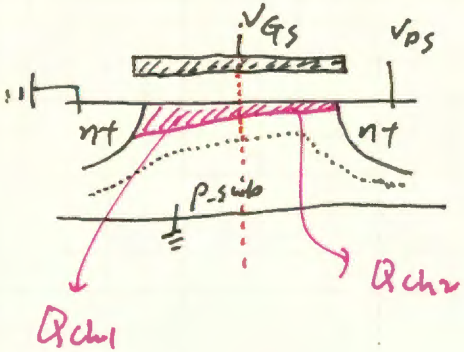
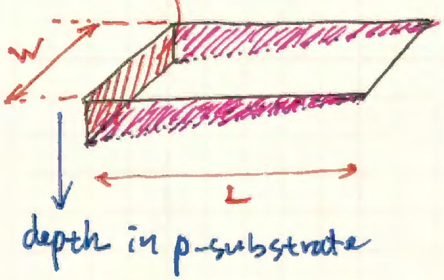
⊛ General triode model including short channel effect



- $V_{GS} \geq V_{th}$, $V_{DS} \leq V_{GS} - V_{th}$ $\left(\Rightarrow V_{GD} \geq V_{th}\right)$ No pinch-off in electron channel
- Non-uniform electron channel

$$\boxed{\begin{aligned} Q_{ch1} &= \frac{C_{ox}}{2}(V_{GS}-V_{th}) \\ Q_{ch2} &= \frac{C_{ox}}{2}(V_{GD}-V_{th}) = \frac{C_{ox}}{2}\left(V_{GS}-V_{DS}-V_{th}\right) \\ \therefore Q_T &= Q_{ch1}+Q_{ch2} \\ &= C_{ox}\left(V_{GS}-V_{th}-\tfrac{1}{2}V_{DS}\right) \end{aligned}}$$

$Q_T \cdot W$ = sheet charge



- $E = \dfrac{V_{DS}}{L}$

- $$I_{DS} = Q_T \cdot W \cdot V_{drift} = Q_T \cdot W\left(\mu_n \frac{E}{1+\frac{E}{E_c}}\right)$$

$$= C_{ox}\left(V_{GS}-V_{th}-\tfrac{1}{2}V_{DS}\right)\frac{W}{L}\,\mu_n \frac{V_{DS}}{1+\frac{V_{DS}}{E_c \cdot L}}$$

$$= \mu_n C_{ox}\frac{W}{L}\left((V_{GS}-V_{th})V_{DS}-\tfrac{1}{2}V_{DS}^2\right)\frac{1}{1+\frac{V_{DS}}{E_c \cdot L}}$$

NOTE: This 3D view of electron distribution in channel is not quite right, but for simple visualization.
In reality, charge population will be decreased exponentially in depth direction.
⇒ Most of electrons are populated in surface area.

⊛ NMOS DC I_{DS}-V_{DS} models (summary)

$$I_{DS} = \mu_n C_{ox} \frac{W}{L}\left((V_{GS}-V_{th})V_{DS} - \frac{1}{2}V_{DS}^2\right)\frac{1}{1+\frac{V_{DS}}{E_c \cdot L}}$$

↓ Long-channeled approximation

$$I_{DS} = \mu_n C_{ox} \frac{W}{L}\left((V_{GS}-V_{th})V_{DS} - \frac{1}{2}V_{DS}^2\right)$$

① $I_{DS} = \mu_n C_{ox} \frac{W}{L}(V_{GS}-V_{th})V_{DS}$

② $I_{DS} = \mu_n C_{ox} \frac{W}{L}\left((V_{GS}-V_{th})V_{DS} - \frac{1}{2}V_{DS}^2\right)\frac{1}{1+\frac{V_{DS}}{E_c \cdot L}}$

$V_{DS} = V_{GS}-V_{th} = V_{DS,sat}$

③ $I_{DS} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS}-V_{th})^2 \frac{1}{1+\frac{V_{GS}-V_{th}}{E_c \cdot L}}$

④ $I_{DS} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS}-V_{th})^2 \frac{1}{1+\frac{V_{GS}-V_{th}}{E_c \cdot L}}(1+\lambda V_{DS})$

① $I_{DS} = \mu_n C_{ox} \frac{W}{L}(V_{GS}-V_{th})V_{DS}$

② $I_{DS} = \mu_n C_{ox} \frac{W}{L}\left((V_{GS}-V_{th})V_{DS} - \frac{1}{2}V_{DS}^2\right)$

$V_{DS} = V_{GS}-V_{th} = V_{DS,sat}$

③ $I_{DS} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS}-V_{th})^2$

④ $I_{DS} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS}-V_{th})^2(1+\lambda V_{DS})$

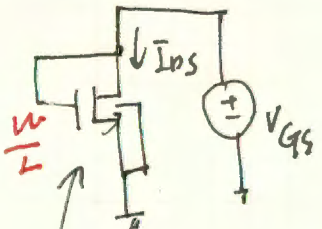
another form

$$I_{DS} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS}-V_{th})^2 \frac{1}{1+\frac{V_{GS}-V_{th}}{E_c \cdot L}}$$

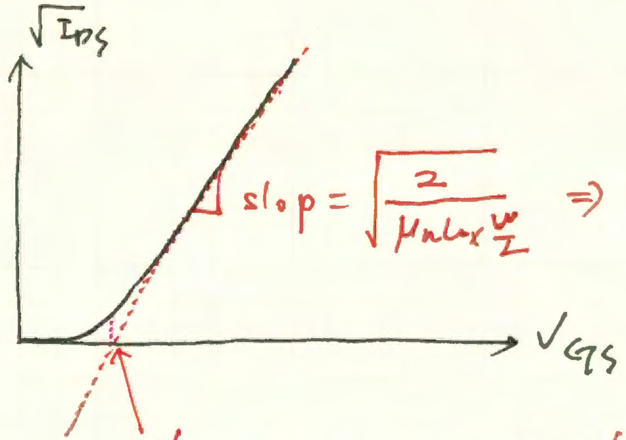
$$= \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS}-V_{th})^2 \frac{E_c \cdot L}{E_c \cdot L + V_{GS}-V_{th}} = C_{ox} W \cdot V_{sat} \frac{(V_{GS}-V_{th})^2}{(V_{GS}-V_{th}) + E_c \cdot L}$$

extreme velocity saturation (very short channel) → $I_{DS} = C_{ox} W \cdot V_{sat} \times (V_{GS}-V_{th})$

(*) Measurement of V_{th}



- No body effect
- Always saturation mode



$$\text{slop} = \sqrt{\frac{2}{\mu_n C_{ox} \frac{W}{L}}} \Rightarrow \text{can measure } \mu_n \cdot C_{ox}$$

$V_{th} \Rightarrow$ can measure V_{th}.

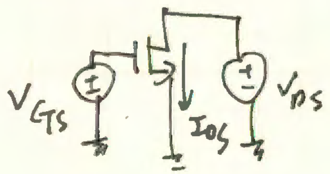
$$I_{DS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})^2$$

$$\Rightarrow V_{GS}-V_{th} = \sqrt{\frac{2 I_{DS}}{\mu_n C_{ox}\frac{W}{L}}}$$

$$\therefore V_{GS} = V_{th} + \sqrt{\frac{2 I_{DS}}{\mu_n C_{ox}\frac{W}{L}}}$$

NOTE:
1. In real measurement, I_{DS} will not be zero when $V_{GS} = V_{th}$.
2. "$V_{GS} = V_{th}$" is a boder line between "subthreshold" region and "above-threshold" region.
3. There is current continuity between the "subthreshold" region and the "above-threshold" region.

⊛ Some comments on square-law modeling



① Subthreshold region

$0 < V_{GS} < V_{th}, \quad V_{DS} > 0$

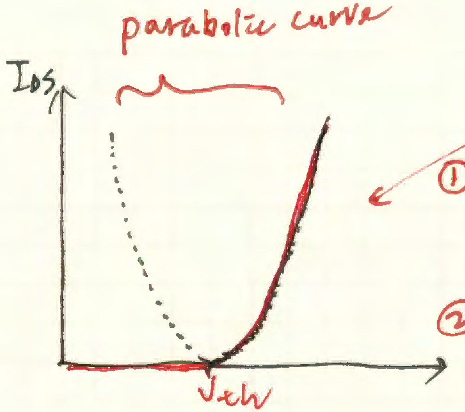
$$I_{DS} = I_s\left(e^{\frac{qV_{GS}}{kT}} - 1\right)\left(1 - e^{-\frac{qV_{DS}}{kT}}\right)$$

② Saturation region

electron channel pinch-off

$V_{GS} \geq V_{th}, \quad V_{DS} \geq V_{GS} - V_{th} \quad (\Rightarrow V_{GD} \leq V_{th})$

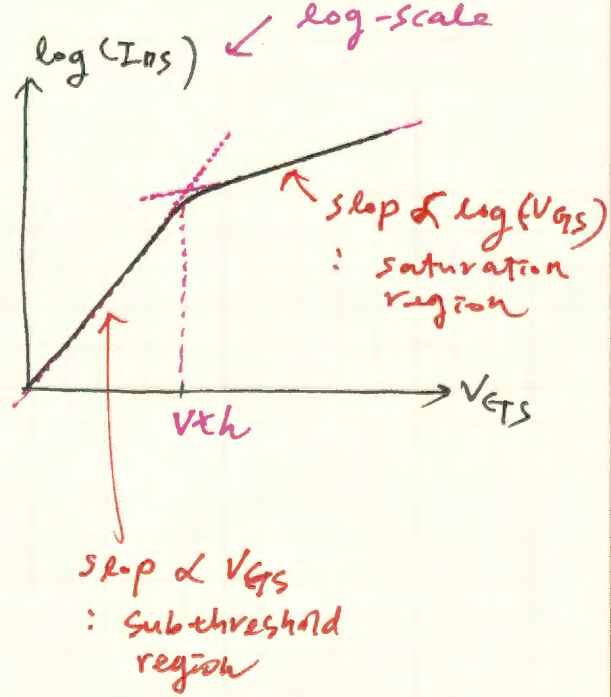
$$I_{DS} = \frac{1}{2}\mu_n C_{ox}(V_{GS} - V_{th})^2$$



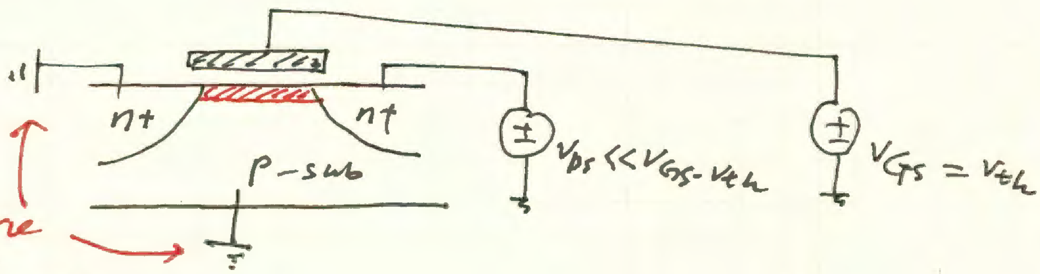
① Square-law model is a rather simplistic one ignoring all the current at $V_{GS} \leq V_{th}$.

② It is important to keep in mind that when $V_{GS} < V_{th}$, there is a subthreshold current.

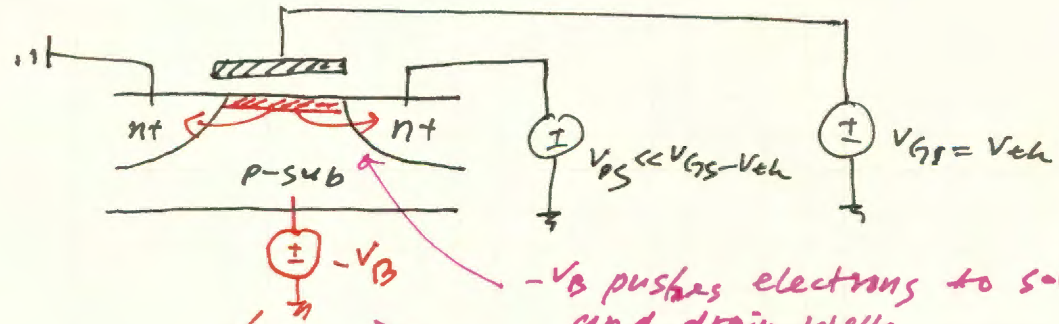
And, $V_{GS} = V_{th}$, the current needs to be continuous.



(*) Body effect (= Backgate effect)



Source and Body are same potential.



$-V_B$ pushes electrons to source and drain wells

Negative Body voltage ($-V_B$) will push electrons to source and drain sides

↓

No electron channel

↓

V_{GS} needs to be increased further from V_{th} to create electron channel

↓

The negative body voltage increases V_{th}

↓

Alternatively, the negative body voltage decreases current.

if Body voltage is positive, V_B, with reference to source voltage

↓

the positive V_B induces more electrons from source and drain.

↓

This effectively reduces V_{th}

↓

Equivalently, the positive body voltage ~~decreases~~ increases current

Body (substrate) plays like another gate. → This is called body effect (or backgate-effect)

⊛ Body effect (= Backgate effect)

→ V_{th} will be modulated by body voltage

→ I_{DS} will be modulated by body voltage

$$V_{th} = V_{th0} + \underbrace{\gamma}_{\text{body factor}} \left(\sqrt{|2\phi_F| - V_{BS}} - \sqrt{|2\phi_F|} \right)$$

V_{th0}: V_{th} without body effect ($V_{BS} = 0$)

$$\phi_F = V_T \ln\left(\frac{N_A}{n_i}\right)$$

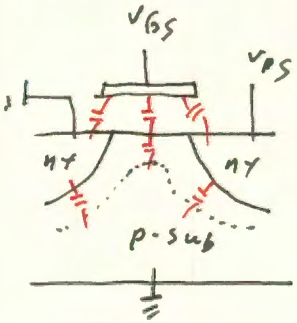
V_T: thermal voltage $= \frac{kT}{q} \simeq 26mV$ @ $T = 300K$

N_A: ~~hole~~ acceptor (hole) concentration

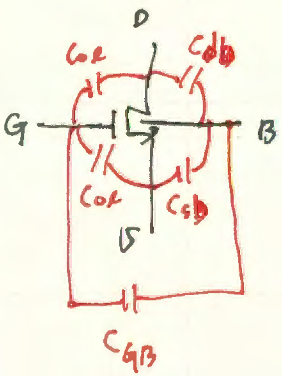
n_i: intrinsic Si carrier concentration.

$$\boxed{\begin{aligned}
g_{mb} &= \frac{\partial I_{DS}}{\partial V_{BS}} = \frac{\partial I_{DS}}{\partial V_{th}} \cdot \frac{\partial V_{th}}{\partial V_{BS}} \\
① \quad I_{DS} &= \frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th})^2 (1 + \lambda V_{DS}) \\
&\simeq \frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th})^2 \\
\Rightarrow \frac{\partial I_{DS}}{\partial V_{th}} &= -\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th}) = -g_m \\
② \quad \frac{\partial V_{th}}{\partial V_{BS}} &= \frac{-\gamma}{2\sqrt{|2\phi_F| - V_{BS}}} \\
\therefore \; g_{mb} &= \left(\frac{\gamma}{2\sqrt{|2\phi_F| - V_{BS}}} \right) \cdot g_m \\
&\simeq (0.1 \sim 0.2)\, g_m \Rightarrow 10 \sim 20\% \text{ of } g_m
\end{aligned}}$$

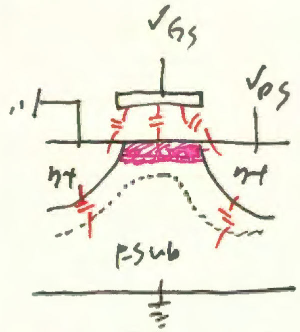
① Subthreshold



· $0 < V_{GS} < V_{th}$

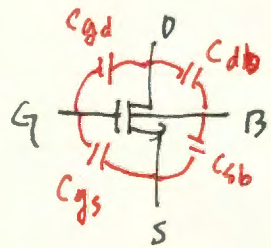


② Linear



· $V_{GS} \geq V_{th}$. $V_{DS} \approx 0$

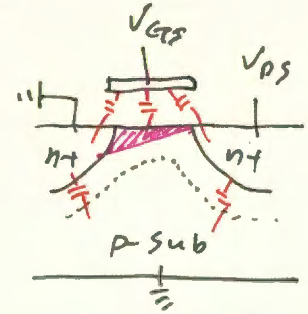
· uniform electron channel



$$C_{gs} = C_{ol} + \frac{1}{2} C_{ox} \cdot W \cdot L$$
$$\simeq \frac{1}{2} C_{ox} \cdot W \cdot L$$

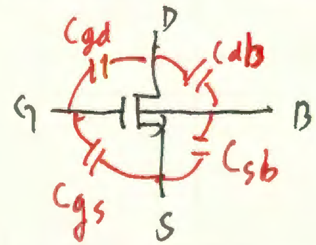
$$C_{gd} = C_{ol} + \frac{1}{2} C_{ox} \cdot W \cdot L$$
$$\simeq \frac{1}{2} C_{ox} \cdot W \cdot L$$

③ Triode



· $V_{GS} \geq V_{th}$, $V_{DS} \leq V_{GS} - V_{th}$

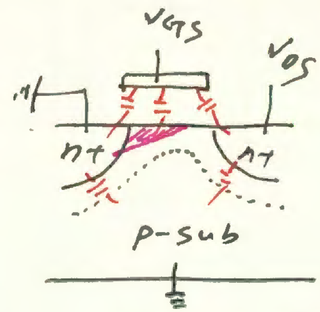
· non uniform electron channel



$$C_{gs} = C_{ol} + \frac{1}{2} C_{ox} \cdot W \cdot L$$
$$\simeq \frac{1}{2} C_{ox} \cdot W \cdot L$$

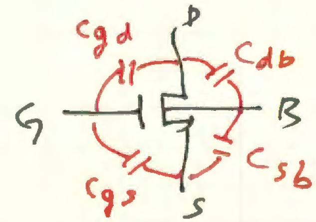
$$C_{gd} = C_{ol} + \frac{1}{2} C_{ox} \cdot W \cdot L$$
$$\simeq \frac{1}{2} C_{ox} \cdot W \cdot L$$

④ saturation



· $V_{GS} \geq V_{th}$, $V_{DS} \geq V_{GS} - V_{th}$

· pinch-off in electron channel



$$C_{gs} \simeq C_{ol} + \frac{2}{3} C_{ox} \cdot W \cdot L$$
$$\simeq \frac{2}{3} C_{ox} \cdot W \cdot L$$
$$C_{gd} = C_{ol}$$