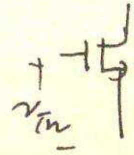
Problem - 1



$$i_o = \alpha_1 v_{in} + \alpha_2 v_{in}^2$$

$$\alpha_1 = \left(\frac{\partial i_o}{\partial v_{in}}\right)_{v_{in}=0}$$

$$\alpha_2 = \frac{\partial^2 i_o}{\partial v_{in}^2} \times \frac{1}{2}$$

1) THD → $\underline{\underline{HD_2}}$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})^2$$

$$\frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th}) = g_m \rightarrow \alpha_1$$

$$\frac{\partial^2 I_D}{\partial V_{GS}^2} = \mu_n C_{ox}\frac{W}{L} \longrightarrow \alpha_2 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}$$

$$\therefore HD_2 = \frac{1}{2}\frac{\alpha_2}{\alpha_1}V_{peak} = \frac{1}{2}\frac{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}{\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})}$$

$$= \frac{1}{4(V_{GS}-V_{th})}$$

2)
$$\left.\begin{aligned} S_{out} &= (g_m V_{in})^2 R_L \\ N_{out} &= \left(4KT\frac{\gamma}{\alpha}g_m\right)\cdot R_L \end{aligned}\right\} \frac{S_{out}}{N_{out}} = \frac{g_m}{4KT\frac{\gamma}{\alpha}}V_{in}^2$$

3) $W/L \uparrow \times 2 \rightarrow g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} \uparrow \times \sqrt{2}$

$$= \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th}) \uparrow \times \sqrt{2}$$

$$\therefore (V_{GS}-V_{th}) \downarrow \times \frac{1}{\sqrt{2}}$$

- g_m increased by a factor of $\sqrt{2}$ ⇒ $(S/N)_{out} \uparrow \times \sqrt{2}$
- $(V_{GS}-V_{th})$ decreased by a factor of $\sqrt{2}$ ⇒ $HD_2 \uparrow \times \sqrt{2}$
- Noise power ⇒ $N_{out} \uparrow \times \sqrt{2}$

~~4)~~ Note: Nout increases by a factor of $\sqrt{2}$.
But signal power also increases by a factor of 2.
$\Rightarrow$ overall S/N increases by a factor of $\sqrt{2}$.

4) $(V_{GS}-V_{th}) \uparrow \times 2 \longrightarrow (W/L) \downarrow \times \frac{1}{4}$

(Why: To maintain same current)

$\rightarrow g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS}-V_{th}) \downarrow \times \frac{1}{2}$

- g_m decreases by a fator of 2.

$\rightarrow$ Noise power will be decreased by a factor of 2

$\rightarrow$ signal power will be decreased by a factor of 4.

$\therefore (S/N)_{out} \downarrow \times \frac{1}{2}$

- $HD_2 \downarrow \times \frac{1}{2}$

NOTE:

From 3) and 4), you can reason that under constant bias current,

1. To achieve better NF $\rightarrow$ Increase W/L ~~small current density per width~~
2. To achieve better linearity $\rightarrow$ Increase $(V_{GS}-V_{th})$. ~~large current density per width~~.

A few more remarks

1. Increasing W/L with constant I_{BIAS} means "small current density" per width for low-noise operation
2. Increasing $(V_{GS}-V_{th})$ with constant I_{BIAS} means "large current density" per width for high-linearity operation.

5) $I_D \uparrow \times 2 \rightarrow g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} \uparrow \times \sqrt{2}$

$= \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th}) \uparrow \times \sqrt{2}$

$\therefore (V_{GS}-V_{th}) \uparrow \times \sqrt{2}$

$\Rightarrow \left.\begin{matrix} (N_{out}) \uparrow \times \sqrt{2} \\ (S_{out}) \uparrow \times 2 \end{matrix}\right\} (S/N)_{out} \uparrow \times \sqrt{2}$

$\Rightarrow HD_2 = \frac{1}{4(V_{GS}-V_{th})} \downarrow \times \sqrt{2}$

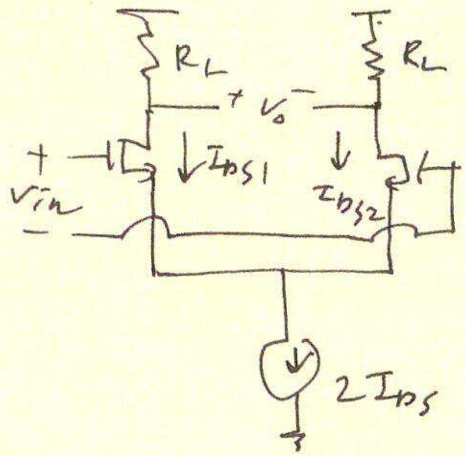
NOTE:
To achieve better NF and linearity at the same time, bias current needs to be increased
$\Rightarrow$ Trade-off between power and NF, linearity

6) $THD = HD_2 = \frac{V_{in,max}}{4(V_{GS}-V_{th})} = 0.01$

$\therefore V_{in,max} = 0.04(V_{GS}-V_{th})$

$\Rightarrow$ 4% of $(V_{GS}-V_{th})$

problem-2



$$i_o = I_{DS1} - I_{DS2}$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS1}-V_{th})^2 - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS2}-V_{th})^2$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left\{\underbrace{(V_{GS1}-V_{GS2})}_{\substack{\text{AC-component}\\ = V_{in}}}\;\underbrace{(V_{GS1}+V_{GS2}-2V_{th})}_{\substack{\text{DC-component}\\ (V_{GS1}-V_{th}) + (V_{GS2}-V_{th})}}\right\}$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS1}-V_{GS2})\left(\sqrt{\frac{2I_{DS1}}{\mu_n C_{ox}\frac{W}{L}}} + \sqrt{\frac{2I_{DS2}}{\mu_n C_{ox}\frac{W}{L}}}\right)$$

$$= \sqrt{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}\,(V_{GS1}-V_{GS2})\left(\sqrt{I_{DS1}} + \sqrt{I_{DS2}}\right) \quad \text{—} \; ①$$

$$= I_{DS1} - I_{DS2} = \left(\sqrt{I_{DS1}} + \sqrt{I_{DS2}}\right)\left(\sqrt{I_{DS1}} - \sqrt{I_{DS2}}\right)$$

$$\therefore \sqrt{I_{DS1}} - \sqrt{I_{DS2}} = \sqrt{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}\,(V_{GS1}-V_{GS2}) \quad \text{—} \; ②$$

From ②,

$$I_{DS1} + I_{DS2} - 2\sqrt{I_{DS1} \cdot I_{DS2}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS1} - V_{GS2})^2$$

$$\therefore 2\sqrt{I_{DS1} \cdot I_{DS2}} = 2I_{DS} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS1} - V_{GS2})^2 \quad — ③$$

apply

From ①,

$$(I_{DS1} - I_{DS2})^2 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS1} - V_{GS2})^2\left(2I_{DS} + 2\sqrt{I_{DS1} \cdot I_{DS2}}\right)$$

$$= 2I_{DS}\mu_n C_{ox}\frac{W}{L}(V_{GS1} - V_{GS2})^2 - \frac{1}{4}\left(\mu_n C_{ox}\frac{W}{L}\right)^2(V_{GS1} - V_{GS2})^4$$

$$\therefore I_{DS1} - I_{DS2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS1} - V_{GS2})\sqrt{\frac{8I_D}{\mu_n C_{ox}\frac{W}{L}} - (V_{GS1} - V_{GS2})^2} \quad — ④$$

From ④, $I_{DS1} - I_{DS2} = i_o$
$V_{GS1} - V_{GS2} = v_{in}$

$$\therefore i_o = \frac{1}{2}\mu_n C_{ox}\frac{W}{L} v_{in}\sqrt{\frac{8I_D}{\mu_n C_{ox}\frac{W}{L}} - v_{in}^2} \quad — ⑤$$

$\therefore$ overall

$$G_m = \left(\frac{\partial i_o}{\partial v_{in}}\right)_{v_{in}=0}$$

$$= \left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{\frac{8I_D}{\mu_n C_{ox}\frac{W}{L}} - 2v_{in}^2}{\sqrt{\frac{8I_D}{\mu_n C_{ox}\frac{W}{L}} - v_{in}^2}}\right)_{v_{in}=0}$$

$$= \sqrt{\mu_n C_{ox}\frac{W}{L} \cdot 2I_D}$$

From ~~4~~ 5,

$$i_o = \frac{1}{2}\mu_n C_{ox}\frac{W}{L} V_{in}\sqrt{\frac{8I_D}{\mu_n C_{ox}\frac{W}{L}} - V_{in}^2}$$

$$V_{GS} = \frac{V_{GS1}+V_{GS2}}{2}$$ ⇐ DC-Bias voltage of NMOS

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})^2$$

apply

$$\Rightarrow i_o = \frac{1}{2}\mu_n C_{ox}\frac{W}{L} V_{in}\sqrt{4(V_{GS}-V_{th})^2 - V_{in}^2}$$

$$= \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th}) V_{in}\sqrt{1-\underbrace{\frac{V_{in}^2}{4(V_{GS}-V_{th})^2}}}$$

if $V_{in} \ll 2(V_{GS}-V_{th})$

$\sqrt{1-x} \simeq 1-\frac{x}{2}$

$$\approx \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th}) V_{in}\left(1-\frac{V_{in}^2}{8(V_{GS}-V_{th})^2}\right)$$

$$= \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})\left(V_{in}-\frac{1}{8(V_{GS}-V_{th})^2}V_{in}^3\right)$$

$$\Rightarrow \boxed{\begin{aligned} &\alpha_1 = g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th}) \\ &\alpha_2 = 0 \\ &|\alpha_3| = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})\frac{1}{8(V_{GS}-V_{th})^2} \end{aligned}}$$

1) $THD = HD_3 = \frac{1}{4} \frac{|\alpha_3|}{\alpha_1} V_{peak}^2$

$$= \frac{1}{4} \cdot \frac{1}{8(V_{GS}-V_{th})^2} = \frac{1}{32(V_{GS}-V_{th})^2}$$

2) $S_{out} = (G_m \cdot V_{in})^2 R_L$

$N_{out} = 4KT \frac{\gamma}{\alpha} G_m \times 2 \times R_L$ ~~Because of 2 transistors.~~

because of two transistors.

$$(S/N)_{out} = \frac{G_m}{8KT \frac{\gamma}{\alpha}} V_{in}^2$$

3) $W/L \uparrow \times 2 \longrightarrow G_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} \uparrow \times \sqrt{2}$

$$= \mu_n C_{ox} \frac{W}{L} (V_{GS}-V_{th}) \uparrow \times \sqrt{2}$$

$$\therefore (V_{GS}-V_{th}) \downarrow \frac{1}{\sqrt{2}}$$

$\Rightarrow N_{out} \uparrow \times \sqrt{2}$

$\Rightarrow (S/N)_{out} \uparrow \times \sqrt{2}$

$\Rightarrow HD_3 \uparrow \times 2$ ← compare this result with that of single-ended case.

To maintain same bias current.

4) $(V_{GS}-V_{th}) \uparrow \times 2 \longrightarrow W/L \downarrow \times \frac{1}{4}$

$$\rightarrow G_m = \mu_n C_{ox} \frac{W}{L} (V_{GS}-V_{th}) \downarrow \frac{1}{2}$$

$\Rightarrow N_{out} \downarrow \times \frac{1}{2}$

$\Rightarrow (S/N)_{out} \downarrow \times \frac{1}{2}$

$\Rightarrow HD_3 \downarrow \times \frac{1}{4}$ ← compare this result with that of single-ended case.

5) $I_D \uparrow \times 2 \longrightarrow G_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} \uparrow \times \sqrt{2}$

$$= \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th}) \uparrow \times \sqrt{2}$$

$$\therefore (V_{GS} - V_{th}) \uparrow \times \sqrt{2}$$

$\Rightarrow N_{out} \uparrow \times \sqrt{2}$

$\Rightarrow (S/N)_{out} \uparrow \times \sqrt{2}$

$\Rightarrow HD_3 \downarrow \times \frac{1}{2}$

6) $THD = HD_3 = \frac{1}{4} \frac{|\alpha_3|}{\alpha_1} \times V_{in,max}^2$

$$= \frac{V_{in,max}^2}{32 (V_{GS} - V_{th})^2} = 0.01$$

$$\therefore V_{in,max} = \sqrt{0.01 \times 32} \cdot (V_{GS} - V_{th})$$

$$\approx 0.57 (V_{GS} - V_{th})$$

$\Rightarrow 57\%$ of $(V_{GS} - V_{th})$

Compare this with single-ended case

NOTE:
Differential topology can achieve much better linearity over wide input range compared with single-ended topology.