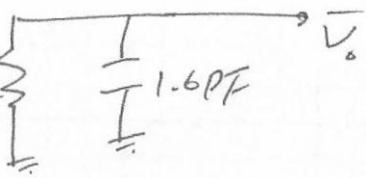
HW-2 solution   problem -1



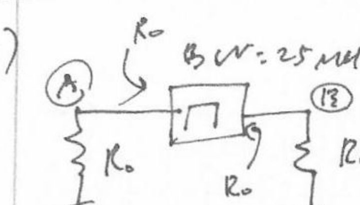
1) $\overline{v_o}^2 = \frac{KT}{C}$

$= \frac{1.38\times10^{-23}\,J/K \cdot 300K}{1.6\times10^{-12}\,F}$

$= 2.5875\times10^{-9}\ V_{rms}^2$

$\therefore \overline{V_o} = 50.87\,\mu V_{rms}$

3)



$T = 373K$ $T = 194K$

~~$KT\Delta f$ @ A~~

$\boxed{\text{at } A}$

noise power

$= KT\Delta f$

$= 1.38\times10^{-23} \times 373 \times 25\times10^{6}$

$= 128.69\ fW$

$\boxed{\text{at } B}$

noise power

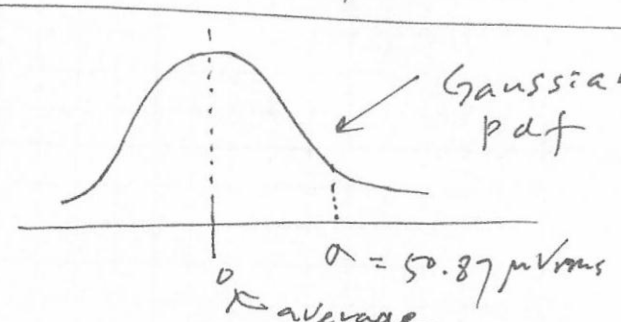
$= KT\Delta f$

$= 1.38\times10^{-23} \times 194 \times 25\times10^{6}$

$= 66.93\ fW$

$\therefore$ 61.76 fW of net power flow from A to B.

2)



normalized random variable for 88 μVrms

$z = \frac{88\,\mu V_{rms} - 0}{50.87\,\mu V_{rms}}$

$= 1.7299 \approx 1.73$

→ Probability ($z \le 1.73$)

$= 0.9582$

→ probability ($z > 1.73$)

$= 1 - 0.9582 = 0.0418$

→ probability ($z < -1.73$)

$= 0.0418$

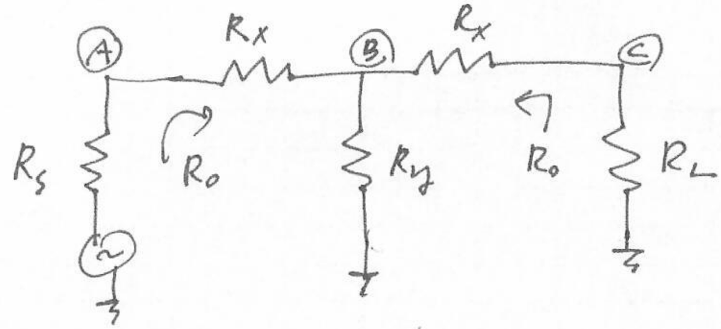
$\therefore$ Probability for $\overline{v_o} > 88\,\mu V$

$=$ Probability ($z > 1.73$)

$+$ Probability ($z < -1.73$)

$= 0.0836$

$= 8.36\%$



$R_S = R_L = R_o$

1) $R_S \longrightarrow \overline{V_{R_S}} = \sqrt{4KTR_S\Delta f}$

$\Rightarrow V_{A} = \frac{1}{2}\overline{V_{R_S}}$

$\Rightarrow V_{B} = \dfrac{R_o - R_X}{R_X + (R_o - R_X)} \cdot V_{A} = \dfrac{R_o - R_X}{R_o} \dfrac{1}{2} \overline{V_{R_S}}$

$\Rightarrow V_{C} = \dfrac{R_L}{R_X + R_L} V_{B} = \dfrac{R_o - R_X}{R_o + R_X} \dfrac{1}{2} \overline{V_{R_S}}$

$\therefore$ Noise power due to R_S across R_L is

$$P_L \Big|_{\text{due to } R_S} = \frac{V_{C}^2}{R_L} = \frac{\overline{V_{R_S}}^2}{4R_L}\left(\frac{R_o - R_X}{R_o + R_X}\right)^2 = KT\Delta f \frac{1}{L}$$

2) ① R_X at left side $\rightarrow \overline{V_{R_X}} = \sqrt{4KTR_X\Delta f}$

$\Rightarrow$ Follow the same steps in 1)

$\Rightarrow$ Noise power across R_L due to R_X at left side is

$$P_L \Big|_{\text{due to } R_X} = \frac{\overline{V_{R_X}}^2}{4R_o}\left(\frac{R_o - R_X}{R_o + R_X}\right)^2 = KT\Delta f \frac{R_X}{R_o} \cdot \frac{1}{L}$$

(2) R_x @ right side $\rightarrow \overline{V_{R_x}} = \sqrt{4kTR_x\Delta f}$

$\Rightarrow V_{Ⓒ}$ due to R_x at right side

$$= \frac{R_L}{R_0+R_L}\overline{V_{R_x}} = \frac{1}{2}\overline{V_{R_x}}$$

$\Rightarrow$ noise power across R_L due to R_x at right side

$$P_L\Big|_{\text{due to } R_x} = \frac{1}{R_L}\left(\frac{R_L}{R_0+R_L}\right)^2\overline{V_{R_x}}^2$$

$$= \frac{1}{4R_0}\overline{V_{R_x}}^2 = kT\Delta f\,\frac{R_x}{R_0}$$

(3) $R_y \longrightarrow \overline{V_{R_y}} = \sqrt{4kTR_y\Delta f}$

$$\Rightarrow V_{Ⓑ} = \frac{\frac{1}{2}(R_0+R_x)}{R_y+\frac{1}{2}(R_0+R_x)}\overline{V_{R_y}}$$

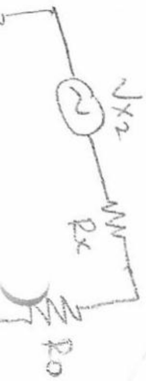
$$\Rightarrow V_{Ⓒ} = \frac{R_L \times V_{Ⓑ}}{R_x+R_L} = \frac{R_0}{R_x+R_0}\,\frac{R_0+R_x}{2R_y+R_0+R_x}\overline{V_{R_y}}$$

$$= \frac{R_0}{2R_y+R_0+R_x}\overline{V_{R_y}}$$

$\Rightarrow$ noise power across R_L due to R_y

$$P_L\Big|_{\text{due to } R_y} = \frac{1}{R_0}\left(\frac{R_0}{2R_y+R_0+R_x}\right)^2\overline{V_{R_y}}^2$$

$$= \frac{\overline{V_{R_y}}^2}{4R_0}\left(\frac{2R_0}{2R_y+R_0+R_x}\right)^2 = kT\Delta f\,\frac{R_y}{R_0}\left(\frac{2R_0}{2R_y+R_0+R_x}\right)^2$$



(3) Noise power at input is

$$P_{N,in} = KT\Delta f \quad \text{(Because attenuator is matched to source)}$$

From (2)

$$P_{Nout} = \frac{1}{R_0}\left\{\frac{\overline{V_s^2}}{4}\frac{(R_0-R_x)^2}{(R_0+R_x)^2} + \frac{\overline{V_{x1}^2}}{4}\frac{(R_0-R_x)^2}{(R_0+R_x)^2} + \frac{\overline{V_{x2}^2}}{4} + \frac{\overline{V_y^2}}{4}\frac{R_0^2}{\left(\frac{R_0+R_x}{2}+R_y\right)^2}\right\}$$

$$= \frac{1}{R_0}\left\{KT\Delta f\,\frac{\cancel{(R_0+R_x)}(R_0-R_x)^2}{(R_0+R_x)^{\cancel{2}}} + KT\Delta f\,R_x + KT\Delta f\,R_y\frac{R_0^2}{\left(\frac{R_0+R_x}{2}+R_y\right)^2}\right\}$$

$$\Rightarrow P_{N,out} = \frac{KT\Delta f}{R_0}\left[\frac{(R_0-R_x)^2}{R_0+R_x} + R_x + \frac{R_y R_0^2}{\left(\frac{R_0+R_x}{2}+R_y\right)^2}\right]$$

From $\quad R_0 = R_x + \dfrac{R_y(R_x+R_0)}{R_y+R_x+R_0} \quad \Rightarrow \quad R_0 = R_x\sqrt{1+\dfrac{2R_y}{R_x}}$

let $\quad R_z = \sqrt{1+\dfrac{2R_y}{R_x}} \quad \Rightarrow \quad R_0 = R_z R_0$

$$\Rightarrow \; = \left[\frac{(R_0-R_x)^2}{R_0+R_x} + R_x + \frac{4R_y R_0^2}{(R_0+R_x+2R_y)^2}\right]$$

$$= R_x\frac{(R_z-1)^2}{R_z+1} + R_x + \frac{\cancel{R_x^2}\,4R_y R_z^2}{\cancel{R_x^2}\left(R_z+\underbrace{1+\frac{2R_y}{R_x}}_{R_z^2}\right)^2}$$

$$= \frac{R_x(R_z-1)^2}{R_z+1} + R_x + \frac{4R_y}{(1+R_z)^2}$$

$$= \frac{R_x(R_z-1)^2}{R_z+1} + R_x + \frac{4R_y}{(1+R_z)^2}$$

$$= \frac{R_x(R_z-1)^2(R_z+1) + R_x(R_z+1)^2 + 4R_y}{(1+R_z)^2}$$

$$= \frac{R_x}{(1+R_z)^2}\left\{ R_z^3 - \cancel{R_z} - \cancel{R_z^2} + 1 + 1 + \cancel{R_z^2} + 2R_z + \frac{4R_y}{R_x} \right\}$$

Since $1 + 1 + \frac{4R_y}{R_x} = 2R_z^2$

$$= \frac{R_x}{(1+R_z)^2}\left\{ R_z^3 + 2R_z^2 + R_z \right\}$$

$$= \frac{R_x R_z}{\cancel{(1+R_z)^2}}\cancel{(R_z^2 + 2R_z + 1)} = R_x R_z$$

$$= R_0$$

$$\Rightarrow \quad P_{N,out} = \frac{kT\Delta f}{R_0} \cdot R_0 = \underline{kT\Delta f}$$

$$\Rightarrow \quad \underline{\underline{P_{N,in} = P_{N,out}}}$$

4) Noise factor

$$= \frac{\text{Total Noise power delivered to load}}{\text{output Noise power due to source noise only}}$$

$\rightarrow$ From 1), $P_L \big|_{\text{due to } R_S}$

$$= \frac{\overline{V_{RS}}^2}{4R_L} \left(\frac{R_o - R_X}{R_o + R_X} \right)^2$$

$$= KT\Delta f \frac{1}{L}$$

$$\therefore F = \frac{KT\Delta f}{KT\Delta f \frac{1}{L}} = L$$

5) $P_L \big|_{\text{due to } R_S} = KT\Delta f \left(\frac{R_o - R_X}{R_o + R_X} \right)^2$

$$= 1.38\times10^{-23} \times 300 \left(\frac{50 - 8.55}{50 + 8.55} \right)^2$$

$$= 2.07 \times 10^{-21} \text{ W/Hz} \quad \text{— (A)}$$

$$P_L \big|_{\text{due to left } R_X} = KT\Delta f \cdot \frac{R_X}{R_o} \left(\frac{R_o - R_X}{R_o + R_X} \right)^2 = 2.07 \times 10^{-21} \times \frac{R_X}{R_o}$$

$$= 354.8 \times 10^{-24} \text{ W/Hz} \quad \text{— (B)}$$

$$P_L \big|_{\text{due to right } R_X} = KT\Delta f \frac{R_X}{R_o}$$

$$= 1.38\times10^{-23} \times 300 \times \frac{8.55}{50}$$

$$= 707.94 \times 10^{-24} \text{ W/Hz} \quad \text{— (C)}$$

$$P_L \big|_{\text{due to } R_y} = KT\Delta f \frac{R_y}{R_o} \left(\frac{2R_o}{2R_y + R_o + R_X} \right)^2$$

$$= 1.38\times10^{-23} \times 300 \cdot \frac{141.9}{50} \left(\frac{2\times50}{2\times141.9 + 50 + 8.55} \right)^2$$

$$= 1 \times 10^{-21} \text{ W/Hz} \quad \text{— (D)}$$

- Total noise power

$$= A + B + C + D = \underline{4.13 \times 10^{-21}\ \mathrm{W/Hz}}$$

- ~~availavat~~ available source noise power

$$= KT\Delta f = 1.38 \times 10^{-23} \times 300$$

$$= \underline{4.14 \times 10^{-21}\ \mathrm{W/Hz}}$$

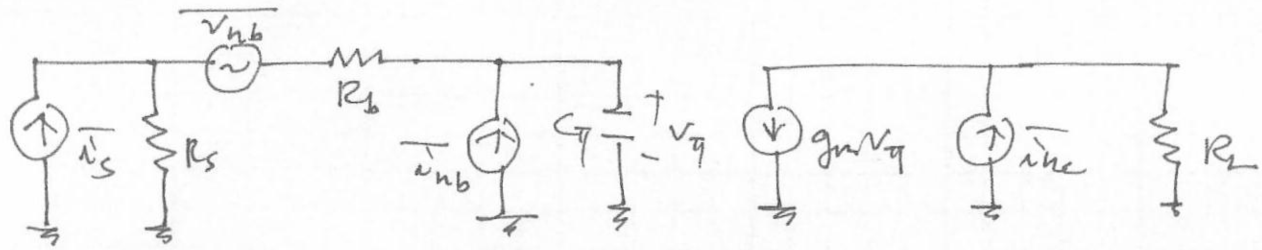
These results are same.
⇒ Noise power keeps constant.

$$F = \frac{4.13 \times 10^{-21}}{2.07 \times 10^{-21}} \simeq 2 \rightarrow 3\,\mathrm{dB}$$

$$L = \left(\frac{50 + 8.55}{50 - 8.55}\right)^2 \simeq 2 \longrightarrow 3\,\mathrm{dB}$$

$$\Rightarrow F \simeq L$$

HW-2 Solution problem-3



1) input referred noise voltage $(= \overline{v_{in}})$

$$\overline{v_{in}} = \overline{v_{nb}} + \overline{i_{nb}}\left(R_s+R_b\right) \,//\, \frac{1}{sC_\pi} + \frac{\overline{i_{nc}}}{g_m}\,\frac{1}{\dfrac{\frac{1}{sC_\pi}}{R_s+R_b+\frac{1}{sC_\pi}}}$$

$$\approx \overline{v_{nb}} + \overline{i_{nb}}\left(R_s+R_b\right) + \frac{\overline{i_{nc}}}{g_m}$$

$$\left.\begin{aligned} \overline{v_{nb}} &= 4KTR_b\Delta f \\ \overline{i_{nb}} &= 2qI_B\Delta f \\ \overline{i_{nc}} &= 2qI_C\Delta f \\ g_m &= \frac{I_C}{V_T} \end{aligned}\right\} \text{apply}$$

$\Rightarrow$ input referred noise power

$$\overline{v_{in}^2} = 4KTR_b\Delta f + 2qI_B\left(R_s+R_b\right)^2\Delta f + 2qI_C\left(\frac{V_T}{I_C}\right)^2\Delta f$$

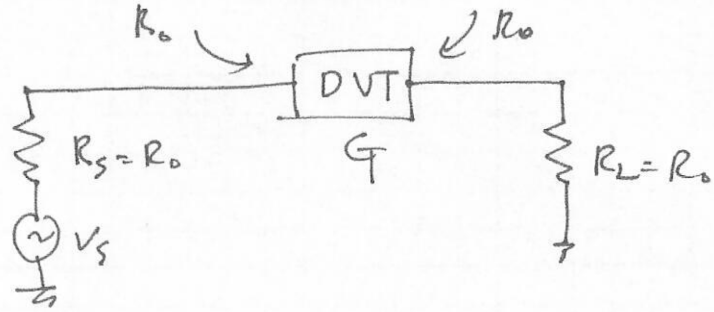
2) Noise factor

$$F = 1 + \frac{4KTR_b\Delta f + 2qI_B\left(R_s+R_b\right)^2\Delta f + 2qI_C\left(\frac{V_T}{I_C}\right)^2\Delta f}{4KTR_s\Delta f}$$

$$= 1 + \frac{R_b}{R_s} + \frac{\left(R_s+R_b\right)^2}{R_s}\,\frac{1}{2}\,\frac{I_B}{V_T} + \frac{1}{R_s}\,\frac{1}{2}\,\frac{V_T}{I_C}$$

HW-2 Solution   Problem-4

NF Measurement (1)



– step 1) $V_S = 0$

$\Rightarrow$ output noise power ($= P_{on}$)

$$P_{on} = kT\Delta f \cdot G + \underbrace{\text{DUT noise power}}$$

$$= kT\Delta f \cdot G + P_{on,DUT}$$

– step 2) Increase V_S until out power becomes $2P_{on}$.

$$\Rightarrow 2P_{on} = \underbrace{\frac{V_S^2}{4R_o} \cdot G}_{\substack{\text{output power}\\\text{due to applied}\\\text{source, } V_S}} + \underbrace{kT\Delta f\, G}_{\substack{\text{output power}\\\text{due to noise}\\\text{in } R_S}} + \underbrace{P_{on,DUT}}_{\substack{\text{DUT output}\\\text{noise power}}}$$

$$= \frac{V_S^2}{4R_o} G + P_{on}$$

$$\therefore P_{on} = \frac{V_S^2}{4R_o} G \Rightarrow P_{on,DUT} = P_{on} - kT\Delta f \cdot G$$

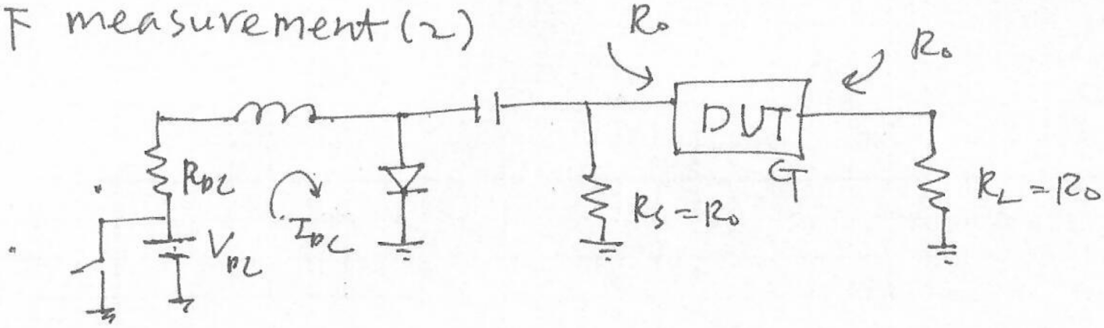
$$= \frac{V_S^2}{4R_o} G - kT\Delta f\, G$$

From step-1) and step-2),

$$F = 1 + \frac{P_{on,DUT}}{kT\Delta f \cdot G} = \frac{\frac{V_S^2}{4R_o} \cdot G}{kT\Delta f \cdot G} = \frac{\frac{V_S^2}{4R_o}}{kT\Delta f}$$

Note: you need to know Δf in this method.

NF measurement (2)



- Step-1) Turn-on switch $\rightarrow$ $I_{DC} = 0$

  $\Rightarrow$ output noise power $(= P_{on})$

$$P_{on} = \underbrace{KT\Delta f \cdot G}_{\text{Noise power from } R_s} + \underbrace{P_{on,DUT}}_{\text{Noise power from DUT}}$$

- Step-2) Turn-off switch and adjust V_{DC} until output power becomes $2 \cdot P_{on}$

$$\Rightarrow 2P_{on} = \underbrace{2qI_{DC}\Delta f\,(R_s // R_o)^2 \cdot G}_{\text{output noise power due to diode noise}}$$

$$+ KT\Delta f\, G + P_{on,DUT}$$

$$= \frac{1}{2} q I_{DC} \Delta f R_s^2 G + P_{on}$$

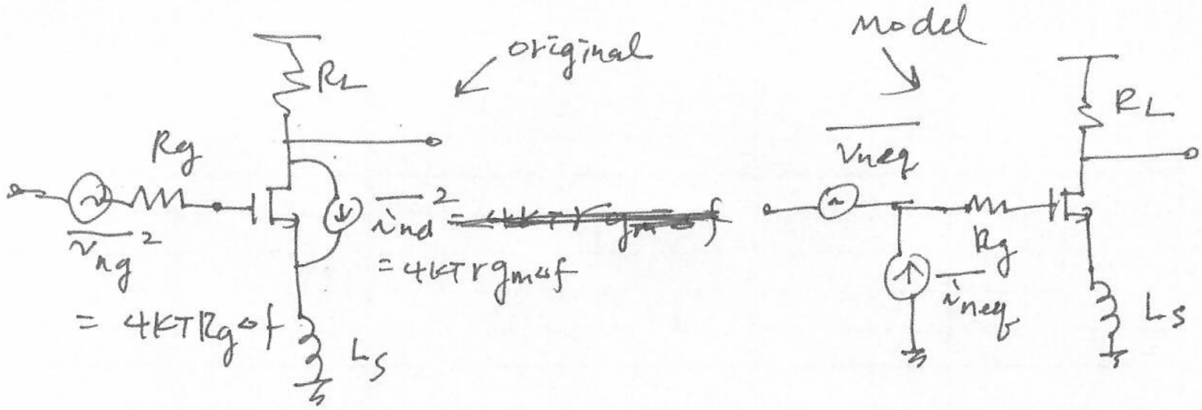
$$\therefore P_{on} = \frac{1}{2} q I_{DC} \Delta f R_s^2 G$$

$$\Rightarrow P_{on,DUT} = q I_{DC} \Delta f R_s^2 G - KT\Delta f \cdot G$$

From step-1) and step-2),

$$\text{Noise factor, } F = 1 + \frac{P_{on,DUT}}{KT\Delta f \cdot G} = \frac{1}{2}\frac{q I_{DC} R_s^2}{KT} = \frac{I_{DC}}{2V_T} \cdot R_s^2$$

Note: In this method, you don't need to know bandwidth of the DUT.



I) Equivalent input noise generators.

① Computing $\overline{v_{neq}}$ ⇒ short input to ground.

From original circuit, output noise ~~voltage~~ current is

$$\overline{i_{on}} = \overline{v_{ng}} \cdot \frac{g_m}{1+g_m \cdot SL_s} + \overline{i_{nd}} \frac{\frac{1}{g_m}}{\frac{1}{g_m}+SL_s}$$

$$= \frac{1}{1+g_m \cdot SL_s}\left(\overline{v_{ng}} \cdot g_m + \overline{i_{nd}}\right) \quad \text{— (A)}$$

From model, output noise current is

$$\overline{i_{on}} = \overline{v_{neq}} \frac{g_m}{1+g_m \cdot SL_s} \quad \text{— (B)}$$

⇒ From (A) and (B),

$$\overline{v_{neq}} = \overline{v_{ng}} + \frac{\overline{i_{nd}}}{g_m}$$

2 Computing $\overline{i_{neq}}$ ⇒ open input node.

From original circuit, output noise current is

$$\overline{i_{on}} = \overline{i_{nd}} \quad \text{— A}$$

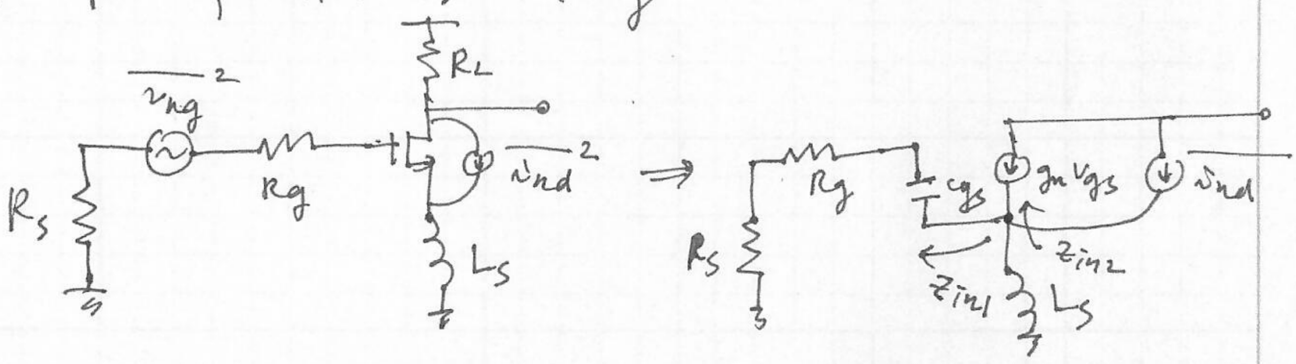
From model, output noise current is

$$\overline{i_{on}} = \overline{i_{neq}} \cdot \frac{1}{sC_{gs}} \cdot g_m \quad \text{— B}$$

⇒ From A and B

$$\overline{i_{neq}} = \overline{i_{nd}} \cdot \frac{sC_{gs}}{g_m} = \overline{i_{nd}} \frac{j\omega C_{gs}}{g_m}$$

2) Input referred noise voltage



From equivalent circuit shown in ~~left~~ right side,

$$Z_{in1} = \frac{1}{sC_{gs}} + R_g + R_s \simeq \frac{1}{sC_{gs}} + R_s$$

$$Z_{in2} = \frac{\frac{1}{sC_{gs}} + R_s + R_g}{\frac{g_m}{sC_{gs}}} \simeq \frac{1 + sC_{gs}R_s}{g_m}$$

⇒ Output noise current due to $\overline{i_{nd}}$ is

$$i_{no}\Big|_{\text{due to } \overline{i_{nd}}} = \frac{Z_{in2}}{Z_{in1} \,//\, sL_s + Z_{in2}} \cdot \overline{i_{nd}}$$

Therefore equivalent input noise voltage to create this noise current is

$$\overline{v_{neq}} \cdot \frac{\frac{g_m}{sC_{gs}}}{R_s + R_g + \frac{1}{sC_{gs}} + \left(1 + \frac{g_m}{sC_{gs}}\right) \cdot sL_s} = i_{no}\Big|_{\text{due to } \overline{i_{nd}}}$$

$$\Rightarrow \overline{v_{neq}} = \frac{\left(\frac{Z_{in2}}{Z_{in1} // sL_s + Z_{in2}} \cdot \overline{i_{nd}}\right)}{\left(\frac{\frac{g_m}{sC_{gs}}}{R_s + R_g + \frac{1}{sC_{gs}} + \left(1 + \frac{g_m}{sC_{gs}}\right) \cdot sL_s}\right)}$$

if, $|Z_{in1}| \gg \omega L_s$

$$= \frac{\frac{1 + sC_{gs}R_s}{g_m sL_s + 1 + sC_{gs}R_s}}{\frac{\frac{g_m}{sC_{gs}}}{R_s + \frac{g_m L_s}{C_{gs}} + \frac{1}{sC_{gs}}}} \cdot \overline{i_{nd}}$$

if $|1 + j\omega C_{gs} \cdot R_s| \gg g_m \cdot \omega L_s$

$$\simeq \frac{1}{g_m}\left(1 + sC_{gs}\left(R_s + \frac{g_m L_s}{C_{gs}}\right)\right) \overline{i_{nd}}$$

$\therefore$ Input referred noise voltage, $\overline{V_{n,in}}$

$$\overline{V_{n,in}} = \overline{V_{ng}} + \overline{V_{neq}}$$

$$\approx \overline{V_{ng}} + \overline{i_{nd}} \cdot \frac{1}{g_m}\left(1 + sC_{gs}\left(R_s + \frac{g_m L_s}{C_{gs}}\right)\right)$$

3) correlation admittance

$$Y_c = \frac{\overline{V_{neq}^{*}}\,\overline{i_{neq}}}{\overline{V_{neq}^{2}}}$$

$$\approx jwC_{gs}$$ ← same as without L_s (see lecture note, page-65)

~~4)~~ Uncorrelate noise current

$$\overline{i_{nu}} = \overline{i_{neq}} - Y_c \overline{V_{neq}}$$

$$= -jwC_{gs}\overline{V_{ng}}$$ ← see lecture note, page-65

correlated noise current

$$\overline{i_{nc}} = \overline{i_{neq}} - \overline{i_{nu}}$$

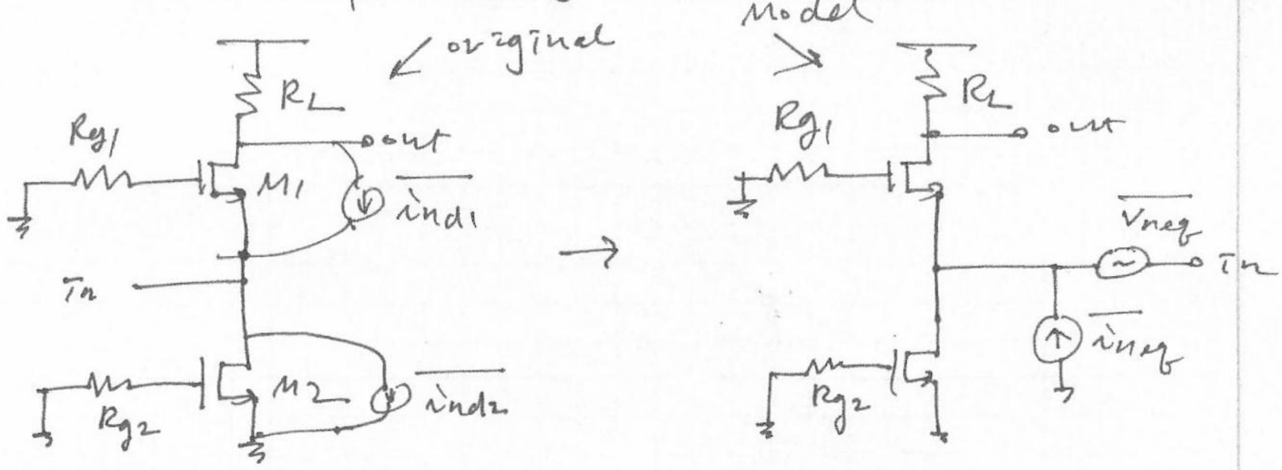
4) optimum admittance

$$Y_{opt} = wC_{gs}\sqrt{\frac{g_m R_g}{r}} - jwC_{gs}$$ ← see lecture note page-66

5) Fmin

$$F_{min} = 1 + 2wC_{gs}\sqrt{r\frac{R_g}{g_m}}$$ ← see lecture note page-67.

Think about why these results are same as in the case of without Source Inductor



1) Input noise generators

① computing $\overline{V_{neq}}$ ⇒ short input to ground

~~$\overline{i_{on}}$~~ For original circuit, output noise current is

$$\overline{i_{on}} = \overline{i_{nd1}} + g_{m1}\overline{V_{ng1}} \quad — (A)$$

For model circuit, output noise current is

$$\overline{i_{on}} = g_{m1}\overline{V_{neq}} \quad — (B)$$

⇒ From (A) and (B),

$$\overline{V_{neq}} = \frac{1}{g_{m1}}\overline{i_{nd1}} + \overline{V_{ng1}}$$

② computing $\overline{i_{neq}}$ ⇒ open input node

For original circuit,

$$\overline{i_{on}} = \overline{i_{nd2}} + g_{m2}\overline{V_{ng2}} \quad — (A)$$

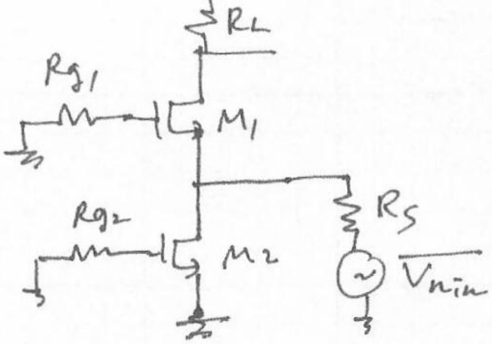
For model circuit,

$$\overline{i_{on}} = \overline{i_{neq}} \quad — (B)$$

⇒ From (A) and (B)

$$\overline{i_{neq}} = \overline{i_{nd2}} + g_{m2}\overline{V_{ng2}}$$

2) Input referred noise voltage, $\overline{v_{nin}}$



$$\underbrace{\underbrace{\overline{v_{nin}} \cdot \frac{\frac{1}{g_{m1}}}{R_S + \frac{1}{g_{m1}}}}_{\text{voltage across } c_{gs} \text{ of } M_1} \cdot g_{m1}}_{\text{equivalent noise current in model circuit}} = \underbrace{\overline{i_{nd1}} \frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}} + R_S} + \overline{v_{ng1}} \frac{g_{m1}}{1+g_{m1}R_S}}_{\text{effective output noise current in original circuit}}$$

$$\therefore \overline{v_{nin}} = \overline{i_{nd1}} \cdot \frac{1}{g_{m1}} + \overline{v_{ng1}}$$

3) Correlation admittance

$$\overline{v_{neq}} = \frac{1}{g_{m1}} \overline{i_{nd1}} + \overline{v_{ng1}}$$

$$\overline{i_{neq}} = \overline{i_{nd2}} + g_{m2} \overline{v_{ng2}}$$

$$Y_C = \frac{\overline{v_{neq}}^{*} \, \overline{i_{neq}}}{\overline{v_{neq}}^{2}} = \frac{0}{\left(\frac{1}{g_{m1}}\right)^2 \overline{i_{nd1}}^{2}} = 0$$

$$\therefore \overline{i_{nc}} = 0$$

$$\therefore \overline{i_{nu}} = \overline{i_{neq}} = \overline{i_{nd2}} + g_{m2} \overline{v_{ng2}}$$

4) optimum admittance

$$R_n = \frac{\overline{V_{neq}^2}}{4KT\Delta f} = \frac{\left(\frac{1}{g_{m1}}\right)^2 4KT\gamma g_{m1}\Delta f + 4KTR_{g1}\Delta f}{4KT\Delta f}$$

$$= \frac{\gamma}{g_{m1}} + R_g$$

$$G_u = \frac{\overline{i_{nu}^2}}{4KT\Delta f} = \frac{4KT\gamma g_{m2}\Delta f + g_{m2}^2 \cdot 4KTR_{g2}\Delta f}{4KT\Delta f}$$

$$= \gamma g_{m2} + g_{m2}^2 R_{g2}$$

$$= g_{m2}(\gamma + g_{m2}R_{g2})$$

$$\therefore G_{s,opt} = \sqrt{\frac{G_u}{R_n}} = \sqrt{\frac{g_{m2}(\gamma + g_{m2} \cdot R_{g2})}{\frac{\gamma}{g_{m1}} + R_{g1}}}$$

$$= \sqrt{\frac{g_{m1} \cdot g_{m2}\left(1 + \frac{1}{\gamma} g_{m2} \cdot R_{g2}\right)}{1 + \frac{1}{\gamma} g_{m1} R_{g1}}}$$

$$B_{s,opt} = 0$$

$$\Rightarrow Y_{s,opt} = G_{s,opt}$$

5) $$F_{min} = 1 + 2R_n\sqrt{\frac{G_u}{R_n}}$$

$$= 1 + 2\left(\frac{\gamma}{g_{m1}} + R_g\right)\sqrt{\frac{g_{m1} g_{m2}\left(1 + \frac{1}{\gamma} g_{m2} \cdot R_{g2}\right)}{1 + \frac{1}{\gamma} g_{m1} R_{g1}}}$$

if, $g_{m1} = g_{m2}$ and $R_{g2} \simeq R_{g1}$, then $F_{min} \simeq$

$$\simeq 1 + 2(\gamma + g_m R_g)$$

$$\simeq 1 + 2\gamma$$