Fundamental Relations and Terminology

Major Forms of the Radar Range Equation

Peak power form, single pulse	$SNR = \dfrac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 \, k T_0 B F L_s R^4}$
Search form:	$\dfrac{P_{avg} A_e}{L_s T_0 F} \geq SNR_{min} 4\pi k \left(\dfrac{R^4}{\sigma}\right)\left(\dfrac{\Omega}{T_{fs}}\right)$
Track form:	$\dfrac{P_{avg} A_e^3 k_m^2}{\lambda^4 L_s T_0 F} = \left(\dfrac{\pi^2}{2}\right)\left(\dfrac{k r N_t R^4}{\sigma \cdot \sigma_\theta^2}\right)\left(\dfrac{1}{\cos^5(\theta_{scan})}\right)$

Definition of Terms:

SNR	Signal-to-noise ratio	B	Receiver bandwidth
SNR_{min}	Minimum detectable SNR	L_s	System losses
P_t	Peak transmitted power	F	Noise figure
P_{avg}	Average transmitted power	k	Boltzmann's constant
G_t	Transmit antenna gain	Ω	Search area solid angle
G_r	Receive antenna gain	T_{fs}	Frame search time
A_e	Effective aperture	k_m	Track measurement error slope
λ	Wavelength	r	Track measurement rate
σ	Target radar cross section	N_i	Number of targets
R	Range to target	σ_θ	Track angle estimate precision (std. dev.)
T_0	Standard temperature (270 K)	θ_{scam}	Scan angle, electronically-scanned array

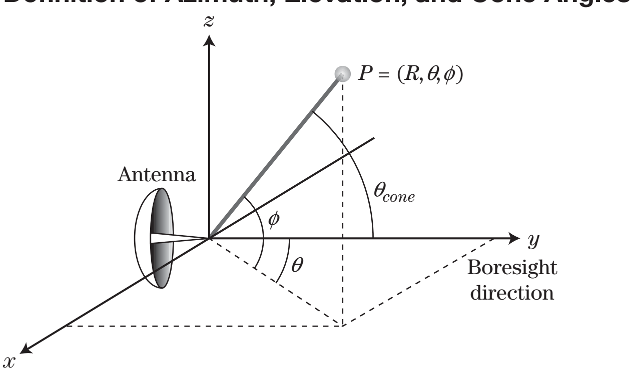
Radar Bands

Band	Frequency Range	ITU Radar Frequency
High frequency (HF)	3–30 MHz	
Very high frequency (VHF)	30–300 MHz	138–144 MHz 216–225 MHz
Ultra high frequency (UHF)	300 MHz–1 GHz	420–450 MHz 890–942 MHz
L	1–2 GHz	1.215–1.400 GHz
S	2–4 GHz	2.3–2.5 GHz 2.7–3.7 GHz
C	4–8 GHz	5.250–5.925 GHz
X	8–12 GHz	8.500–10.680 GHz
Ku ("under" K-band)	12–18 GHz	13.4–14.0 GHz 15.7–17.7 GHz
K	18–27 GHz	24.05–24.25 GHz 24.65–24.75 GHz
Ka ("above" K-band)	27–40 GHz	33.4–36.0 GHz
V	40–75 GHz	59.0–64.0 GHz
W	75–110 GHz	76.0–81.0 GHz 92.0–100.0 GHz
mm	100–300 GHz	126.0–142.0 GHz 144.0–149.0 GHz 231.0–235.0 GHz 238.0–248.0 GHz

Time Delay

A time delay of . . .	. . . is approximately equivalent to a range of . . .
1 nanosecond (ns)	0.15 meters (m)
	15 centimeters (cm)
	0.5 feet (ft)
	6 inches (in)
1 microsecond (μs)	0.15 km
	150 meters (m)
	0.1 (0.093) miles
	500 (492) feet (ft)

Definition of Azimuth, Elevation, and Cone Angles



Antenna Directivity, Gain, and Beamwidth

Maximum Directivity D_{max} and Gain G

$$D_{\max} \approx \begin{cases} \dfrac{4\pi}{\theta_3 \phi_3}\alpha^2, & \theta_3, \phi_3 \text{ in radians} \\ \dfrac{129,600}{\pi^2 \theta_3 \phi_3}\alpha^2, & \theta_3, \phi_3 \text{ in degrees} \end{cases}$$

Gain $G\,(dB) = D_{\max}$ (dB) – antenna losses (dB)

3 dB Beamwidth θ_3

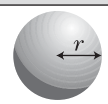
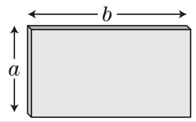
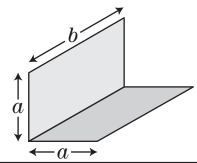
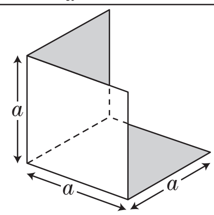
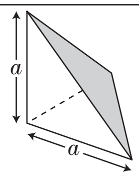
$$\theta_3 \approx \begin{cases} \dfrac{\alpha\lambda}{D} \text{ radians} \\ \dfrac{180\alpha\lambda}{\pi D} \text{ degrees} \end{cases}$$

D = aperture size
α = aperture factor
θ_3, ϕ_3 = azimuth and elevation 3 dB beamwidths

Peak Sidelobe Level, dB	Aperture Factor α
−13	0.88
−12	0.98
−25	1.05
−30	1.12
−35	1.18
−40	1.25
−45	1.30

Radar Phenomenology

Maximum RCS of Simple Shapes, $\lambda \ll$ Object Size

Shape		RCS
Sphere, radius r		πr^2
Flat plate, edge lengths a and b		$4\pi(ab)^2/\lambda^2$
Dihedral, edge lengths a and b		$8\pi(ab)^2/\lambda^2$
Trihedral, square sides, edge length a		$12\pi a^4/\lambda^2$
Trihedral, triangular sides, edge length a		$4\pi a^4/3\lambda^2$

Swerling Models

Probability Density Function of RCS σ	Decorrelation	
	Scan-to-Scan	Pulse-to-Pulse
Exponential, $p_\sigma(\sigma) = \frac{1}{\bar{\sigma}} \exp\left[\frac{-\sigma}{\bar{\sigma}}\right]$	Case 1	Case 2
Chi-square, degree 4, $p(\sigma) = \frac{4\sigma}{\bar{\sigma}^2} \exp\left[\frac{-2\sigma}{\bar{\sigma}}\right]$	Case 3	Case 4

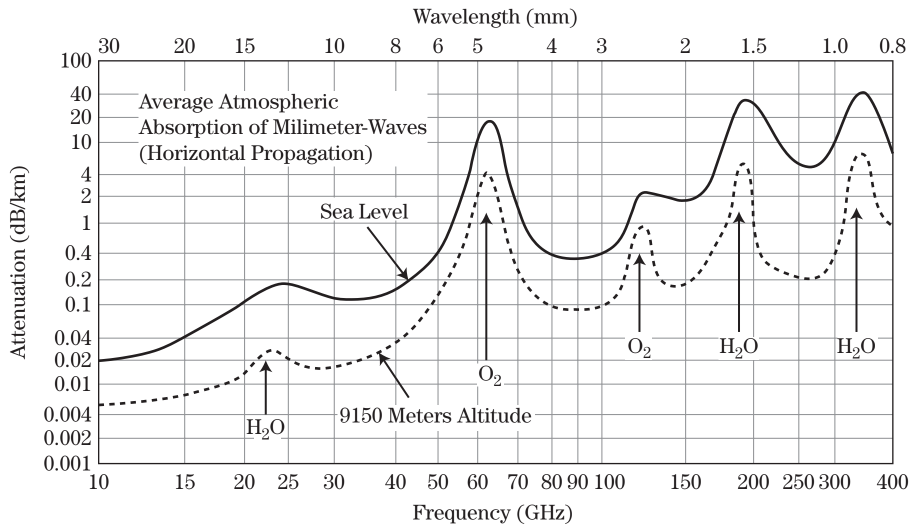
RCS Decorrelation

Variable	Required Change	Comment
Aspect angle (rad)	$\frac{c}{2Lf} = \frac{\lambda}{2L}$	L = width as viewed along radar line of sight
Frequency (Hz)	$\frac{c}{2L}$	L = depth as viewed along radar line of sight

Values of Doppler Shift

Radio frequency f		Doppler Shift f_d (Hz)		
Band	Frequency (GHz)	1 m/s	1 knot	1 mph
L	1	6.67	3.43	2.98
S	3	20.0	10.3	8.94
C	6	40.0	20.5	17.9
X	10	66.7	34.3	29.8
K_u	16	107	54.9	47.7
K_a	35	233	120	104
W	95	633	326	283

Atmospheric Attenuation



Signal Analysis and Processing

Select Fourier Transforms and Properties

Continuous Time	
$x(t)$	$X(f)$
$\begin{cases} A, & -\frac{\tau}{2} \le t \le \frac{\tau}{2} \\ 0, & \text{otherwise} \end{cases}$	$A\tau \frac{\sin(\pi f \tau)}{\pi f \tau} \equiv A\tau \operatorname{sinc}(\pi f \tau)$
$\begin{cases} A\cos(2\pi f_0 t), & -\frac{\tau}{2} \le t \le \frac{\tau}{2} \\ 0, & \text{otherwise} \end{cases}$	$\frac{A\tau}{2} \operatorname{sinc}[\pi(f - f_0)\tau] + \frac{A\tau}{2} \operatorname{sinc}[\pi(f + f_0)\tau]$
$\sum_{n=-\infty}^{\infty} \delta_D(t - nT)$	$\sum_{k=-\infty}^{\infty} \delta_D(f - k \cdot PRF)$
$AB \frac{\sin(\pi B t)}{\pi B t} \equiv AB \operatorname{sinc}(\pi B t)$	$\begin{cases} A, & -\frac{B}{2} \le t \le \frac{B}{2} \\ 0, & \text{otherwise} \end{cases}$
$x(t - t_0)$	$e^{-j2\pi f t_0} X(f)$
$e^{+j2\pi f_0 t} x(t)$	$X(f - f_0)$
Discrete Time	
$x[n]$	$X(\hat{f})$
$Ae^{j2\pi \hat{f}_0 n}, \quad n = 0, \ldots, N-1$	$A \frac{1 - e^{-j2\pi(\hat{f} - \hat{f}_0)N}}{1 - e^{-j2\pi(\hat{f} - \hat{f}_0)}} \equiv NAe^{-j\pi(\hat{f} - \hat{f}_0)(N-1)} \operatorname{asinc}(\hat{f} - \hat{f}_0, N)$
$A\delta[n - n_0]$	$e^{-j2\pi \hat{f} n_0}$
$\sum_{n=-\infty}^{\infty} \delta_D(t - nT)$	$\sum_{k=-\infty}^{\infty} \delta_D(f - k \cdot PRF)$
$A\hat{B} \frac{\sin\left[\pi \hat{B} n\right]}{\pi \hat{B} n} \equiv A\hat{B} \operatorname{sinc}\left[\pi \hat{B} n\right]$	$\begin{cases} A, & \left\|\hat{f}\right\| < \hat{B} \\ \hat{B} < \left\|\hat{f}\right\| < \pi, & \text{otherwise} \end{cases}$
$x[n - n_0]$	$e^{-j2\pi \hat{f} n_0} X\left(\hat{f}\right)$
$e^{+j2\pi \hat{f}_0 n} x[n]$	$X\left(\hat{f} - \hat{f}_0\right)$

Window Properties

Window	3 dB Mainlobe Width, relative to rectangular	Peak Sidelobe (dB)	Sidelobe Rolloff (dB per octave)	SNR Loss (dB)	Maximum Straddle Loss (dB)
Rectangular	1.0	−13.2	6	0	3.92
Hann	1.68	−31.5	18	−1.90	1.33
Hamming	1.50	−41.7	6	−1.44	1.68
Taylor, 35 dB, $\bar{n} = 5$	1.34	−35.2	0/6	−0.93	2.11
Taylor, 50 dB, $\bar{n} = 5$	1.52	−46.9	0/6	−1.49	1.64
Dolph-Chebyshev (50 dB equiripple)	1.54	−50.0	0	−1.54	1.61
Dolph-Chebyshev (70 dB equiripple)	1.78	−70.0	0	−2.21	1.19

Resolution

Dimension	Resolution with Matched Filter	Comments
Range (ΔR)	$\dfrac{c\tau}{2}$	simple pulse, length τ
	$\dfrac{c}{2B}$	arbitrary waveform, bandwidth B Hz
Cross-range (ΔCR)	$R\theta_3$	real beam imaging
	$\dfrac{\lambda R}{2vT_a} = \dfrac{\lambda R}{2D_{SAR}}$	synthetic aperture imaging v = platform velocity T_a = aperture time D_{SAR} = synthetic aperture size
	$\dfrac{\lambda}{2\gamma}$	synthetic aperture imaging γ = integration angle

Simplified Probability of Detection Estimates

Nonfluctuating target, Noncoherent integration of N samples (Albersheim's equation)	$P_D = \dfrac{1}{1+e^{-B}}, \quad A = \ln\left(\dfrac{0.62}{P_{FA}}\right),$ $Z = \dfrac{SNR_{1_{dB}} + 5\log_{10} N}{6.2 + \dfrac{4.54}{\sqrt{N+0.44}}}, \quad B = \dfrac{10^Z - A}{1.7 + 0.12A}$
Swerling 1 Fluctuating Target, No Noncoherent Integration	$P_D = (P_{FA})^{\frac{1}{1+SNR}}$

Miscellaneous Relations

Linear ←→ dB Scale

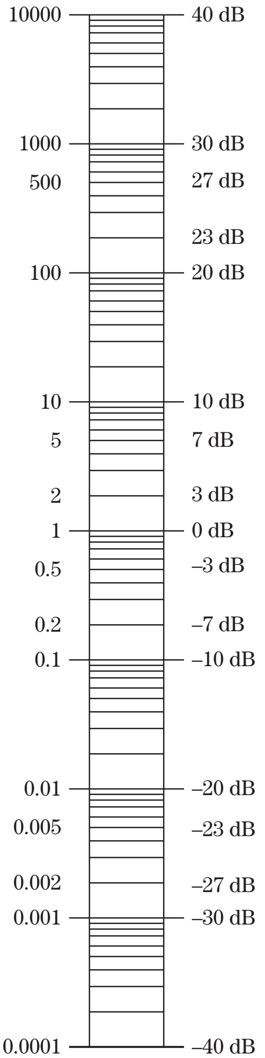


Table of Constants

Constant	Symbol	Value
Speed of light	c	2.99792458×10^8 m/s $\approx 3 \times 10^8$ m/s
Permittivity of free space	ε_0	8.85×10^{-12} F/m
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H/m
Impedance of free space	η	377 Ω
Boltzmann's constant	k	1.38×10^{-23} J/K

Subset of AN Nomenclature Applicable to US Radar Systems

First Letter (Type of Installation)		Second Letter (Type of Equipment)		Third Letter (Purpose)	
A	Piloted aircraft	L	Countermeasures	D	Direction finger, reconnaissance, or surveillance
F	Fixed Ground	P	Radar	G	Fire control or searchlight directing
M	Ground, mobile (installed as operating unit in a vehicle which has no function other than transporting the equipment	Y	Signal/data processing	K	Computing
P	Pack or portable (animal or man)			N	Navigational aids (including altimeter, beacons, compasses, racons, depth sounding, approach, and landing)
S	Water surface craft			Q	Special, or combination of purposes
T	Ground, transportable			R	Receiving, passive detecting
U	Ground utility			S	Detecting or range and bearing, search
V	Ground, vehicular (installed in vehicle designed for functions other than carrying electronic equipment, etc., such as tanks			Y	Surveillance (search, detect, and multiple target tracking) and control (both fire control and air control)

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