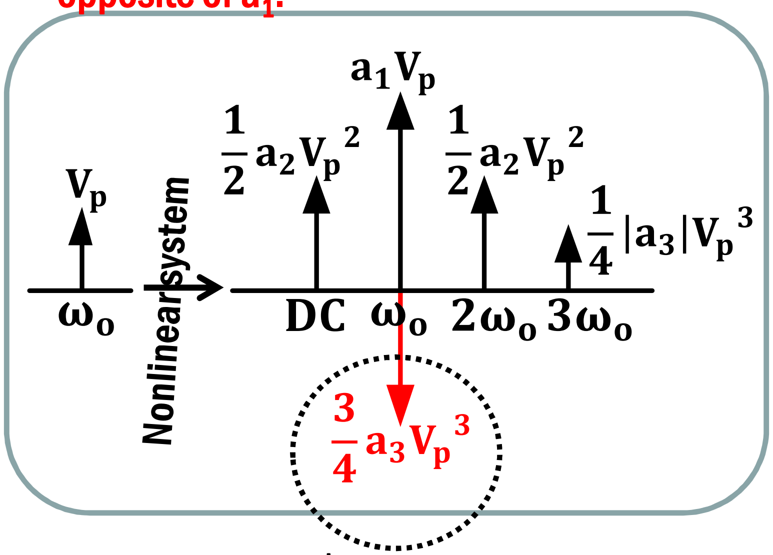
$a_1 {V_p}^2 \cos^2\omega_o t$

$$\frac{1}{2} a_2 {V_p}^2 (\cos 2\omega_o t + 1)$$
2^{nd} –harmonic + DC

$a_3 {V_p}^3 \cos^3\omega_o t$

$$\frac{1}{4} a_3 {V_p}^3 (3\cos\omega_o t + \cos 3\omega_o t)$$
fundamental+ 3^{rd} -harmonic

For most systems, polarity of a_3 is the opposite of a_1.

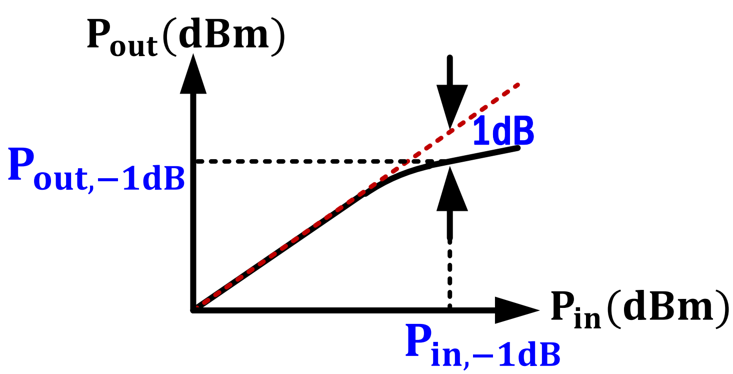


Because of this 3^{rd}-harmonic term, gain will be decreased from ideal value.
("gain desensitization" or "self-jamming effect")

Input & Output P1dB Compression Point

- We need a parameter to describe the self-jamming effect.

$$\mathrm{fundamental} = a_1 V_p \underbrace{\left(1 + \frac{3}{4}\frac{a_3}{a_1} V_p^{\ 2}\right)}_{\text{Self-jamming factor}} \cos\omega_o t$$



$$20\log\left(\frac{a_1 V_p\left(1 + \frac{3}{4}\frac{a_3}{a_1} V_p^{\ 2}\right)}{a_1 V_p}\right) = -1$$

$$\rightarrow 1 + \frac{3}{4}\frac{a_3}{a_1} V_p^{\ 2} = 0.89$$

$$\rightarrow \boxed{V_{p,-1dB} = 0.38\sqrt{\left|\frac{a_1}{a_3}\right|}}$$

- The input 1dB compression point $P_{in,-1dB}$ is the input power at which output power drops by 1dB from the predicted output power based on the small signal power gain.
- The output 1dB compression point $P_{out,-1dB}$ is the output power at which output power drops by 1dB from the predicted output power based on the small signal power gain.

$$\boxed{P_{out,-1dB}(\mathrm{dBm}) = G_P(\mathrm{dB}) + P_{in,-1dB}(\mathrm{dBm}) - 1\ \mathrm{dB}}$$

Gain Compression (Adjacent Ch. Jamming)

- When a large interference signal is received along with the desired weak signal, the output of the desired signal gets compressed similar to the case of self-jamming. This behavior is referred to as "adjacent channel jamming".

$V_{in} \rightarrow$ Nonlinear system $\rightarrow V_{out}$

$$V_{out} = a_1 V_{in} + a_2 {V_{in}}^2 + a_3 {V_{in}}^3 + \text{higher orders}$$
$$\approx a_1 V_{in} + a_2 {V_{in}}^2 + a_3 {V_{in}}^3$$

$$V_{in} = V_{small} \cos\omega_s t + V_{large} \cos\omega_{adj} t$$

$$a_3 {V_{in}}^3 = a_3 (V_{small} \cos\omega_s t)^3 + a_3 (V_{large} \cos\omega_{adj} t)^3$$
$$+ 3a_3 (V_{small} \cos\omega_s t)^2 V_{large} \cos\omega_{adj} t + 3a_3 (V_{large} \cos\omega_{adj} t)^2 V_{small} \cos\omega_s t$$

$$3a_3 {V_{large}}^2 V_{small} \left(\frac{1 + \cos 2\omega_{adj} t}{2} \right) \cos\omega_s t$$

$$\boxed{\frac{3}{2} a_3 {V_{large}}^2 V_{small} \cos\omega_s t}$$

Fundamental component due to adjacent channel interference

Gain Compression (Adjacent Ch. Jamming)

- Jamming threshold: signal level of ω_{adj} where the gain of wanted signal gets reduced by 3dB.

$V_{in} \rightarrow$ **Nonlinear system** $\rightarrow V_{out}$

$$V_{out} = a_1 V_{in} + a_2 {V_{in}}^2 + a_3 {V_{in}}^3 + \text{higher orders}$$
$$\approx a_1 V_{in} + a_2 {V_{in}}^2 + a_3 {V_{in}}^3$$

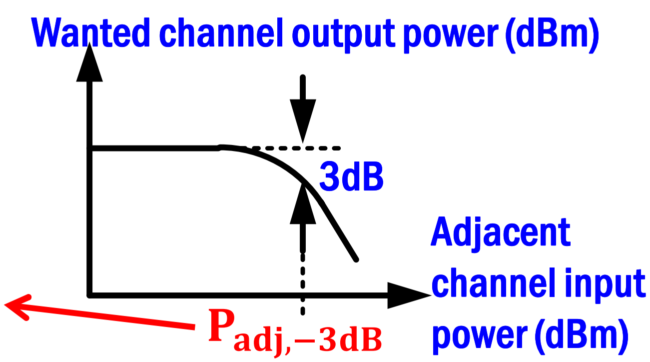
$$V_{in} = V_{small} \cos\omega_s t + V_{large} \cos\omega_{adj} t$$

$$V_{out,interested} = a_1 V_{small} \cos\omega_s t + \frac{3}{2} a_3 {V_{large}}^2 V_{small} \cos\omega_s t$$

$$20\log\left(\frac{a_1 V_{small} + \frac{3}{2} a_3 {V_{large}}^2 V_{small}}{a_1 V_{small}}\right) = 20\log\left(1 + \frac{3}{2}\frac{a_3}{a_1}{V_{large}}^2\right) = -3$$

$$\rightarrow 1 + \frac{3}{2}\frac{a_3}{a_1}{V_{large}}^2 = 0.71$$

$$\rightarrow \boxed{V_{large,-3dB} = 0.44\sqrt{\left|\frac{a_1}{a_3}\right|}}$$



Jamming threshold input power of adjacent channel

Cross Modulation

- Cross modulation is a form of intermodulation, where the amplitude modulation of a strong signal modulates the amplitude of a weak signal at another frequency.

$V_{in} \rightarrow$ **Nonlinear system** $\rightarrow V_{out}$

$$V_{out} = a_1 V_{in} + a_2 {V_{in}}^2 + a_3 {V_{in}}^3 + \text{higher orders}$$
$$\approx a_1 V_{in} + a_2 {V_{in}}^2 + a_3 {V_{in}}^3$$

$$V_{in} = V_{small}\cos\omega_s t + V_{large}(1 + M_{index}\cos\omega_m t)\cos\omega_c t$$

$$V_{out,interested} = a_1 V_{small}\cos\omega_s t + \frac{3}{2} a_3 {V_{large}}^2 (1 + M_{index}\cos\omega_m t)^2 V_{small}\cos\omega_s t$$

This cross modulated signal is superimposed on top of wanted signal due to 3rd -order nonlinear term of a_3.

- This is commonly experienced by listener s of AM radio as they drive past a large powerful transmitting station while listening to another AM station. The amplitude modulation of the large transmitting station gets superimposed on top of the station the listener is tuned to.

Fractional 2^{nd}–order Harmonic Distortion (HD_2)

- HD_2 is defined as the amplitude ratio of 2^{nd} –order harmonic to fundamental component.

$V_{in} \rightarrow$ Nonlinear system $\rightarrow V_{out}$

$$V_{out} = a_1 V_{in} + a_2 {V_{in}}^2 + a_3 {V_{in}}^3 + \text{higher orders}$$

$$\approx a_1 V_{in} + a_2 {V_{in}}^2 + a_3 {V_{in}}^3$$

Apply

$$V_{in} = V_p \cos\omega_o t$$

$$a_1 V_p \cos\omega_o t$$

fundamental

$$a_1 {V_p}^2 \cos^2\omega_o t$$

$$\frac{1}{2} a_2 {V_p}^2 (\cos 2\omega_o t + 1)$$

2^{nd} –harmonic + DC

$$a_3 {V_p}^3 \cos^3\omega_o t$$

$$\frac{1}{4} a_3 {V_p}^3 (3\cos\omega_o t + \cos 3\omega_o t)$$

fundamental+ 3^{rd} -harmonic

$$HD_2 = \frac{\textit{Amplitude of the 2nd} - \textit{order harmonic output}}{\textit{Amplitude of the fundamental output}}$$

$$= \frac{\frac{1}{2} a_2 {V_p}^2}{a_1 V_p} = \boxed{\frac{a_2 V_p}{2 a_1}}$$

Fractional 3rd–order Harmonic Distortion (HD_3)

- HD_3 is defined as the amplitude ratio of 3rd –order harmonic to fundamental component.

V_{in} → Nonlinear system → V_{out}

$$V_{out} = a_1 V_{in} + a_2 V_{in}^{\ 2} + a_3 V_{in}^{\ 3} + \text{higher orders}$$
$$\approx a_1 V_{in} + a_2 V_{in}^{\ 2} + a_3 V_{in}^{\ 3}$$

Apply

$$V_{in} = V_p \cos\omega_o t$$

$$a_1 V_p \cos\omega_o t$$
fundamental

$$a_1 V_p^{\ 2} \cos^2\omega_o t$$

$$\frac{1}{2} a_2 V_p^{\ 2} (\cos 2\omega_o t + 1)$$
2nd –harmonic + DC

$$a_3 V_p^{\ 3} \cos^3\omega_o t$$

$$\frac{1}{4} a_3 V_p^{\ 3} (3\cos\omega_o t + \cos 3\omega_o t)$$
fundamental+ 3rd -harmonic

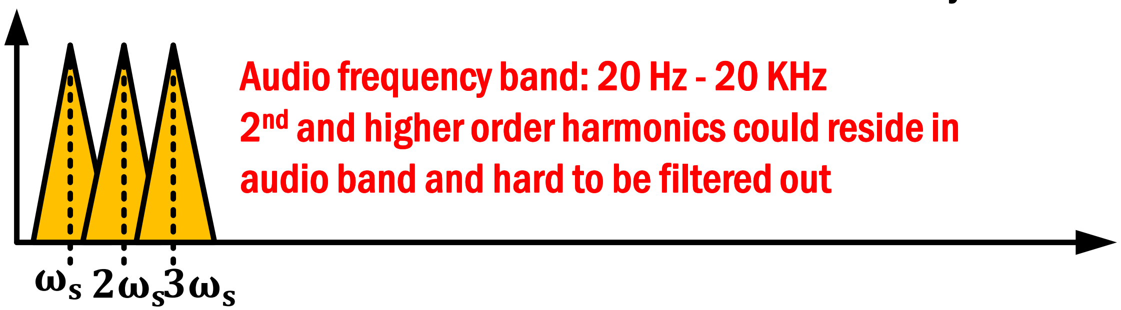
$$HD_3 = \frac{\textit{Amplitude of the 3rd} - \textit{order harmonic output}}{\textit{Amplitude of the fundamental output}}$$
$$= \frac{\frac{1}{4} a_3 V_p^{\ 3}}{a_1 V_p} = \boxed{\frac{a_3 V_p^{\ 2}}{4 a_1}}$$

Total Harmonic Distortion (THD)

- Total Harmonic Distortion (THD) is defined as the sum of all the fractional harmonic distortion terms, i.e.,

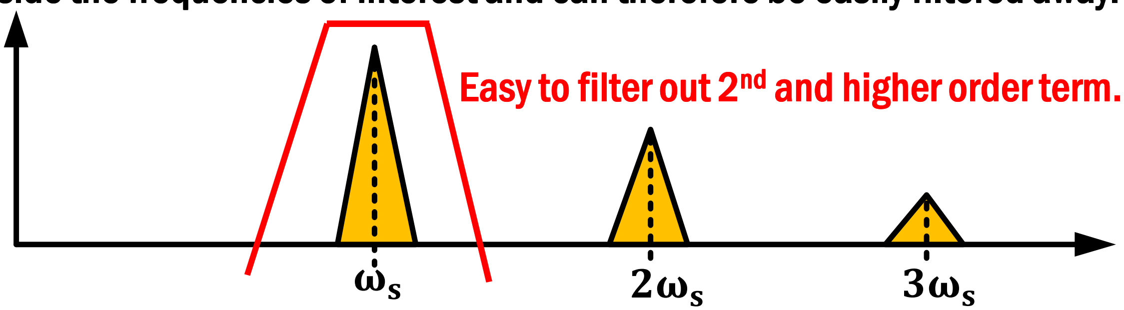
$$\mathrm{THD} = \sum_{n=2}^{\infty} \mathrm{HD_n}$$

- THD is often quoted by audio manufacturers who “spec” their amplifiers for spectral purity. The THD values less than 0.1%=-60 dB are hard to discern by the human ear.



Audio frequency band: 20 Hz - 20 KHz
2nd and higher order harmonics could reside in audio band and hard to be filtered out

- In most communication systems, a very narrow bandwidth signal is modulated by very high frequency carrier. In such systems, the second and higher order harmonics lie outside the frequencies of interest and can therefore be easily filtered away.



Intermodulation Distortion (IMD)

- Until now we injected single frequency tone into a nonlinear system. But in reality, several other signals closely spaced to the desired signal are received and processed by the front end RF systems; LNAs and mixers.
- Typical way of characterizing nonlinearity of RF system is two-tone test; apply two closely spaced frequency tones which have same amplitude into the nonlinear system.

$$V_{in} \rightarrow \boxed{\text{Nonlinear system}} \rightarrow V_{out}$$

$$V_{out} = a_1 V_{in} + a_2 V_{in}{}^2 + a_3 V_{in}{}^3 + \text{higher orders}$$
$$\approx a_1 V_{in} + a_2 V_{in}{}^2 + a_3 V_{in}{}^3$$

$$V_{in} = V_p \cos\omega_{s1} t + V_p \cos\omega_{s2} t$$

A: $$a_1 V_p \cos\omega_{s1} t + a_1 V_p \cos\omega_{s2} t$$

B: $$a_2 V_p{}^2 \cos^2\omega_{s1} t + a_2 V_p{}^2 \cos^2\omega_{s2} t + 2a_2 V_p{}^2 \cos\omega_{s1} t \cos\omega_{s2} t$$

C: $$a_3 V_p{}^3 \cos^3\omega_{s1} t + a_3 V_p{}^3 \cos^3\omega_{s2} t$$
$$+3a_3 V_p{}^3 \cos^2\omega_{s1} t \cos\omega_{s2} t + 3a_3 V_p{}^3 \cos^2\omega_{s2} t \cos\omega_{s1} t$$

Intermodulation Distortion (2-tone test)

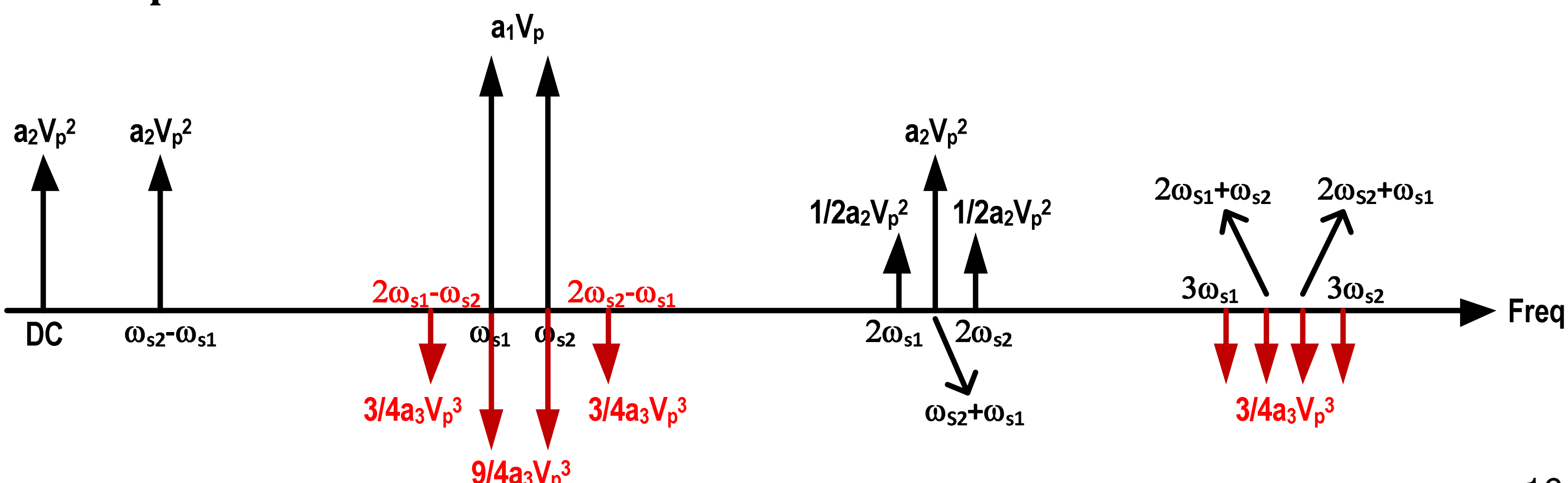
A : $a_1 V_p \cos\omega_{s1}t + a_1 V_p \cos\omega_{s2}t$

B : $a_2 V_p^2 + \frac{a_2 V_p^2}{2}(\cos 2\omega_{s1}t + \cos 2\omega_{s2}t) + a_2 V_p^2(\cos(\omega_{s1}+\omega_{s2})t + \cos(\omega_{s1}-\omega_{s2})t)$

C : $\frac{9a_3 V_p^3}{4}(\cos\omega_{s1}t + \cos\omega_{s2}t) + \frac{a_3 V_p^3}{4}(\cos 3\omega_{s1}t + \cos 3\omega_{s2}t)$

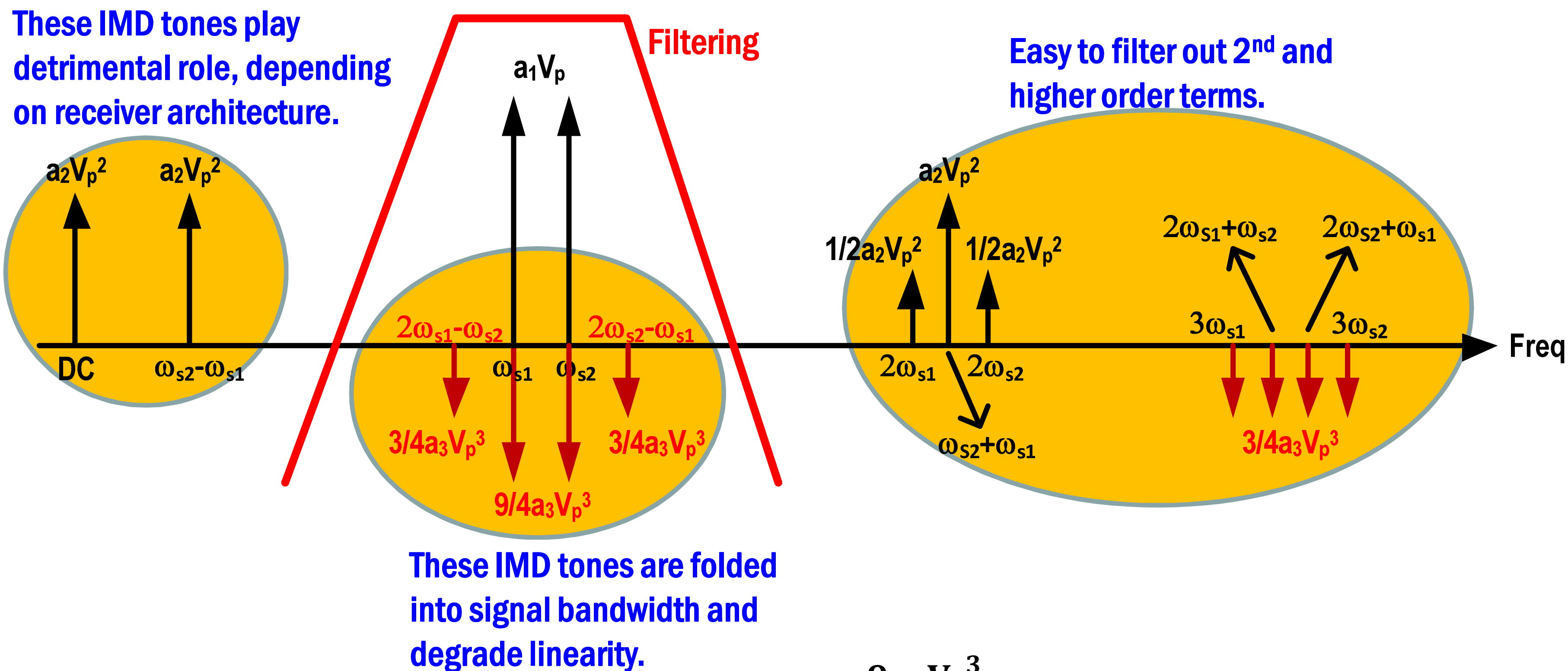
$+\frac{3a_3 V_p^3}{4}(\cos(2\omega_{s1}-\omega_{s2})t + \cos(2\omega_{s2}-\omega_{s1})t)$

$+\frac{3a_3 V_p^3}{4}(\cos(2\omega_{s1}+\omega_{s2})t + \cos(2\omega_{s2}+\omega_{s1})t)$



VirginiaTech
Invent the Future

Intermodulation Distortion (2-tone test)



1. Fundamental tone $(\omega_{s1}, \omega_{s2})$: $a_1 V_p + \dfrac{9a_3{V_p}^3}{4}$

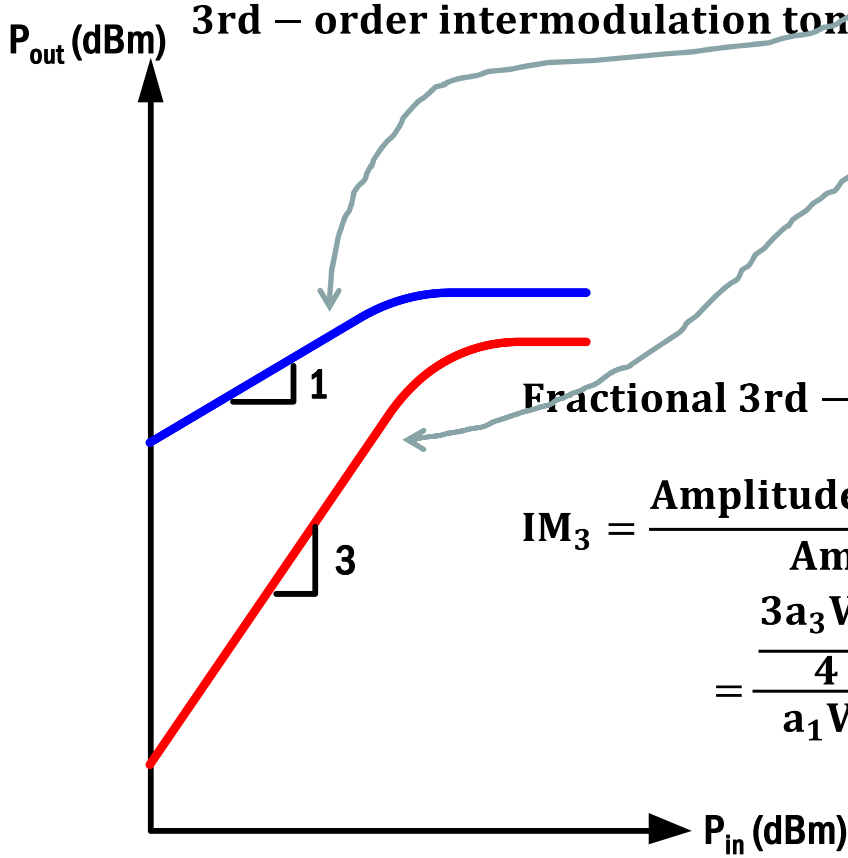
2. 2nd − order intermodulation tone $(\omega_{s2} - \omega_{s1})$: $a_2{V_p}^2$

3. 3rd − order intermodulation tone $(2\omega_{s2} - \omega_{s1}, 2\omega_{s1} - \omega_{s2})$: $\dfrac{3a_3{V_p}^3}{4}$

3rd –order Intermodulation Distortion (IM3)

Fundamental tone $(\omega_{s1,}\ \omega_{s2})$: $a_1 V_p + \frac{9a_3 {V_p}^3}{4} \approx a_1 V_p$ @well below sat. point

3rd − order intermodulation tone $(2\omega_{s2} - \omega_{s1}, 2\omega_{s1} - \omega_{s2})$: $\frac{3a_3 {V_p}^3}{4}$



Fractional 3rd − order intermodulation (IM3) is given by:

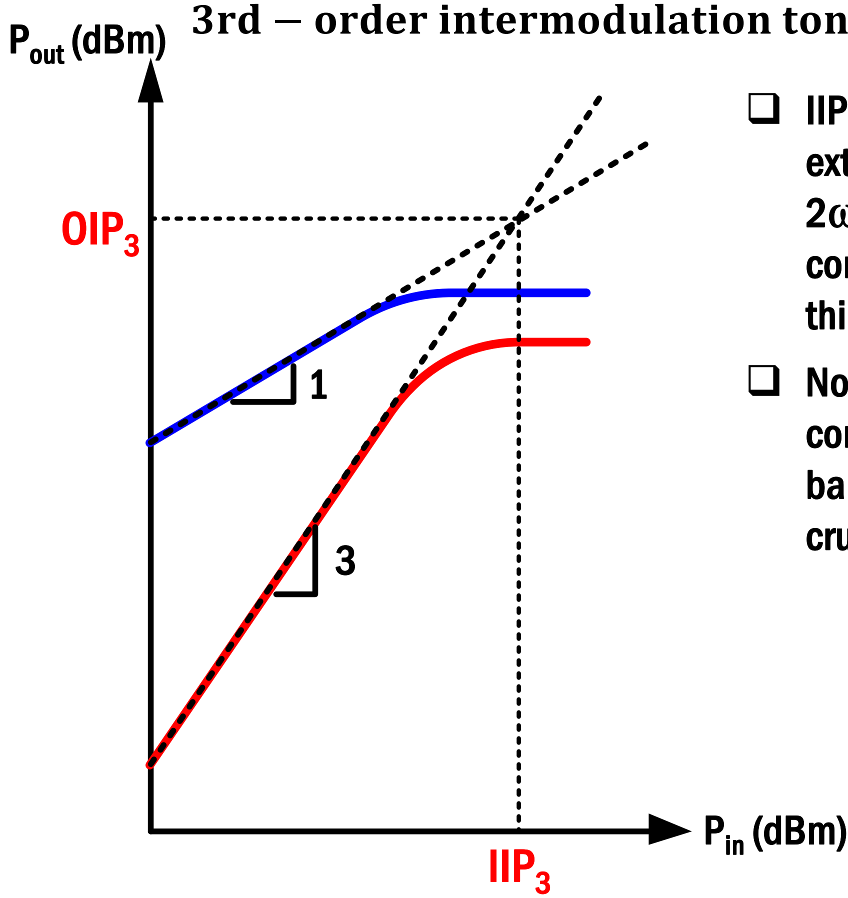
$$IM_3 = \frac{\text{Amplitude of the 3rd order intermodulation output}}{\text{Amplitude of the fundamental output}}$$

$$= \frac{\frac{3a_3 {V_p}^3}{4}}{a_1 V_p} = \boxed{\frac{3a_3 {V_p}^2}{4a_1}}$$

3rd –order Input Intercept Point (IIP3)

$$\textbf{Fundamental tone } (\omega_{s1}, \omega_{s2}): a_1 V_p + \frac{9a_3 V_p^{\ 3}}{4} \approx a_1 V_p \ \text{@well below sat. point}$$

$$\textbf{3rd − order intermodulation tone } (2\omega_{s2} - \omega_{s1}, 2\omega_{s1} - \omega_{s2}): \frac{3a_3 V_p^{\ 3}}{4}$$



- IIP3 is defined as the signal level at which the extrapolated output signal levels at ω_{s1} (or ω_{s2}) and $2\omega_{s1}$-ω_{s2} (or $2\omega_{s2}$-ω_{s1}) equal each other. The corresponding output power level is referred to as the third-order output intercept point (OIP3).
- Note that the 3rd –order intermodulation distortion corrupts the signal since it falls in the same frequency band as the signal of interest. IIP3 is therefore very crucial for high frequency communication systems.

$$IM_3 = \frac{3a_3 V_p^{\ 2}}{4a_1} = 1 \ \text{@IIP3}$$

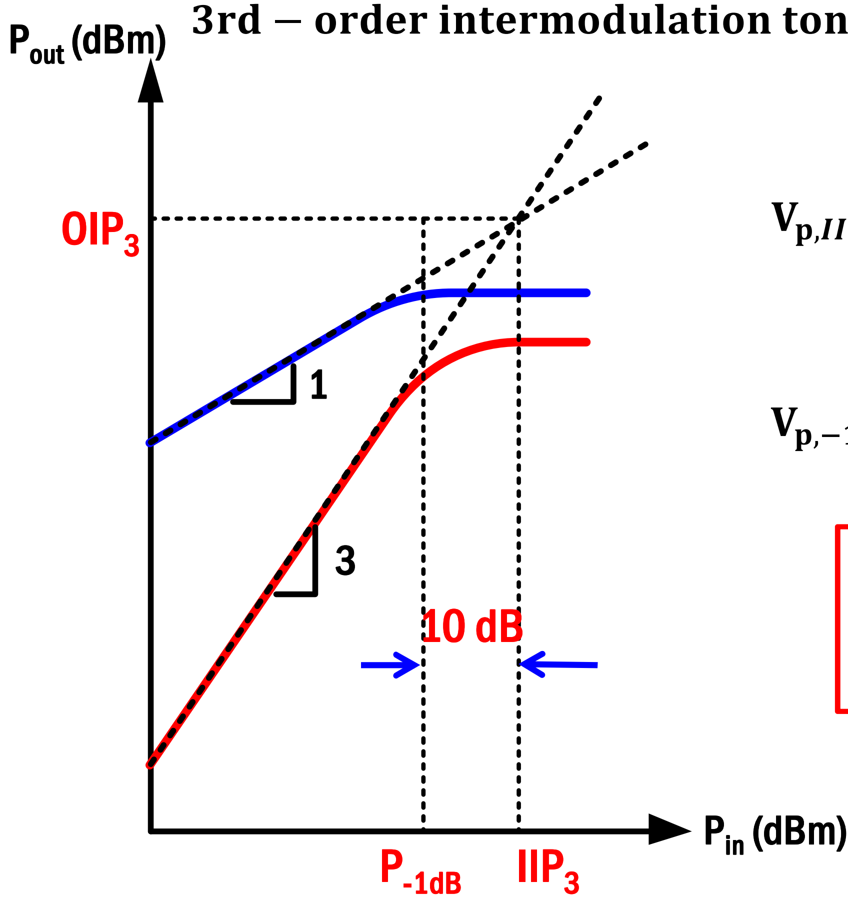
$$\rightarrow V_{p,IIP3} = \sqrt{\frac{4}{3}\left|\frac{a_1}{a_3}\right|} \approx \boxed{1.15\sqrt{\left|\frac{a_1}{a_3}\right|}}$$

$$\rightarrow V_{p,OIP3} = a_1\sqrt{\frac{4}{3}\left|\frac{a_1}{a_3}\right|} \approx 1.15\sqrt{\left|\frac{a_1^{\ 3}}{a_3}\right|}$$

Relating IIP3 to Input P1dB

Fundamental tone $(\omega_{s1}, \omega_{s2})$: $a_1 V_p + \frac{9a_3 V_p^{\ 3}}{4} \approx a_1 V_p$ @well below sat. point

3rd − order intermodulation tone $(2\omega_{s2} - \omega_{s1}, 2\omega_{s1} - \omega_{s2})$: $\frac{3a_3 V_p^{\ 3}}{4}$



$$V_{p,IIP3} = \sqrt{\frac{4}{3}\left|\frac{a_1}{a_3}\right|} \approx \boxed{1.15\sqrt{\left|\frac{a_1}{a_3}\right|}} \approx 1.2\sqrt{\left|\frac{a_1}{a_3}\right|}$$

$$V_{p,-1dB} = \boxed{0.38\sqrt{\left|\frac{a_1}{a_3}\right|}} \approx 0.4\sqrt{\left|\frac{a_1}{a_3}\right|}$$

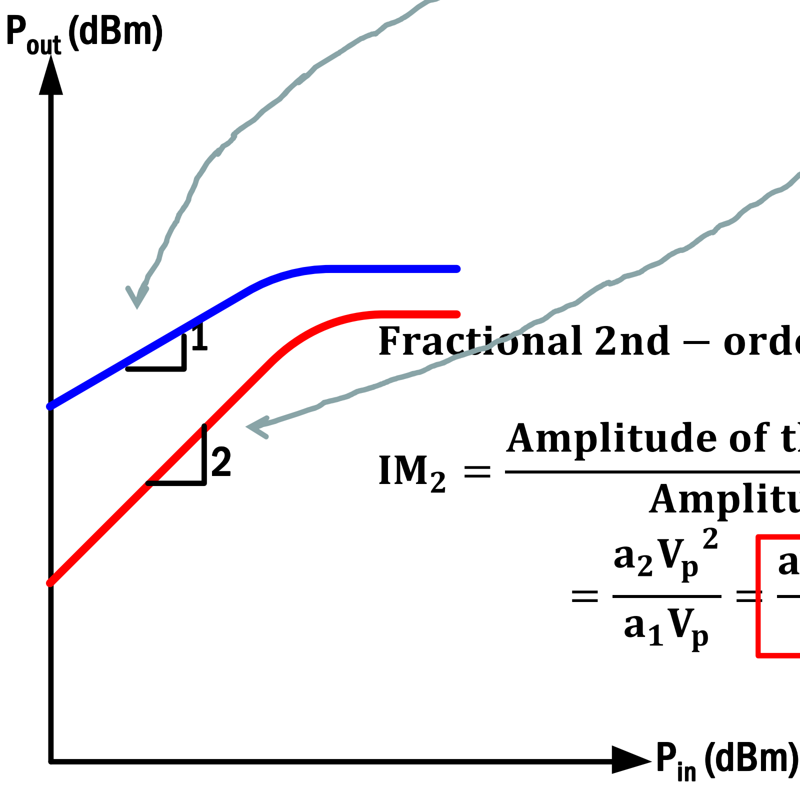
$$\boxed{\begin{aligned} &V_{p,IIP3} \approx 3V_{p,-1dB} \\ &V_{p,IIP3}(dB) \approx V_{p,-1dB}(dB) + 10dB \end{aligned}}$$

- ❑ Note: IIP3 is two-tone test, while input P1dB is single-tone test.

2^{nd} –order Intermodulation Distortion (IM2)

Fundamental tone $(\omega_{s1,}\ \omega_{s2})$: $a_1 V_p + \frac{9a_3 V_p{}^3}{4} \approx a_1 V_p$ @well below sat. point

2nd − order intermodulation tone $(\omega_{s2} - \omega_{s1})$: $a_2 V_p{}^2$



Fractional 2nd − order intermodulation (IM2) is given by:

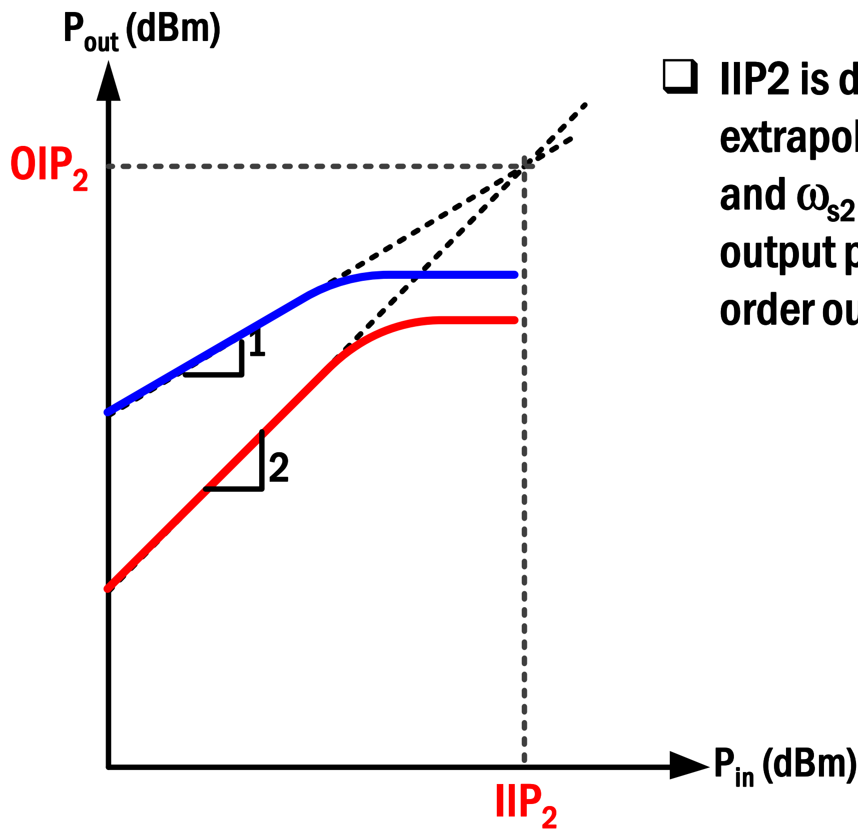
$$IM_2 = \frac{\text{Amplitude of the 2nd order intermodulation output}}{\text{Amplitude of the fundamental output}}$$

$$= \frac{a_2 V_p{}^2}{a_1 V_p} = \boxed{\frac{a_2 V_p}{a_1}}$$

2^{nd} –order Intermodulation Distortion (IM2)

$$\text{Fundamental tone } (\omega_{s1}, \omega_{s2}): a_1 V_p + \frac{9 a_3 V_p^3}{4} \approx a_1 V_p \text{ @well below sat. point}$$

$$\text{2nd} - \text{order intermodulation tone } (\omega_{s2} - \omega_{s1}): a_2 V_p^2$$



- IIP2 is defined as the signal level at which the extrapolated output signal levels at ω_{s1} (or ω_{s2}) and ω_{s2}-ω_{s1} equal each other. The corresponding output power level is referred to as the second-order output intercept point (OIP2).

$$IM_2 = \frac{a_2 V_p}{a_1} = 1 \text{ @IIP2}$$

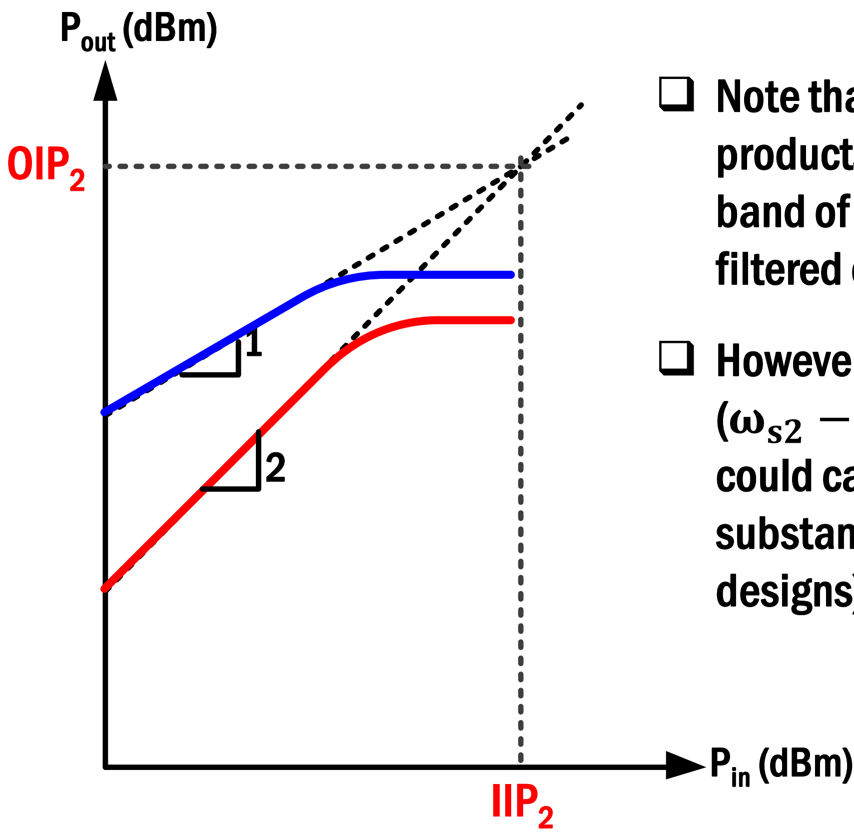
$$\rightarrow V_{p,IIP2} = \boxed{\left|\frac{a_1}{a_2}\right|}$$

$$\rightarrow V_{p,OIP2} = a_1 \left|\frac{a_1}{a_2}\right| = \left|\frac{a_1^2}{a_2}\right|$$

2^{nd} –order Intermodulation Distortion (IM2)

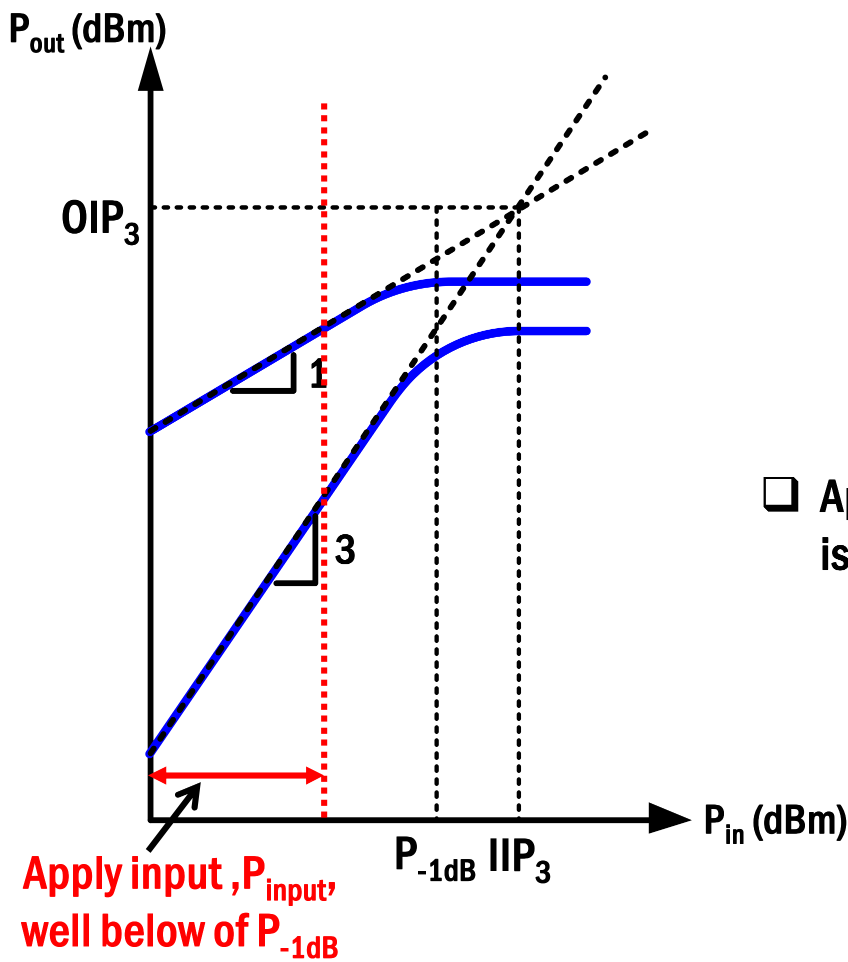
$$\textbf{Fundamental tone } (\omega_{s1,}\ \omega_{s2}):\ a_1 V_p + \frac{9a_3 V_p{}^3}{4} \approx a_1\ V_p\ \text{@well below sat. point}$$

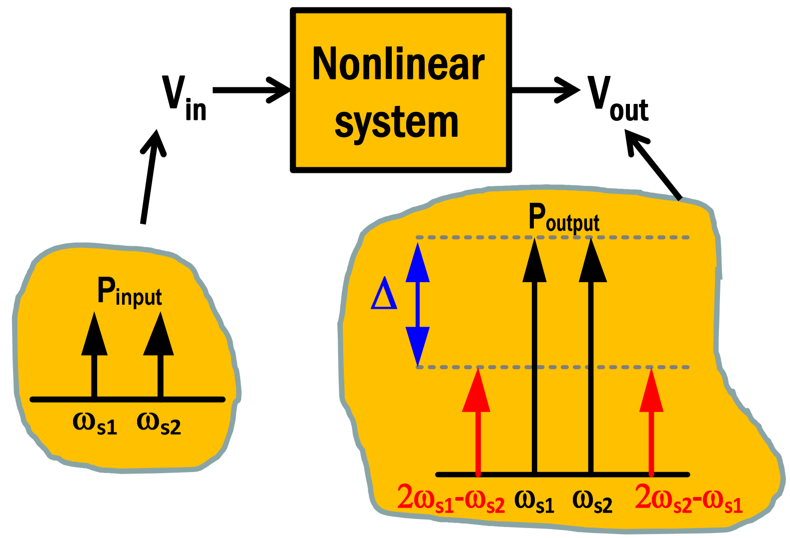
$$\textbf{2nd} - \textbf{order intermodulation tone } (\omega_{s2} - \omega_{s1}):\ a_2 V_p{}^2$$



- Note that the second –order intermodulation products are well separated from the frequency band of interest, and therefore they can be easily filtered out.
- However, for certain types of receiver architectures, $(\omega_{s2} - \omega_{s1})$ can not be filtered out, and IM2 could cause linearity degradation of a system substantially (we will visit this issue in mixer designs).

Estimating IIP3 Experimentally





- Apply weak two-tone input signals so that output is not saturated, then IIP3 will be calculate as:

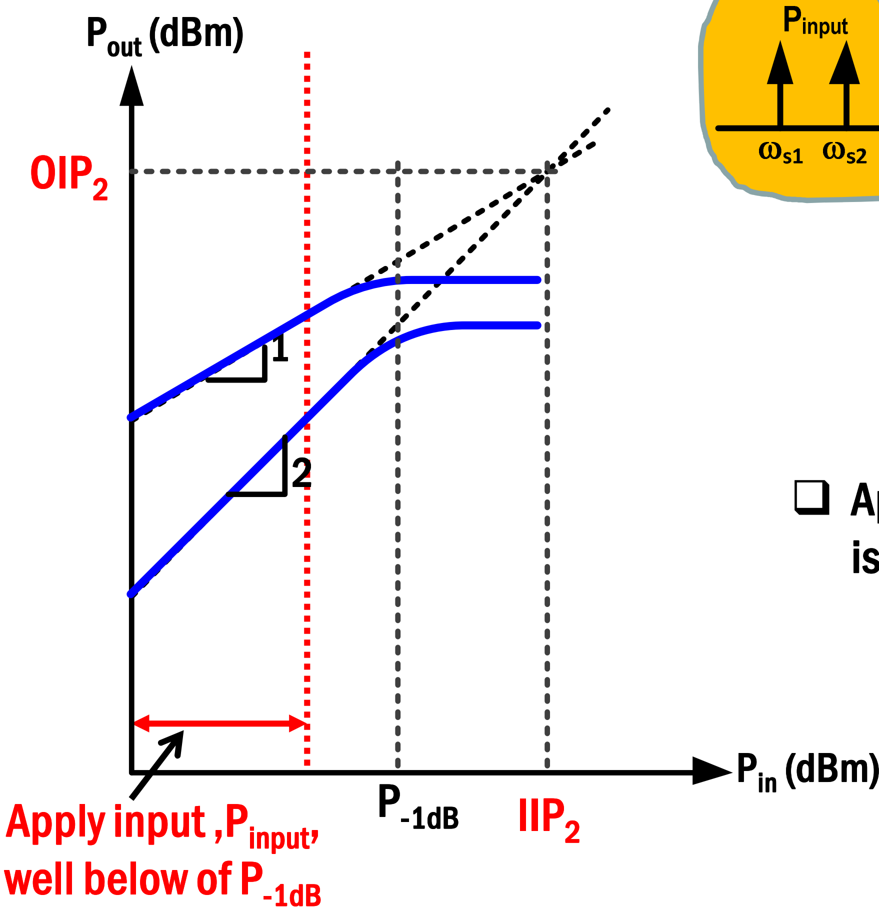
$$\mathrm{IIP_3(dB)} = \boxed{\frac{\Delta}{2} + \mathrm{P_{input}}}$$

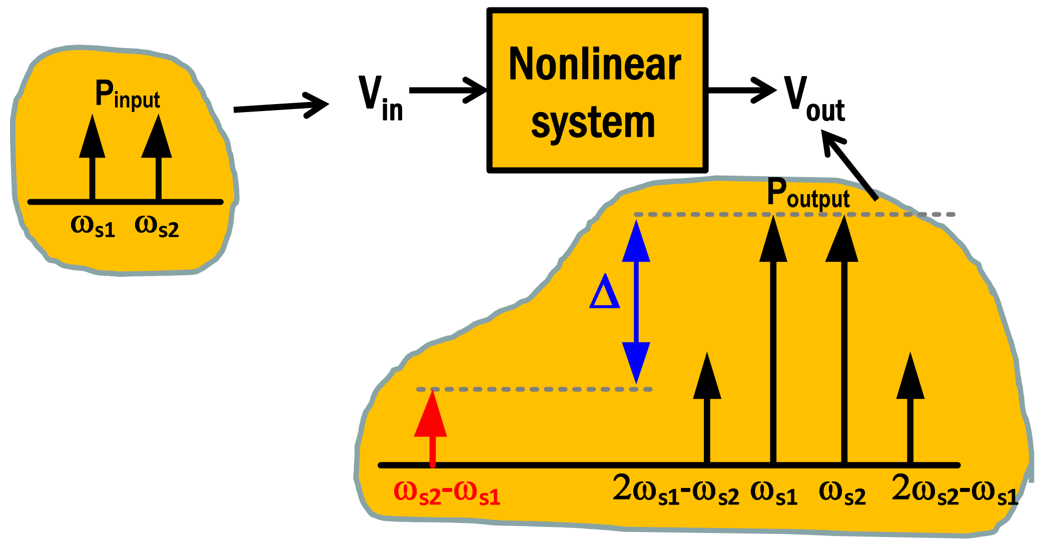
Q) Why?

$$\begin{aligned}\mathrm{OIP_3(dB)} &= \mathrm{IIP_3} + \mathrm{Gain} \\ &= \frac{\Delta}{2} + \mathrm{P_{input}} + (\mathrm{P_{output}} - \mathrm{P_{input}}) \\ &= \boxed{\frac{\Delta}{2} + \mathrm{P_{output}}}\end{aligned}$$

VirginiaTech
Invent the Future

Estimating IIP2 Experimentally





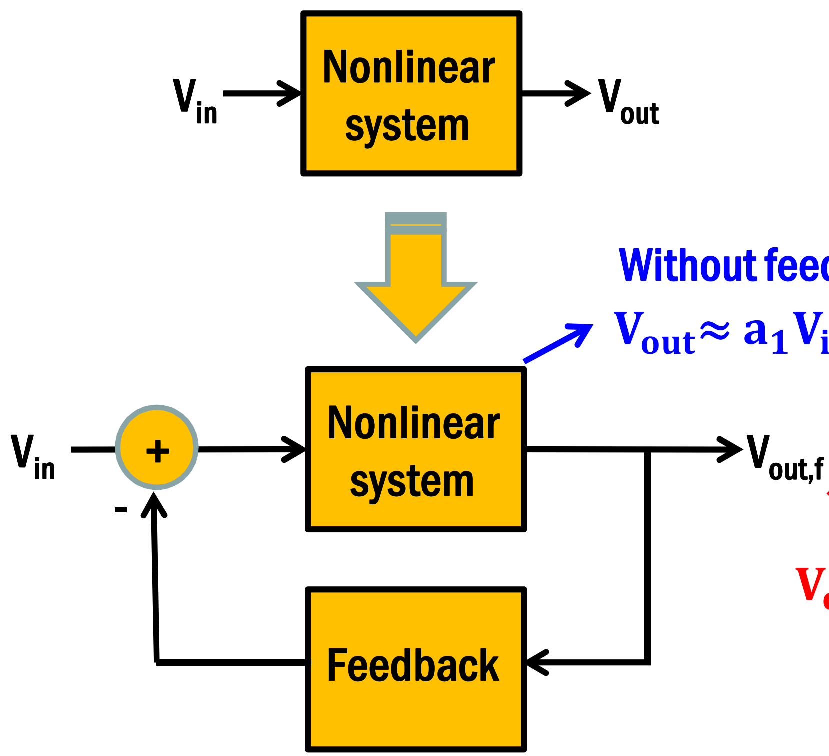
- Apply weak two-tone input signals so that output is not saturated, then IIP2 will be calculate as:

$$IIP_2(dB) = \boxed{\Delta + P_{input}}$$ Q) Why?

$$\begin{aligned} OIP_2(dB) &= IIP_3 + Gain \\ &= \Delta + P_{input} + (P_{output} - P_{input}) \\ &= \boxed{\Delta + P_{output}} \end{aligned}$$

Effect of Feedback on Nonlinearity

- Negative feedback is often applied to improve linearity of a system.



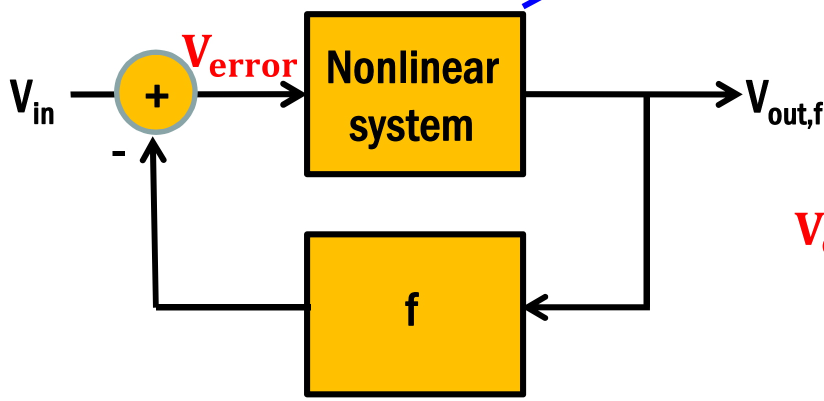
Without feedback:

$$V_{out} \approx a_1 V_{in} + a_2 V_{in}^{\ 2} + a_3 V_{in}^{\ 3}$$

With feedback:

$$V_{out,f} \approx a_{1f} V_{in} + a_{2f} V_{in}^{\ 2} + a_{3f} V_{in}^{\ 3}$$

Effect of Feedback on Nonlinearity



Without feedback :

$$V_{out} \approx a_1 V_{in} + a_2 {V_{in}}^2 + a_3 {V_{in}}^3$$

With feedback :

$$V_{out,f} \approx a_{1f} V_{in} + a_{2f} {V_{in}}^2 + a_{3f} {V_{in}}^3$$

- $V_{error} = V_{in} - fV_{out,f}$
  $= V_{in} - f\left(a_{1f} V_{in} + a_{2f} {V_{in}}^2 + a_{3f} {V_{in}}^3 \right)$

- $V_{out,f} = a_1 V_{error} + a_2 {V_{error}}^2 + a_3 {V_{error}}^3$
  $= a_1 \left(V_{in} - fa_{1f} V_{in} - fa_{2f} {V_{in}}^2 - fa_{3f} {V_{in}}^3 \right)$
  $+ a_2 \left(V_{in} - fa_{1f} V_{in} - fa_{2f} {V_{in}}^2 - fa_{3f} {V_{in}}^3 \right)^2$
  $+ a_3 \left(V_{in} - fa_{1f} V_{in} - fa_{2f} {V_{in}}^2 - fa_{3f} {V_{in}}^3 \right)^3$
  $= a_{1f}\ V_{in} + a_{2f} {V_{in}}^2 + a_{3f} {V_{in}}^3$

Effect of Feedback on Nonlinearity

- $a_{1f}V_{in} + a_{2f}V_{in}^2 + a_{3f}V_{in}^3 = a_1\left(V_{in} - fa_{1f}V_{in} - fa_{2f}V_{in}^2 - fa_{3f}V_{in}^3\right)$
$$+ a_2\left(V_{in} - fa_{1f}V_{in} - fa_{2f}V_{in}^2 - fa_{3f}V_{in}^3\right)^2$$
$$+ a_3\left(V_{in} - fa_{1f}V_{in} - fa_{2f}V_{in}^2 - fa_{3f}V_{in}^3\right)^3$$

$$a_{1f} = a_1(1 - fa_{1f}) \rightarrow \boxed{a_{1f} = \frac{a_1}{1+fa_1}}$$

$$a_{2f} = a_1(-fa_{2f}) + a_2(1 - fa_{1f})^2 \rightarrow \boxed{a_{2f} = \frac{a_2}{(1+fa_1)^3}}$$

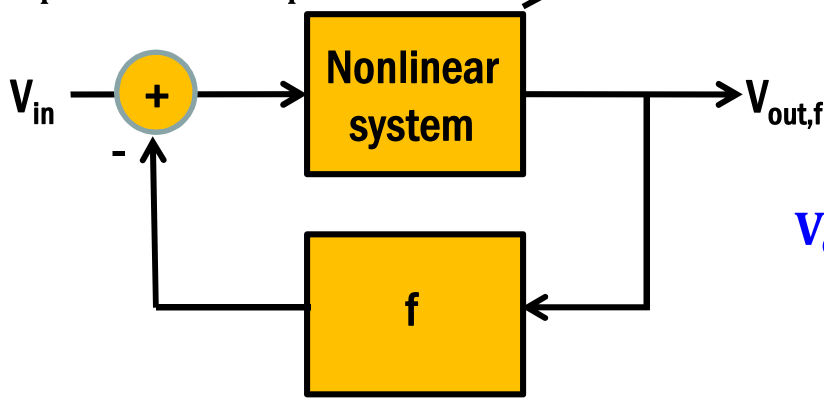
$$a_{3f} = a_1(-fa_{3f}) - 2a_2(1 - fa_{1f})a_{2f} + a_3(1 - fa_{1f})^3 \rightarrow \boxed{a_{3f} = \frac{a_3(1+fa_1) - 2f\,a_2^2}{(1+fa_1)^5}}$$

Effect of Feedback on IIP2

$V_{in} = V_p \cos\omega_{s1} t + V_p \cos\omega_{s2} t$

Without feedback :

$V_{out} \approx a_1 V_{in} + a_2 {V_{in}}^2 + a_3 {V_{in}}^3$



With feedback :

$V_{out,f} \approx a_{1f} V_{in} + a_{2f} {V_{in}}^2 + a_{3f} {V_{in}}^3$

$$a_{1f} = \frac{a_1}{1+fa_1}$$

$$a_{2f} = \frac{a_2}{(1+fa_1)^3}$$

$$V_{p,IIP2,f} = \left|\frac{a_{1f}}{a_{2f}}\right| = \left|\frac{a_1}{a_2}\right| (1+fa_1)^2$$

IIP2 without feedback

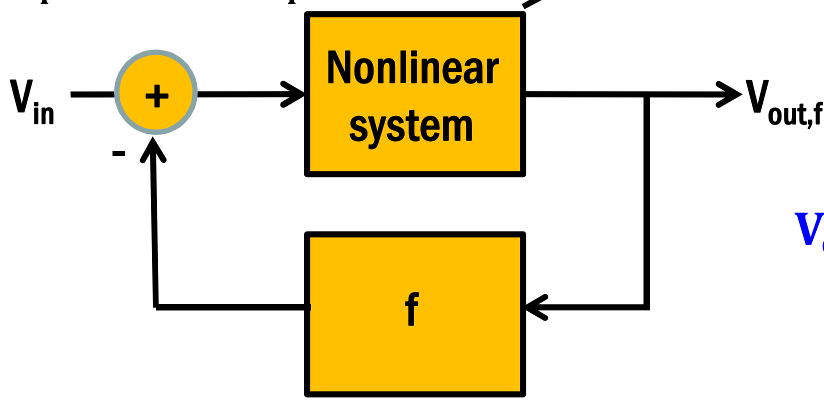
IIP2 increment factor due to feedback

Effect of Feedback on IIP3

$$V_{in} = V_p \cos\omega_{s1} t + V_p \cos\omega_{s2} t$$

Without feedback :

$$V_{out} \approx a_1 V_{in} + a_2 {V_{in}}^2 + a_3 {V_{in}}^3$$



With feedback : c

$$V_{out,f} \approx a_{1f} V_{in} + a_{2f} {V_{in}}^2 + a_{3f} {V_{in}}^3$$

$$a_{1f} = \frac{a_1}{1+fa_1}$$

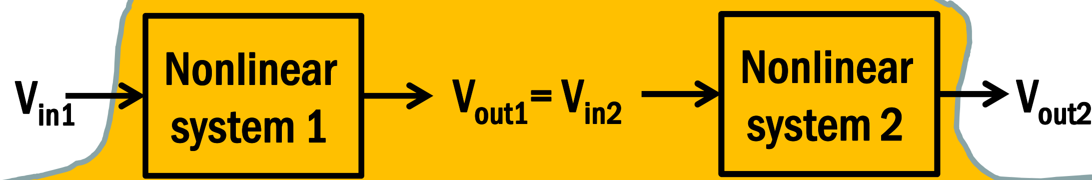
$$a_{3f} = \frac{a_3(1+fa_1) - 2f\,{a_2}^2}{(1+fa_1)^5}$$

$$V_{p,IIP3,f} = \sqrt{\frac{4}{3}\left|\frac{a_{1f}}{a_{3f}}\right|} = \sqrt{\frac{4}{3}\left|\frac{a_1}{a_3}\right|} \frac{(1+fa_1)^2}{\sqrt{\left|1+fa_1-\frac{2f\,{a_2}^2}{a_3}\right|}}$$

IIP3 without feedback

IIP3 increment factor due to feedback

Cascaded IIP3



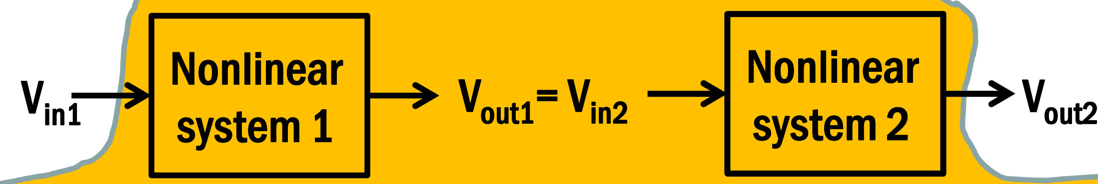
$$V_{out1} \approx a_1 V_{in1} + a_2 V_{in1}{}^2 + a_3 V_{in1}{}^3 \qquad V_{out2} \approx b_1 V_{out1} + a_2 V_{out1}{}^2 + a_3 V_{out1}{}^3$$

Overall cascaded nonlinear response: $V_{out2} \approx c_1 V_{in1} + c_2 V_{in1}{}^2 + c_3 V_{in1}{}^3$

$$\begin{aligned} V_{out2} &\approx c_1 V_{in1} + c_2 V_{in1}{}^2 + c_3 V_{in1}{}^3 \\ &= b_1 \left(a_1 V_{in1} + a_2 V_{in1}{}^2 + a_3 V_{in1}{}^3\right) \\ &+ b_2\left(a_1 V_{in1} + a_2 V_{in1}{}^2 + a_3 V_{in1}{}^3\right)^2 \\ &+ b_3\left(a_1 V_{in1} + a_2 V_{in1}{}^2 + a_3 V_{in1}{}^3\right)^3 \end{aligned}$$

$$\boxed{\begin{aligned} c_1 &= a_1 b_1 \\ c_2 &= a_1{}^2 b_2 + a_2 b_1 \\ c_3 &= a_1{}^3 b_3 + a_3 b_1 + 2 b_2 a_1 a_2 \end{aligned}}$$

Cascaded IIP3



$$V_{out1} \approx a_1 V_{in1} + a_2 V_{in1}^{\ 2} + a_3 V_{in1}^{\ 3} \qquad V_{out2} \approx b_1 V_{out1} + b_2 V_{out1}^{\ 2} + b_3 V_{out1}^{\ 3}$$

Overall cascaded nonlinear response: $V_{out2} \approx c_1 V_{in1} + c_2 V_{in1}^{\ 2} + c_3 V_{in1}^{\ 3}$

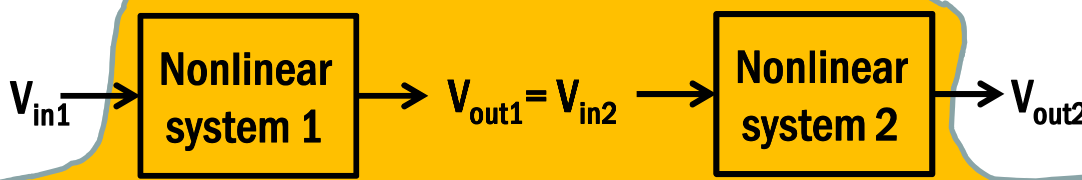
$$\boxed{c_1 = a_1 b_1} \quad \boxed{c_2 = a_1^{\ 2} b_2 + a_2 b_1} \quad \boxed{c_3 = a_1^{\ 3} b_3 + a_3 b_1 + 2 b_2 a_1 a_2}$$

$$V_{p,IIP3,sys-1} = \sqrt{\frac{4}{3}\left|\frac{a_1}{a_3}\right|} \qquad V_{p,IIP3,sys-2} = \sqrt{\frac{4}{3}\left|\frac{b_1}{b_3}\right|} \qquad V_{p,IIP2,sys-2} = \left|\frac{b_1}{b_2}\right|$$

$$V_{p,IIP3,overall} = \sqrt{\frac{4}{3}\left|\frac{c_1}{c_3}\right|} = \sqrt{\frac{4 a_1 b_1}{3(a_1^{\ 3} b_3 + a_3 b_1 + 2 b_2 a_1 a_2)}} = \sqrt{\frac{1}{\frac{3a_3}{4a_1} + a_1^{\ 2}\frac{3b_3}{4b_1} + \frac{3a_2}{2}\frac{b_2}{b_1}}}$$

$$\boxed{\left(\frac{1}{V_{p,IIP3,overall}}\right)^2 = \left(\frac{1}{V_{p,IIP3,sys-1}}\right)^2 + \left(\frac{a_1}{V_{p,IIP3,sys-2}}\right)^2 + \left(\frac{1.5 a_2}{V_{p,IIP2,sys-2}}\right)}$$

Cascaded IIP2



$$V_{out1} \approx a_1 V_{in1} + a_2 V_{in1}{}^2 + a_3 V_{in1}{}^3 \qquad V_{out2} \approx b_1 V_{out1} + b_2 V_{out1}{}^2 + b_3 V_{out1}{}^3$$

Overall cascaded nonlinear response: $V_{out2} \approx c_1 V_{in1} + c_2 V_{in1}{}^2 + c_3 V_{in1}{}^3$

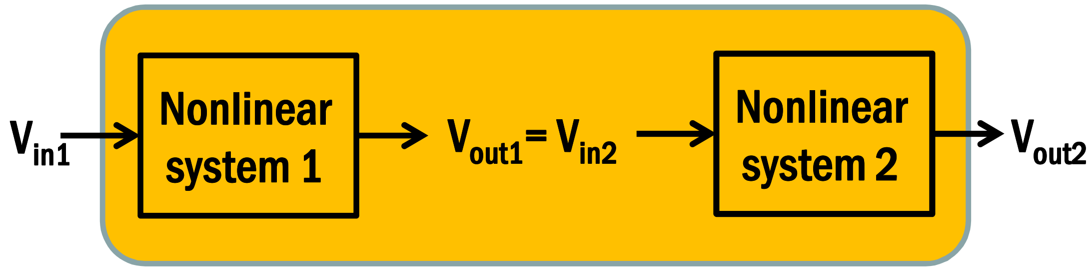
$$\boxed{c_1 = a_1 b_1} \quad \boxed{c_2 = a_1{}^2 b_2 + a_2 b_1} \quad \boxed{c_3 = a_1{}^3 b_3 + a_3 b_1 + 2 b_2 a_1 a_2}$$

$$V_{p,IIP2,sys-1} = \left|\frac{a_1}{a_2}\right| \qquad V_{p,IIP2,sys-2} = \left|\frac{b_1}{b_2}\right|$$

$$V_{p,IIP2,overall} = \left|\frac{c_1}{c_2}\right| = \frac{a_1 b_1}{a_1{}^2 b_2 + a_2 b_1} = \frac{1}{\frac{a_2}{a_1} + a_1 \frac{b_2}{b_1}}$$

$$\boxed{\left(\frac{1}{V_{p,IIP2,overall}}\right)^2 = \left(\frac{1}{V_{p,IIP2,sys-1}}\right)^2 + \left(\frac{a_1}{V_{p,IIP2,sys-2}}\right)^2}$$

Comments on Cascaded Input Intercept Points



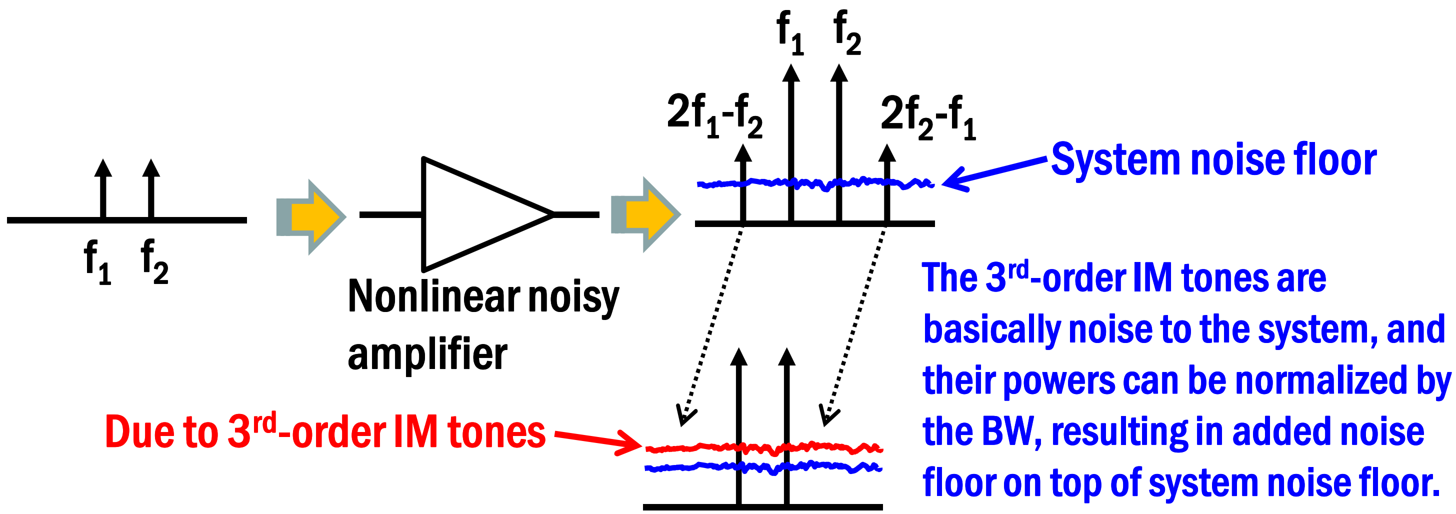
$$\left(\frac{1}{V_{p,IIP3,overall}}\right)^2 = \left(\frac{1}{V_{p,IIP3,sys-1}}\right)^2 + \left(\frac{a_1}{V_{p,IIP3,sys-2}}\right)^2 + \left(\frac{1.5a_2}{V_{p,IIP2,sys-2}}\right)^2$$

$$\left(\frac{1}{V_{p,IIP2,overall}}\right)^2 = \left(\frac{1}{V_{p,IIP2,sys-1}}\right)^2 + \left(\frac{a_1}{V_{p,IIP2,sys-2}}\right)^2$$

- The second stage IIP2 and IIP3 are divided by gain of the first stage. Therefore, linearity of the second stage plays dominant role for the cascaded system.
- In general IIP2 & IIP3 of latter stage are divided by all the cascaded gain of preceding stages, and dominate overall linearity performance.
- By the same reason, in LNA & mixer cascaded system, mixer linearity tends to be more emphasized than LNA.

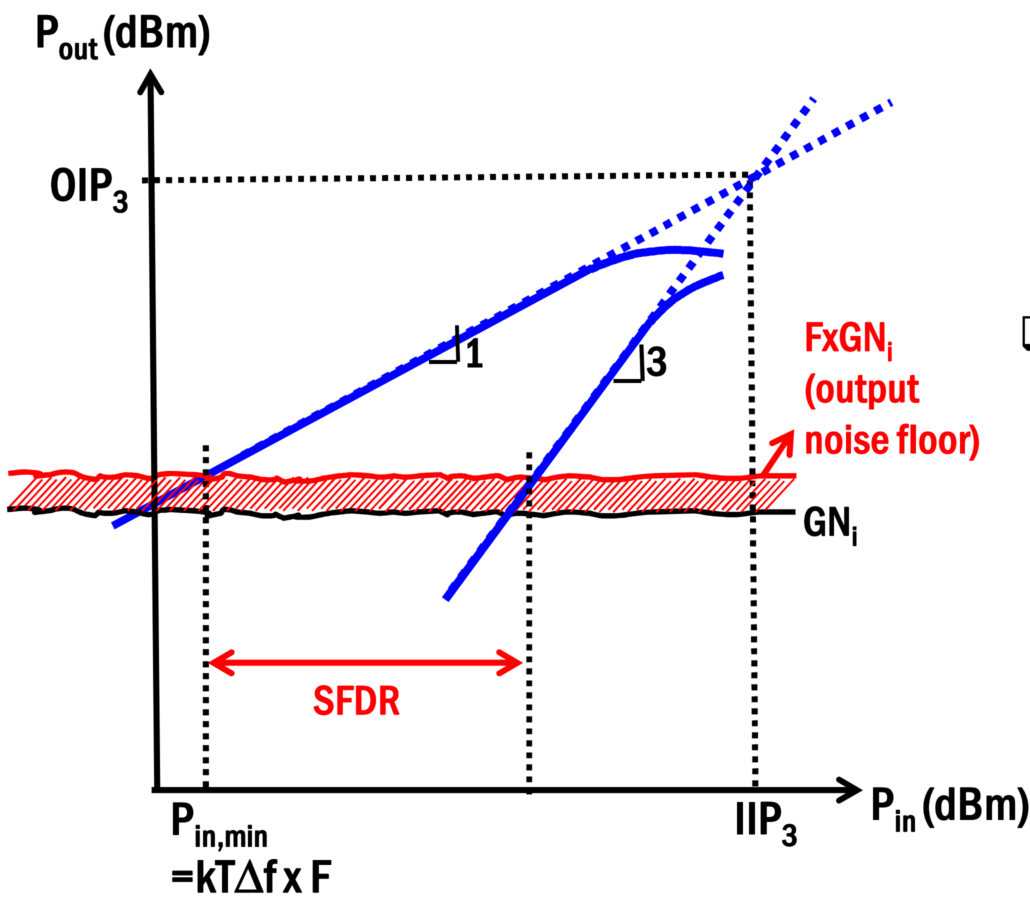
Effective Noise Floor Due to Nonlinearity

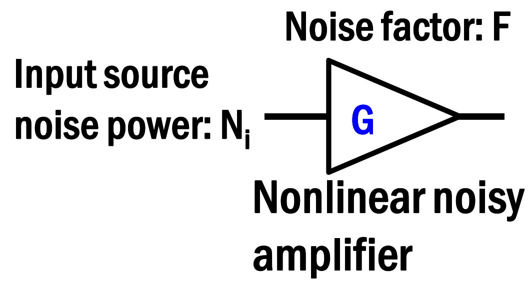
- The 3rd-order intermodulation outputs contribute additional noise; "unwanted signal" which would degrade the SNR at the output if they become larger than the noise floor.



- Net effect of spurious tone is degrading SNR of a system.

Spurious Free Dynamic Range (SFDR)





- "SFDR" is the difference between the system input noise floor and the largest signal that can be accommodated before the 3rd-order IM product rises above the noise floor.

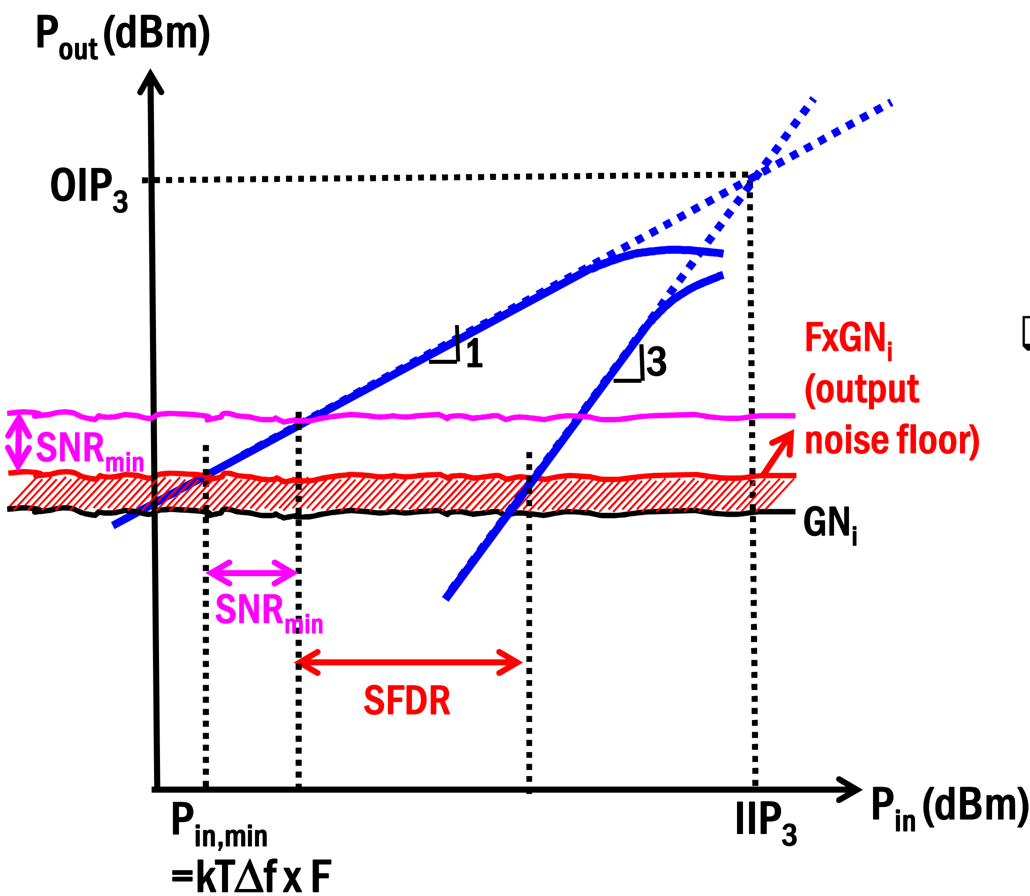
$$\mathrm{IIP_3} - \mathrm{P_{in,min}} = 3(\mathrm{IIP_3} - (\mathrm{P_{in,min}} + SFDR))$$

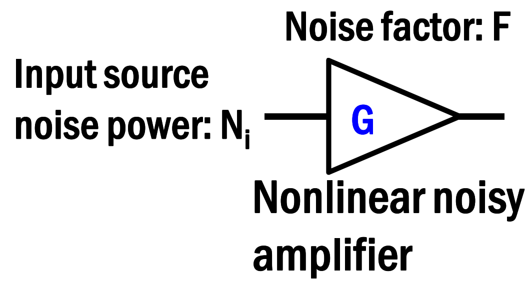
$$\therefore SFDR = \frac{2}{3}(\mathrm{IIP_3} - \mathrm{P_{in,min}})$$

In this expression, we generally assume that $\mathrm{SNR_{min}}$=1 (0 dB).

VirginiaTech
Invent the Future

Spurious Free Dynamic Range (SFDR)





- "SFDR" is the difference between **the system input noise floor +SNR_{min}** and the largest signal that can be accommodated before the 3rd-order IM product rises above the noise floor.

$$\boxed{\therefore SFDR = \frac{2}{3}\left(\mathrm{IIP_3} - \mathrm{P_{in,min}}\right) - \mathrm{SNR_{min}}}$$

This is more general definition of SFDR.