

# Fundamental Relations and Terminology

## Major Forms of the Radar Range Equation

Peak power form, single pulse	$SNR = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 k T_0 B F L_s R^4}$		
Search form:	$\frac{P_{avg} A_e}{L_s T_0 F} \geq SNR_{min} 4\pi k \left( \frac{R^4}{\sigma} \right) \left( \frac{\Omega}{T_{fs}} \right)$		
Track form:	$\frac{P_{avg} A_e^3 k_m^2}{\lambda^4 L_s T_0 F} = \left( \frac{\pi^2}{2} \right) \left( \frac{k r N_t R^4}{\sigma \cdot \sigma_\theta^2} \right) \left( \frac{1}{\cos^5(\theta_{scan})} \right)$		
<b>Definition of Terms:</b>			
$SNR$	Signal-to-noise ratio	$B$	Receiver bandwidth
$SNR_{min}$	Minimum detectable SNR	$L_s$	System losses
$P_t$	Peak transmitted power	$F$	Noise figure
$P_{avg}$	Average transmitted power	$k$	Boltzmann's constant
$G_t$	Transmit antenna gain	$\Omega$	Search area solid angle
$G_r$	Receive antenna gain	$T_{fs}$	Frame search time
$A_e$	Effective aperture	$k_m$	Track measurement error slope
$\lambda$	Wavelength	$r$	Track measurement rate
$\sigma$	Target radar cross section	$N_i$	Number of targets
$R$	Range to target	$\sigma_\theta$	Track angle estimate precision (std. dev.)
$T_0$	Standard temperature (270 K)	$\theta_{scan}$	Scan angle, electronically-scanned array

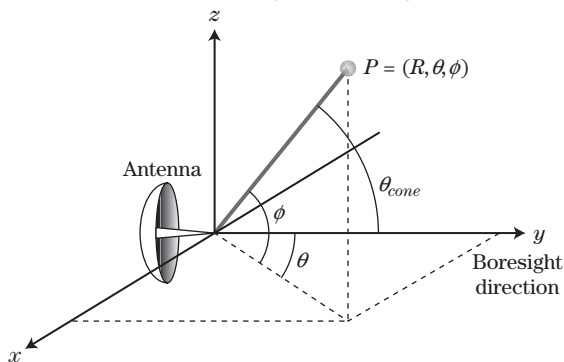
## Radar Bands

Band	Frequency Range	ITU Radar Frequency
High frequency (HF)	3–30 MHz	
Very high frequency (VHF)	30–300 MHz	138–144 MHz 216–225 MHz
Ultra high frequency (UHF)	300 MHz–1 GHz	420–450 MHz 890–942 MHz
L	1–2 GHz	1.215–1.400 GHz
S	2–4 GHz	2.3–2.5 GHz 2.7–3.7 GHz
C	4–8 GHz	5.250–5.925 GHz
X	8–12 GHz	8.500–10.680 GHz
Ku (“under” K-band)	12–18 GHz	13.4–14.0 GHz 15.7–17.7 GHz
K	18–27 GHz	24.05–24.25 GHz 24.65–24.75 GHz
Ka (“above” K-band)	27–40 GHz	33.4–36.0 GHz
V	40–75 GHz	59.0–64.0 GHz
W	75–110 GHz	76.0–81.0 GHz 92.0–100.0 GHz
mm	100–300 GHz	126.0–142.0 GHz 144.0–149.0 GHz 231.0–235.0 GHz 238.0–248.0 GHz

## Time Delay

A time delay of ...	... is approximately equivalent to a range of ...
1 nanosecond (ns)	0.15 meters (m)
	15 centimeters (cm)
	0.5 feet (ft)
	6 inches (in)
1 microsecond ( $\mu$ s)	0.15 km
	150 meters (m)
	0.1 (0.093) miles
	500 (492) feet (ft)

## Definition of Azimuth, Elevation, and Cone Angles




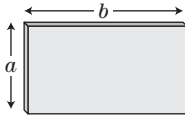
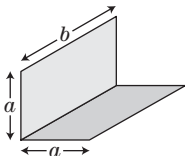
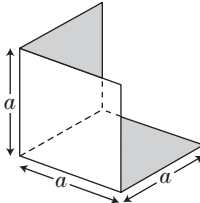
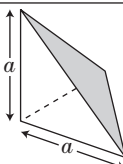
## Antenna Directivity, Gain, and Beamwidth

Maximum Directivity $D_{max}$ and Gain $G$
$D_{max} \approx \begin{cases} \frac{4\pi}{\theta_3 \phi_3} \alpha^2, & \theta_3, \phi_3 \text{ in radians} \\ \frac{129,600}{\pi^2 \theta_3 \phi_3} \alpha^2, & \theta_3, \phi_3 \text{ in degrees} \end{cases}$
Gain $G$ (dB) = $D_{max}$ (dB) – antenna losses (dB)
3 dB Beamwidth $\theta_3$
$\theta_3 \approx \begin{cases} \frac{\alpha \lambda}{D} \text{ radians} \\ \frac{180 \alpha \lambda}{\pi D} \text{ degrees} \end{cases}$
$D$ = aperture size
$\alpha$ = aperture factor
$\theta_3, \phi_3$ = azimuth and elevation 3 dB beamwidths

Peak Sidelobe Level, dB	Aperture Factor $\alpha$
–13	0.88
–12	0.98
–25	1.05
–30	1.12
–35	1.18
–40	1.25
–45	1.30

# Radar Phenomenology

## Maximum RCS of Simple Shapes, $\lambda \ll \text{Object Size}$

Shape		RCS
Sphere, radius $r$		$\pi r^2$
Flat plate, edge lengths $a$ and $b$		$4\pi (ab)^2 / \lambda^2$
Dihedral, edge lengths $a$ and $b$		$8\pi (ab)^2 / \lambda^2$
Trihedral, square sides, edge length $a$		$12\pi a^4 / \lambda^2$
Trihedral, triangular sides, edge length $a$		$4\pi a^4 / 3\lambda^2$

## Swerling Models

Probability Density Function of RCS $\sigma$	Decorrelation	
	Scan-to-Scan	Pulse-to-Pulse
Exponential, $p_{\sigma}(\sigma) = \frac{1}{\bar{\sigma}} \exp \left[ \frac{-\sigma}{\bar{\sigma}} \right]$	Case 1	Case 2
Chi-square, degree 4, $p(\sigma) = \frac{4\sigma}{\bar{\sigma}^2} \exp \left[ \frac{-2\sigma}{\bar{\sigma}} \right]$	Case 3	Case 4

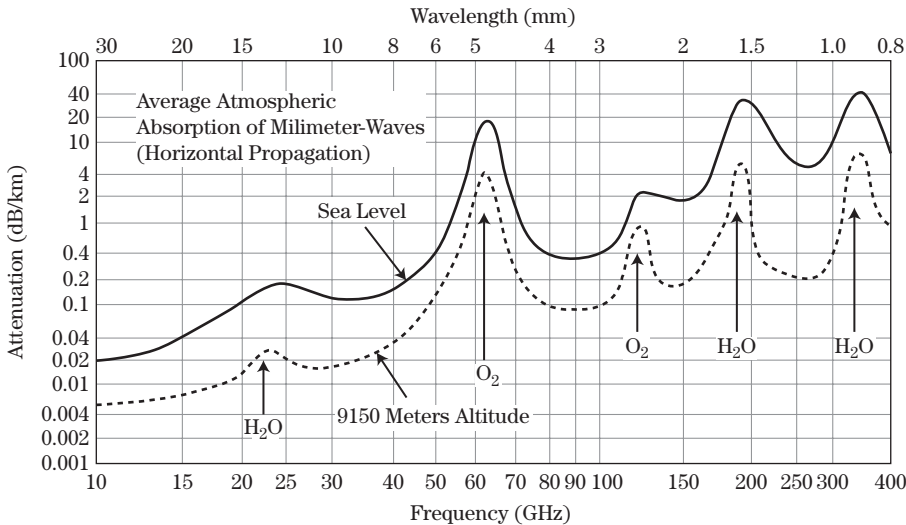
## RCS Decorrelation

Variable	Required Change	Comment
Aspect angle (rad)	$\frac{c}{2Lf} = \frac{\lambda}{2L}$	$L$ = width as viewed along radar line of sight
Frequency (Hz)	$\frac{c}{2L}$	$L$ = depth as viewed along radar line of sight

## Values of Doppler Shift

Radio frequency $f$		Doppler Shift $f_d$ (Hz)		
Band	Frequency (GHz)	1 m/s	1 knot	1 mph
L	1	6.67	3.43	2.98
S	3	20.0	10.3	8.94
C	6	40.0	20.5	17.9
X	10	66.7	34.3	29.8
K <sub>u</sub>	16	107	54.9	47.7
K <sub>a</sub>	35	233	120	104
W	95	633	326	283

## Atmospheric Attenuation



# Signal Analysis and Processing

## Select Fourier Transforms and Properties

Continuous Time	
$x(t)$	$X(f)$
$\begin{cases} A, & -\frac{\tau}{2} \leq t \leq \frac{\tau}{2} \\ 0, & \text{otherwise} \end{cases}$	$A\tau \frac{\sin(\pi f \tau)}{\pi f \tau} \equiv A\tau \text{sinc}(\pi f \tau)$
$\begin{cases} A \cos(2\pi f_0 t), & -\frac{\tau}{2} \leq t \leq \frac{\tau}{2} \\ 0, & \text{otherwise} \end{cases}$	$\frac{A\tau}{2} \text{sinc}[\pi(f - f_0)\tau] + \frac{A\tau}{2} \text{sinc}[\pi(f + f_0)\tau]$
$\sum_{n=-\infty}^{\infty} \delta_D(t - nT)$	$\sum_{k=-\infty}^{\infty} \delta_D(f - k \cdot PRF)$
$AB \frac{\sin(\pi Bt)}{\pi Bt} \equiv AB \text{sinc}(\pi Bt)$	$\begin{cases} A, & -\frac{B}{2} \leq t \leq \frac{B}{2} \\ 0, & \text{otherwise} \end{cases}$
$x(t - t_0)$	$e^{-j2\pi f t_0} X(f)$
$e^{+j2\pi f_0 t} x(t)$	$X(f - f_0)$
Discrete Time	
$x[n]$	$X(\hat{f})$
$Ae^{j2\pi \hat{f}_0 n}, \quad n = 0, \dots, N - 1$	$A \frac{1 - e^{-j2\pi(\hat{f} - \hat{f}_0)N}}{1 - e^{-j2\pi(\hat{f} - \hat{f}_0)}} \equiv NAe^{-j\pi(\hat{f} - \hat{f}_0)(N-1)} \text{asinc}(\hat{f} - \hat{f}_0, N)$
$A\delta[n - n_0]$	$e^{-j2\pi \hat{f} n_0}$
$\sum_{n=-\infty}^{\infty} \delta_D(t - nT)$	$\sum_{k=-\infty}^{\infty} \delta_D(f - k \cdot PRF)$
$A\hat{B} \frac{\sin[\pi \hat{B}n]}{\pi \hat{B}n} \equiv A\hat{B} \text{sinc}[\pi \hat{B}n]$	$\begin{cases} A, &  \hat{f}  < \hat{B} \\ \hat{B} <  \hat{f}  < \pi, & \text{otherwise} \end{cases}$
$x[n - n_0]$	$e^{-j2\pi \hat{f} n_0} X(\hat{f})$
$e^{+j2\pi \hat{f}_0 n} x[n]$	$X(\hat{f} - \hat{f}_0)$

## Window Properties

Window	3 dB Mainlobe Width, relative to rectangular	Peak Sidelobe (dB)	Sidelobe Rolloff (dB per octave)	SNR Loss (dB)	Maximum Straddle Loss (dB)
Rectangular	1.0	-13.2	6	0	3.92
Hann	1.68	-31.5	18	-1.90	1.33
Hamming	1.50	-41.7	6	-1.44	1.68
Taylor, 35 dB, $\bar{n} = 5$	1.34	-35.2	0/6	-0.93	2.11
Taylor, 50 dB, $\bar{n} = 5$	1.52	-46.9	0/6	-1.49	1.64
Dolph-Chebyshev (50 dB equiripple)	1.54	-50.0	0	-1.54	1.61
Dolph-Chebyshev (70 dB equiripple)	1.78	-70.0	0	-2.21	1.19

## Resolution

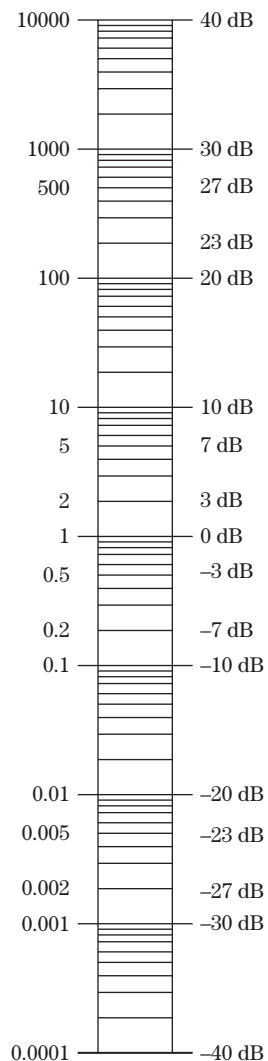
Dimension	Resolution with Matched Filter	Comments
Range ( $\Delta R$ )	$\frac{c\tau}{2}$	simple pulse, length $\tau$
	$\frac{c}{2B}$	arbitrary waveform, bandwidth $B$ Hz
Cross-range ( $\Delta CR$ )	$R\theta_3$	real beam imaging
	$\frac{\lambda R}{2vT_a} = \frac{\lambda R}{2D_{SAR}}$	synthetic aperture imaging $v$ = platform velocity $T_a$ = aperture time $D_{SAR}$ = synthetic aperture size
	$\frac{\lambda}{2\gamma}$	synthetic aperture imaging $\gamma$ = integration angle

## Simplified Probability of Detection Estimates

Nonfluctuating target, Noncoherent integration of $N$ samples (Albersheim's equation)	$P_D = \frac{1}{1 + e^{-B}}, \quad A = \ln \left( \frac{0.62}{P_{FA}} \right),$ $Z = \frac{SNR_{1dB} + 5 \log_{10} N}{6.2 + \frac{4.54}{\sqrt{N + 0.44}}}, \quad B = \frac{10^Z - A}{1.7 + 0.12A}$
Swerling 1 Fluctuating Target, No Noncoherent Integration	$P_D = (P_{FA})^{\frac{1}{1+SNR}}$

# Miscellaneous Relations

## Linear $\longleftrightarrow$ dB Scale



## Table of Constants

Constant	Symbol	Value
Speed of light	$c$	$2.99792458 \times 10^8$ m/s $\approx 3 \times 10^8$ m/s
Permittivity of free space	$\epsilon_0$	$8.85 \times 10^{-12}$ F/m
Permeability of free space	$\mu_0$	$4\pi \times 10^{-7}$ H/m
Impedance of free space	$\eta$	$377 \Omega$
Boltzmann's constant	$k$	$1.38 \times 10^{-23}$ J/K

## Subset of AN Nomenclature Applicable to US Radar Systems

First Letter (Type of Installation)		Second Letter (Type of Equipment)		Third Letter (Purpose)	
A	Piloted aircraft	L	Countermeasures	D	Direction finder, reconnaissance, or surveillance
F	Fixed Ground	P	Radar	G	Fire control or searchlight directing
M	Ground, mobile (installed as operating unit in a vehicle which has no function other than transporting the equipment)	Y	Signal/data processing	K	Computing
P	Pack or portable (animal or man)			N	Navigational aids (including altimeter, beacons, compasses, racons, depth sounding, approach, and landing)
S	Water surface craft			Q	Special, or combination of purposes
T	Ground, transportable			R	Receiving, passive detecting
U	Ground utility			S	Detecting or range and bearing, search
V	Ground, vehicular (installed in vehicle designed for functions other than carrying electronic equipment, etc., such as tanks)			Y	Surveillance (search, detect, and multiple target tracking) and control (both fire control and air control)

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