

1) Under mismatched conditions,

$$Z_s = R_s + jX_s$$

$$Z_L = (R_s + \Delta R_s) - j(X_s + \Delta X_s)$$

$$\Rightarrow V_L = \frac{Z_L}{Z_s + Z_L} V_s$$

$$= \frac{R_s + \Delta R_s - j(X_s + \Delta X_s)}{2R_s + \Delta R_s - j\Delta X_s} V_s$$

$$\Rightarrow i_L = \frac{1}{2R_s + \Delta R_s - j\Delta X_s} V_s$$

- Voltage across  $R_L \Rightarrow V_{RL} = \frac{R_s + \Delta R_s}{2R_s + \Delta R_s - j\Delta X_s} V_s$

- Current through  $R_L \Rightarrow i_{RL} = \frac{1}{2R_s + \Delta R_s - j\Delta X_s} V_s$

$$\therefore P_{RL} = \operatorname{Re}(V_{RL} \cdot i_{RL}^*)$$

$$= \frac{(R_s + \Delta R_s) V_s^2}{(2R_s + \Delta R_s)^2 + (\Delta X_s)^2}$$

$$= \boxed{\frac{V_s^2}{4R_s} \frac{\left(1 + \frac{\Delta R_s}{R_s}\right)}{\left(1 + \frac{\Delta R_s}{2R_s}\right)^2 + \left(\frac{\Delta X_s}{2R_s}\right)^2}} \quad (\text{Ans})$$

(if  $V_s$  is peak value, then

$$P_{RL} = \boxed{\frac{1}{2} \cdot \frac{V_s^2}{4R_s} \frac{\left(1 + \frac{\Delta R_s}{R_s}\right)}{\left(1 + \frac{\Delta R_s}{2R_s}\right)^2 + \left(\frac{\Delta X_s}{2R_s}\right)^2}} \quad (\text{Ans})$$

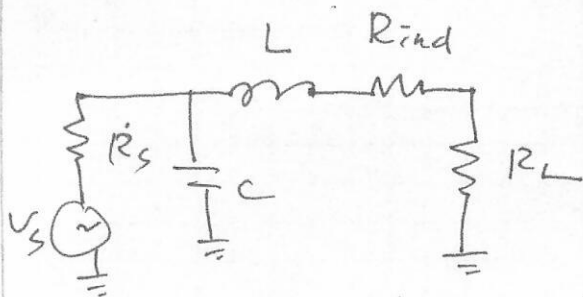
For both cases

$$P_L = P_{max} \frac{1 + \frac{\Delta R_s}{R_s}}{\left(1 + \frac{\Delta R_s}{2R_s}\right)^2 + \left(\frac{\Delta X_s}{2R_s}\right)^2}$$

$$2) \frac{P_L}{P_{max}} = \frac{1 + \frac{\Delta R_s}{R_s}}{\left(1 + \frac{\Delta R_s}{2R_s}\right)^2 + \left(\frac{\Delta X_s}{2R_s}\right)^2} \quad (\text{Ans})$$

$$3) \text{ (if) } \Delta X_s = 0$$

$\frac{\Delta R_s}{R_s}$	$\frac{P_L}{P_{max}}$
5% <del>(0.05)</del>	
$\rightarrow \frac{\Delta R_s}{R_s} = 0.05$	$\rightarrow 0.9994 \rightarrow 99.94\%$
$\rightarrow \frac{\Delta R_s}{R_s} = -0.05$	$\rightarrow 0.9993 \rightarrow 99.93\%$
10%	
$\rightarrow \frac{\Delta R_s}{R_s} = 0.1$	$\rightarrow 0.9977 \rightarrow 99.77\%$
$\rightarrow \frac{\Delta R_s}{R_s} = -0.1$	$\rightarrow 0.9972 \rightarrow 99.72\%$
20%	
$\rightarrow \frac{\Delta R_s}{R_s} = 0.2$	$\rightarrow 0.9917 \rightarrow 99.17\%$
$\rightarrow \frac{\Delta R_s}{R_s} = -0.2$	$\rightarrow 0.9877 \rightarrow 98.77\%$
50%	
$\rightarrow \frac{\Delta R_s}{R_s} = 0.5$	$\rightarrow 0.96 \rightarrow 96\%$
$\rightarrow \frac{\Delta R_s}{R_s} = -0.5$	$\rightarrow 0.8888 \rightarrow 88.88\%$



Under matched conditions

$$1) V_L = \frac{1}{2} V_s \frac{R_L}{SL + R_{ind} + R_L}$$

$$\hat{V}_L = \frac{1}{2} V_s \frac{1}{SL + R_{ind} + R_L}$$

$$\therefore P_L = \operatorname{Re}(V_L \cdot I_L^*) = \frac{V_s^2}{4R_L} \frac{1}{\left(1 + \frac{R_{ind}}{R_L}\right)^2 + \left(\frac{\omega_0 L}{R_L}\right)^2}$$

$$\frac{R_s}{R_L} = Q_T^2 + 1$$

$$\frac{R_{ind}}{R_L} = \frac{Q_T}{Q_{ind}}$$

$$\frac{\omega_0 L}{R_L} = Q_T$$

apply

$$= \frac{V_s^2}{4R_s} \cdot \frac{R_s}{R_L} \frac{1}{\left(1 + \frac{R_{ind}}{R_L}\right)^2 + \left(\frac{\omega_0 L}{R_L}\right)^2}$$

$$= \frac{V_s^2}{4R_s} (Q_T^2 + 1) \frac{1}{\left(1 + \frac{Q_T}{Q_{ind}}\right)^2 + Q_T^2}$$

$$2) \hat{V}_{ind} = \frac{1}{2} V_s \frac{R_{ind}}{SL + R_{ind} + R_L}, \quad \hat{V}_{ind} = \hat{V}_L = \frac{1}{2} V_s \frac{1}{SL + R_{ind} + R_L}$$

$$\therefore P_{ind} = \operatorname{Re}(V_{ind} \cdot I_{ind}^*) = \frac{V_s^2}{4R_L} \frac{R_{ind}/R_L}{\left(1 + \frac{R_{ind}}{R_L}\right)^2 + \left(\frac{\omega_0 L}{R_L}\right)^2}$$

$$= \frac{V_s^2}{4R_s} \cdot \frac{R_s}{R_L} \cdot \frac{R_{ind}}{R_L} \frac{1}{\left(1 + \frac{R_{ind}}{R_L}\right)^2 + \left(\frac{\omega_0 L}{R_L}\right)^2}$$

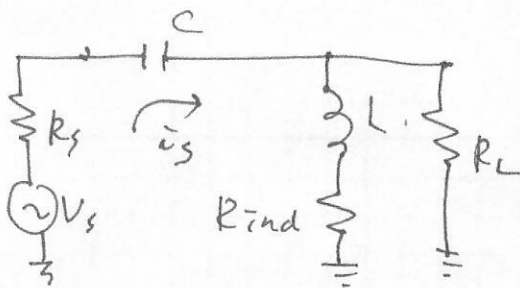
$$= \frac{V_s^2}{4R_s} (Q_T^2 + 1) \frac{Q_T}{Q_{ind}} \frac{1}{\left(1 + \frac{Q_T}{Q_{ind}}\right)^2 + Q_T^2}$$

3) 
$$\eta = \frac{P_L}{P_L + P_{ind}} = \frac{1}{1 + \frac{Q_T}{Q_{ind}}} = \frac{Q_{ind}}{Q_T + Q_{ind}}$$

4) ~~same as 3)~~ or

$$\eta = \frac{1}{1 + \frac{R_{ind}}{R_L}} = \frac{R_L}{R_{ind} + R_L}$$

4) ~~for~~ 
$$\eta = \frac{Q_{ind}}{Q_T + Q_{ind}}$$



Under matched conditions,

$$\hat{i}_s = \frac{V_s}{2R_s}$$

$$1) \bar{V}_{RL} = \frac{V_s}{2R_s} \cdot \frac{SL + R_{ind}}{SL + R_{ind} + R_L}$$

$$V_{RL} = \hat{i}_{RL} \cdot R_L$$

$$\therefore P_{RL} = \text{Re}(V_{RL} \cdot \hat{i}_{RL}^*) = R_L \cdot \hat{i}_{RL} \cdot \hat{i}_{RL}^*$$

$$= \frac{V_s^2}{4R_s^2} \cdot R_L \frac{R_{ind}^2 + \omega_0^2 L^2}{(R_L + R_{ind})^2 + \omega_0^2 L^2}$$

$$= \frac{V_s^2}{4R_s} \cdot \frac{R_L}{R_s} \frac{(R_{ind}/R_L)^2 + (\omega_0 L/R_L)^2}{(1 + R_{ind}/R_L)^2 + (\omega_0 L/R_L)^2}$$

$$\frac{R_L}{R_s} = Q_T^2 + 1$$

$$\frac{\omega_0 L}{R_L} = \frac{1}{Q_T}$$

$$\frac{R_{ind}}{R_L} = \frac{1}{Q_T \cdot Q_{ind}}$$

apply

$$= \frac{V_s^2}{4R_s} (Q_T^2 + 1) \frac{\left(\frac{1}{Q_T \cdot Q_{ind}}\right)^2 + \left(\frac{1}{Q_T}\right)^2}{\left(1 + \frac{1}{Q_T \cdot Q_{ind}}\right)^2 + \left(\frac{1}{Q_T}\right)^2}$$

$$= \frac{V_s^2}{4R_s} (Q_T^2 + 1) \frac{1 + Q_{ind}^2}{(1 + Q_T \cdot Q_{ind})^2 + Q_{ind}^2}$$



$$2) \hat{I}_{R_{ind}} = \frac{V_s}{2R_s} \cdot \frac{R_L}{sL + R_{ind} + R_L}$$

$$V_{R_{ind}} = \hat{I}_{R_{ind}} \cdot R_{ind}$$

$$\therefore P_{ind} = R_L (V_{R_{ind}} \cdot \hat{I}_{R_{ind}}^*)$$

$$= \frac{V_s^2}{4R_s^2} \cdot R_{ind} \cdot \frac{R_L}{(R_{ind} + R_L)^2 + \omega_0^2 L^2}$$

$$= \frac{V_s^2}{4R_s} \cdot \frac{R_L}{R_s} \cdot \frac{R_{ind}/R_L}{\left(1 + \frac{R_{ind}}{R_L}\right)^2 + \left(\frac{\omega_0 L}{R_L}\right)^2}$$

$$= \frac{V_s^2}{4R_s} (Q_T^2 + 1) \cdot \frac{\frac{1}{Q_T \cdot Q_{ind}}}{\left(1 + \frac{1}{Q_T \cdot Q_{ind}}\right)^2 + \left(\frac{1}{Q_T}\right)^2}$$

$$= \frac{V_s^2}{4R_s} (Q_T^2 + 1) \cdot \frac{Q_T \cdot Q_{ind}}{(1 + Q_T \cdot Q_{ind})^2 + Q_{ind}^2}$$

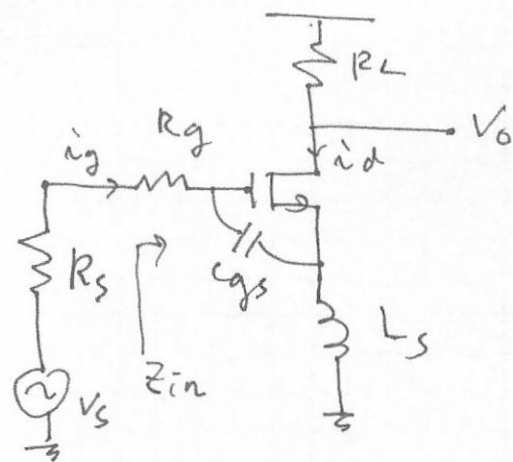
$$3) \eta = \frac{P_L}{P_L + P_{ind}} = \frac{\left(\frac{R_{ind}}{R_L}\right)^2 + \left(\frac{\omega_0 L}{R_L}\right)^2}{\left(\frac{R_{ind}}{R_L}\right)^2 + \left(\frac{\omega_0 L}{R_L}\right)^2 + R_{ind}/R_L}$$

$$= \frac{1 + Q_{ind}^2}{1 + Q_{ind}^2 + Q_T \cdot Q_{ind}}$$

if  $Q_{ind} \gg 1$

$$\approx \frac{Q_{ind}}{Q_{ind} + Q_T} \leftarrow \text{same as problem-2.}$$

$$4) \eta = \frac{Q_{ind}}{Q_{ind} + Q_T}$$



current gain,  $\beta$

$$\beta = \frac{\hat{i}_d}{\hat{i}_g} = \frac{g_m}{s C_{gs}}$$

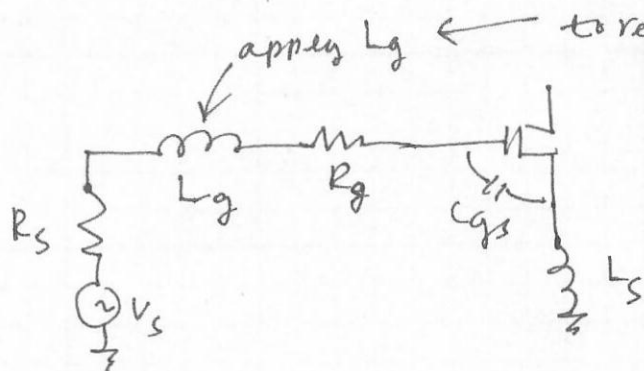
$$\begin{aligned} 1) \quad Z_{in} &= R_g + \frac{1}{s C_{gs}} + (1 + \beta) s L_s \\ &= R_g + \frac{1}{s C_{gs}} + \left(1 + \frac{g_m}{s C_{gs}}\right) s L_s \\ &= R_g + \frac{1}{s C_{gs}} + s L_s + \frac{g_m L_s}{C_{gs}} \end{aligned}$$

$$\begin{aligned} 2) \quad V_{gs} &= \hat{i}_g \cdot \frac{1}{s C_{gs}} \\ &= \frac{V_s}{R_s + Z_{in}} \cdot \frac{1}{s C_{gs}} = \frac{\frac{1}{s C_{gs}}}{R_s + \left(R_g + \frac{g_m L_s}{C_{gs}}\right) + s L_s + \frac{1}{s C_{gs}}} \cdot V_s \end{aligned}$$

$$\begin{aligned} i_{out} &= g_m V_{gs} = \hat{i}_g \cdot \beta \\ &= \frac{g_m / s C_{gs}}{R_s + \left(R_g + \frac{g_m L_s}{C_{gs}}\right) + s L_s + \frac{1}{s C_{gs}}} \cdot V_s \end{aligned}$$

$$3) \quad A_V = \frac{V_{out}}{V_s} = \frac{-i_{out} \cdot R_L}{V_s} = - \frac{g_m / s C_{gs} \cdot R_L}{\left(R_s + R_g + \frac{g_m L_s}{C_{gs}}\right) + s L_s + \frac{1}{s C_{gs}}}$$

$$4) \quad Z_{in} = \underbrace{R_g + \frac{g_m L_s}{s c_{gs}}}_{= R_s} + \underbrace{s L_s + \frac{1}{s c_{gs}}}_{\text{Since } \omega_0 L_s < \frac{1}{\omega_0 c_{gs}}, \text{ we need inductance to resonate the capacitance.}}$$



$$\omega_0 (L_g + L_s) = \frac{1}{\omega_0 c_{gs}}$$

$$\therefore L_g = \frac{1}{\omega_0^2 c_{gs}} - L_s$$

5) Under matched condition

$$\hat{i}_g = \frac{V_s}{2R_s} \rightarrow V_{gs} = \hat{i}_g \frac{1}{s c_{gs}} = \frac{1}{2R_s s c_{gs}} \cdot V_s$$

$$\rightarrow \hat{i}_{out} = g_m V_{gs} = \hat{i}_g \cdot \beta$$

$$= \frac{V_s}{2R_s} \cdot \frac{g_m}{s c_{gs}}$$

$$6) \quad A_v = \frac{V_{out}}{V_s} = \frac{-\hat{i}_{out} \cdot R_L}{V_s} = - \frac{1}{2R_s} \cdot \frac{g_m}{s c_{gs}} \cdot R_L$$

$\Rightarrow$  Under matched condition, gain will be increased by a factor of  $\left(1 + \frac{s L_s + \frac{1}{s c_{gs}}}{2R_s}\right)$ .

$$\Rightarrow 1 + \frac{s L_s + \frac{1}{s c_{gs}}}{2R_s} = 1 - \frac{s L_g}{2R_s} = 1 - j \frac{1}{2} \frac{\omega_0 L_g}{R_s} \approx 1 - j \frac{1}{2} Q$$

$\therefore$  if  $Q$  is large enough, the gain factor is  $\frac{1}{2} Q$ , where  $Q$  is quality factor of matching circuit.

7)  $L_s$  provides real impedance for matching.