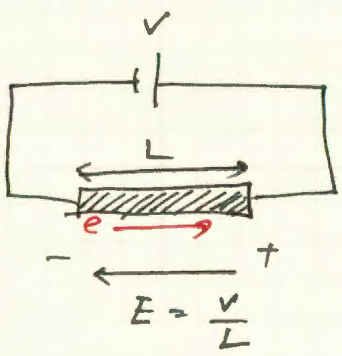
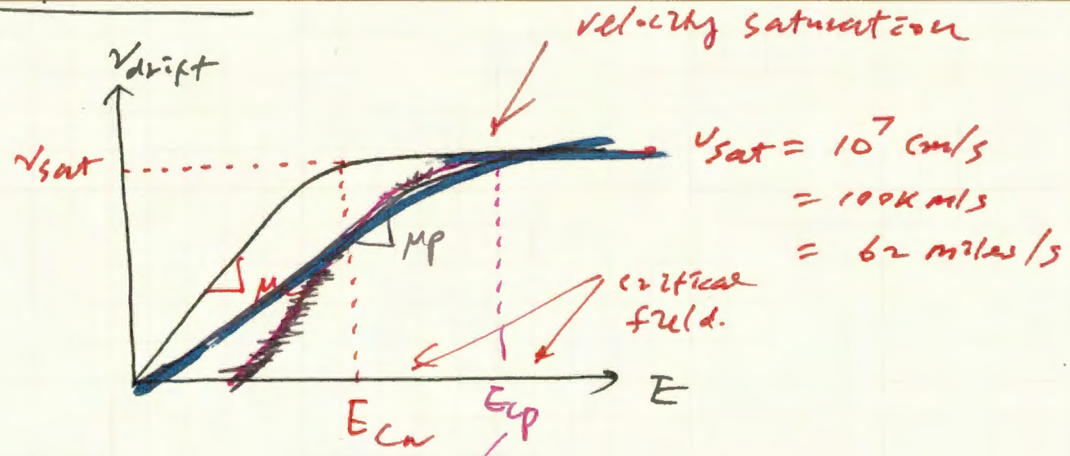
Carrier velocity vs E-field





$E_{cn} = 6\times10^4$ V/cm (electron) = 6V/μm

$E_{cp} = 24\times10^4$ V/cm (hole) = 24 V/μm

$E_{cn} = 600$ mV/0.1μm = 60 mV/10nm

$E_{cp} = 2.4$ V/0.1μm = 240 mV/10nm

(*) $$v_{drift} = \mu_n \frac{E}{1+\frac{E}{E_c}}$$

(if) $E \ll E_c$, $v_{drift} \simeq \mu_n E$

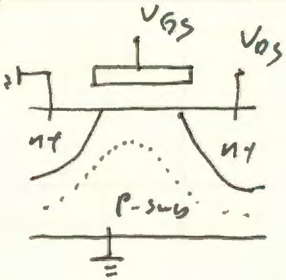
(if) $E \geq E_c$, $v_{drift} = \frac{1}{2}\mu_n E_c = v_{sat}$

(*) $I_e = q \cdot v_{drift}$

(if) $E \ll E_c$, $I_e = q \cdot \mu_n E$

(if) $E \geq E_c$, $I_e = q \cdot v_{sat}$

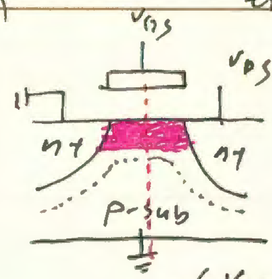
① Subthreshold



- $0 < V_{GS} < V_{th}$
- No electron channel
- Subthreshold current

$$I_{DS} = I_S\left(e^{\frac{qV_{GS}}{nkT}} - 1\right)\left(1 - e^{\frac{-qV_{DS}}{kT}}\right)$$

② Linear (= deep triode)



$0 \le V_{DS} \ll V_{GS} - V_{th}$

- $V_{GS} \geq V_{th}$, ~~$V_{DS}\approx 0$~~
- Uniform electron channel

① $Q_{ch1} = \frac{C_{ox}}{2}(V_{GS} - V_{th})$

$Q_{ch2} = \frac{C_{ox}}{2}(V_{GD} - V_{th})$

$V_{GD} = V_{GS} + V_{SD}$
$= V_{GS} - V_{DS}$
$\approx V_{GS}$

$Q_{ch1} \approx Q_{ch2}$

② $Q_T = Q_{ch1} + Q_{ch2}$
$= C_{ox}(V_{GS} - V_{th})$

③ $E = \frac{V_{DS}}{L}$

④ $I_{DS} = Q_T \cdot W \cdot \mu_n E$
$= \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{th})V_{DS}$

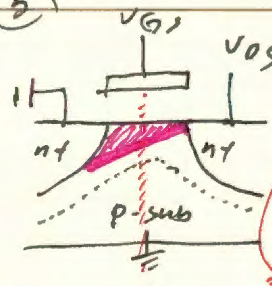
Linear

$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \mu_n C_{ox}\frac{W}{L} V_{DS}$

$g_{ds} = \frac{\partial I_{DS}}{\partial V_{DS}} = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{th})$

$r_o = \frac{1}{g_{ds}} = \frac{1}{\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{th})}$

③ Triode



$V_{GD} \geq V_{th}$
$V_{GS} - V_{DS} \geq V_{th}$
$\Rightarrow V_{DS} \le V_{GS} - V_{th}$

- $V_{GS} \geq V_{th}$, $V_{DS} \le V_{GS} - V_{th}$
- Non-uniform electron channel

$Q_{ch1} = \frac{C_{ox}}{2}(V_{GS} - V_{th})$

$Q_{ch2} = \frac{C_{ox}}{2}(V_{GD} - V_{th})$
$= \frac{C_{ox}}{2}(V_{GS} - V_{DS} - V_{th})$

$Q_T = Q_{ch1} + Q_{ch2}$
$= C_{ox}\left(V_{GS} - V_{th} - \frac{1}{2}V_{DS}\right)$

$E = \frac{V_{DS}}{L}$

$I_{DS,max} \Rightarrow @\ V_{DS} = V_{GS} - V_{th} = V_{DS,sat}$

$I_{DS} = Q_T \cdot W \cdot \mu_n E$
$= \mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{th} - \frac{1}{2}V_{DS}\right)V_{DS}$

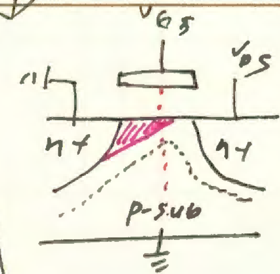
parabolic

$g_m = \mu_n C_{ox}\frac{W}{L} V_{DS}$

g_{ds}, $r_o \Rightarrow$ Non-linear

$V_{DS,sat} = V_{GS} - V_{th}$
$= \sqrt{\frac{2 I_{DS}}{\mu_n C_{ox}\frac{W}{L}}}$

④ Saturation (long channel approximation)



$V_{GD} < V_{th}$
$V_{GS} + V_{SD} < V_{th}$
$V_{GS} - V_{DS} < V_{th}$
$V_{GS} - V_{th} < V_{DS}$

- $V_{GS} \geq V_{th}$, $V_{DS} > V_{GS} - V_{th}$
- pinch-off in electron channel

$Q_{ch1} = \frac{C_{ox}}{2}(V_{GS} - V_{th})$

$Q_{ch2} \approx 0$

$Q_T = Q_{ch1}$
$= \frac{C_{ox}}{2}(V_{GS} - V_{th})$

$E = \frac{V_{GS} - V_{th}}{L'}$ — channel length modulation

$I_{DS} = Q_T \cdot W \cdot \mu_n E$
$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L'}(V_{GS} - V_{th})^2$

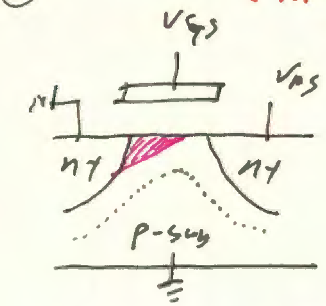
$\approx \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{th})^2(1 + \lambda V_{DS})$

channel length mod. factor

$g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{th})$
$= \sqrt{2\mu_n C_{ox}\frac{W}{L} I_{DS}}$
$= \frac{2 I_{DS}}{V_{GS} - V_{th}}$

$r_o = \frac{1}{\lambda I_{DS}}$

⑤ Velocity saturation (short channel effect)



- $V_{GS} \geq V_{th}$. $V_{DS} > V_{GS}-V_{th}$
- pinch off in electron channel

$$\begin{cases} Q_{ch1} = \frac{C_{ox}}{2}(V_{GS}-V_{th}) \\ Q_{ch2} \approx 0 \end{cases}$$

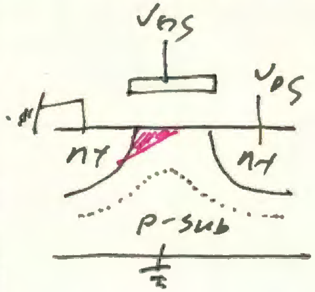
- $Q_T \approx Q_{ch1}$

$$\boxed{E = \frac{V_{GS}-V_{th}}{L'} \geq E_c}$$

$$I_{DS} \approx Q_T W \mu_n E_c = \frac{C_{ox}}{2} \cdot W (V_{GS}-V_{th}) \mu_n E_c = C_{ox} \cdot W (V_{GS}-V_{th}) v_{sat}$$

$g_m = C_{ox} \cdot W \cdot v_{sat}$
$\Rightarrow$ No dependent on L

⑥ General saturation model including short channel effect.



- $V_{GS} \geq V_{th}$, $V_{DS} > V_{GS}-V_{th}$
- pinch off in electron channel

$$\begin{cases} Q_{ch1} = \frac{C_{ox}}{2}(V_{GS}-V_{th}) \\ Q_{ch2} \approx 0 \end{cases}$$

- $Q_T \approx Q_{ch1}$

$$\boxed{E = \frac{V_{GS}-V_{th}}{L'} < E_c}$$

$$I_{DS} = Q_T \cdot W \left(\mu_n \frac{E}{1+\frac{E}{E_c}}\right)$$

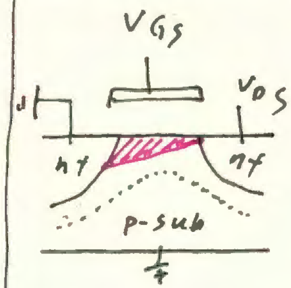
$$\boxed{= \frac{1}{2}\mu_n C_{ox} \frac{W}{L'} (V_{GS}-V_{th})^2 \left(\frac{1}{1+\frac{V_{GS}-V_{th}}{E_c \cdot L'}}\right)} \quad ★$$

At $V_{DS} = V_{DS,sat}$ these two current is same.

$\Downarrow$

$V_{DS,sat} = (V_{GS}-V_{th})$

⑦ General triode model including short channel effect



- $V_{GS} \geq V_{th}$, $V_{DS} \leq V_{GS}-V_{th}$
- Non-uniform electron channel

$$\begin{cases} Q_{ch1} = \frac{C_{ox}}{2}(V_{GS}-V_{th}) \\ Q_{ch2} = \frac{C_{ox}}{2}(V_{GD}-V_{th}) = \frac{C_{ox}}{2}(V_{GS}-V_{DS}-V_{th}) \end{cases}$$

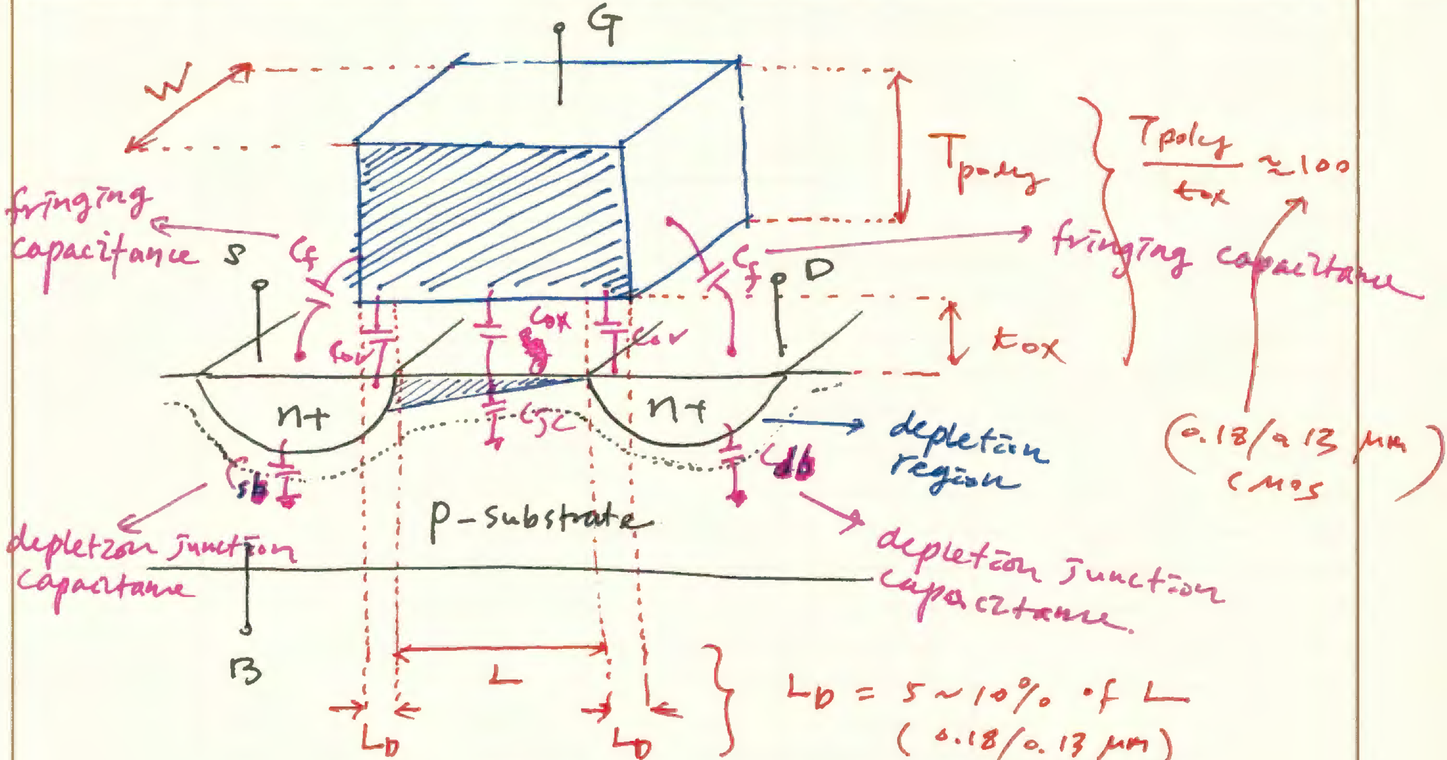
$$Q_T = Q_{ch1}+Q_{ch2} = C_{ox}\left(V_{GS}-V_{th}-\frac{1}{2}V_{DS}\right)$$

$$E = \frac{V_{DS}}{L}$$

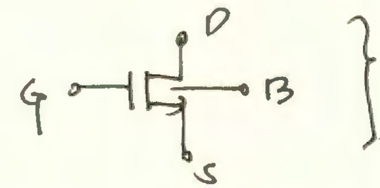
$$I_{DS} = Q_T \cdot W \cdot \left(\mu_n \frac{E}{1+\frac{E}{E_c}}\right)$$

$$\boxed{= \mu_n C_{ox} \frac{W}{L}\left(V_{GS}-V_{th}-\frac{1}{2}V_{DS}\right)\left(\frac{V_{DS}}{1+\frac{V_{DS}}{E_c \cdot L}}\right)}$$

NMOS Device



L_D = 5 ~ 10% of L
(0.18/0.13 μm)
CMOS

⇒ 

4-terminal device

⇒ L_D is extra extension of gate metal (poly silicon) to ~~gate~~ source and drain region to prevent channel discontinuity

⇒ Overlap capacitance

$$C_{ov} = C_{ox} \cdot W \cdot L_D$$

oxide capacitance per unit area, $C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{\varepsilon_0 \varepsilon_r}{t_{ox}}$

ε_0: 8.854×10^{-14} F/cm

ε_r: 4 for SiO_2

$C_g \simeq$ constant for process generations

ex) L = 5 μm, t_{ox} = 110 Å

L = 0.35 μm, t_{ox} = 75 Å

L = 0.18 μm, t_{ox} = 35 Å

L = 0.13 μm, t_{ox} = 22 Å

	0.18 μm NMOS	0.13 μm NMOS
L	0.18 μm	0.13 μm
t_{ox}	35 Å	22 Å
C_{ox}	10 fF/μm²	16 fF/μm²
[illegible]	[illegible]	[illegible]

$$\boxed{\begin{aligned} C_G &= C_{ox} \cdot W \cdot L \\ &= (C_{ox} \cdot L) \cdot W \\ &= C_g \cdot W \\ C_g &\simeq 2\,fF/\mu m \end{aligned}}$$

(*) I-V characteristics

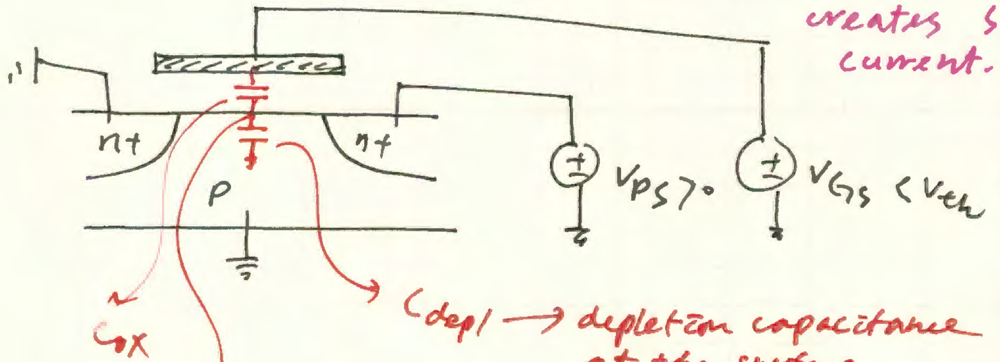
① $0 < V_{GS} < V_{th}$, $V_{DS} > 0$

⇒ subthreshold region

No electron channel but lateral bipolar NPN creates subthreshold current.

↓

Can be used for ultra-low-power systems. (ex) digital watch, Bio-medical systems).



C_{depl} → depletion capacitance at the surface

$V_{surface}$ = surface ~~potential~~ voltage

$$= V_{GS} \times \frac{C_{ox}}{C_{ox} + C_{depl}}$$

⇒ V_{GS} depletes holes at the surface of p-substrate under gate area.

⇒ This creates a depletion capacitance, C_{depl}

⇒ surface voltage, $V_{surface} = V_{GS} \cdot \dfrac{C_{ox}}{C_{ox}+C_{depl}} = V_{GS} \dfrac{1}{1+\frac{C_{depl}}{C_{ox}}} = \dfrac{V_{GS}}{n}$

⇒ $V_{surface}$ will be developed on the surface of p-substrate.

⇒ This forms N-p-N transistor (lateral bipolar transistor)

This gm can be larger than standard gm of saturation mode.

$$I_{DS} = I_S\left(e^{\frac{V_{GS}}{nV_T}} - 1\right)\left(1 - e^{-\frac{V_{DS}}{V_T}}\right)$$

NOTE: (if) $V_{DS} \gg V_T$

① gm (current driving capability)

$$= \frac{\partial I_{DS}}{\partial V_{GS}} \approx \frac{I_S}{nV_T} e^{\frac{V_{GS}}{nV_T}}$$

② No base current

- $V_T = \frac{kT}{q} = 26\,mV$, @ $T = 300\,^{\circ}K$
- $n = 1 + \dfrac{C_{depl}}{C_{ox}}$
- I_S = reverse saturation current.

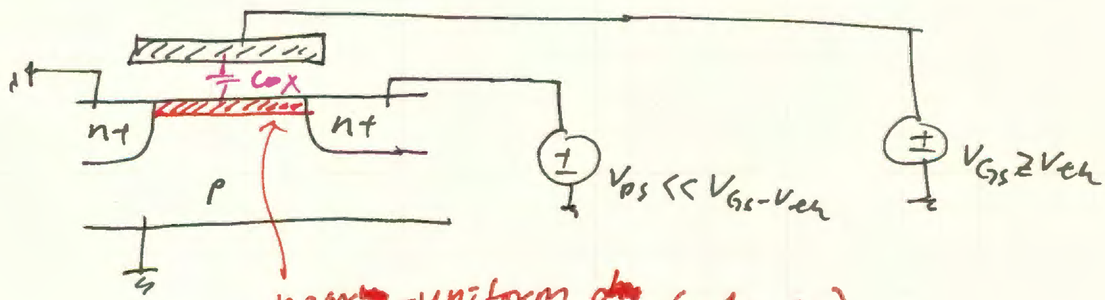
⇒ Subthreshold Current. ←
- leakage current
- typically nA, pA order,
- could cause standby power dissipation in VLSI systems

③ $V_{GS} \geq V_{th}$, $V_{DS} \ll V_{GS} - V_{th}$

⇒ linear region ⇒ $V_D - V_S \ll V_G - V_S - V_{th}$

⇒ $V_{th} \ll V_G - V_D$

⇒ $\boxed{V_{th} \ll V_{GD}}$



nearly-uniform electron channel (why?)

overdriving voltage

⇒ Channel charge per unit area

$Q_{ch} = -(V_{GS} - V_{th}) C_{ox}$ ← oxide capacitance per unit area

channel charge per unit length ← $Q_{ch}' = -(V_{GS} - V_{th}) C_{ox} \cdot W$

⇒ $I_{DS} = Q_{ch}' \cdot v_{drift}$ ← electron drift velocity

$= -Q_{ch}' \mu_n E$

$= -Q_{ch}' \mu_n \frac{V_{DS}}{L}$

$= \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th}) V_{DS}$

$\left(v_{drift} = -\mu_n E = -\mu_n \left(\frac{V}{L} \right) \right.$

μ_n: electron mobility

E: applied E-field

V: applied voltage

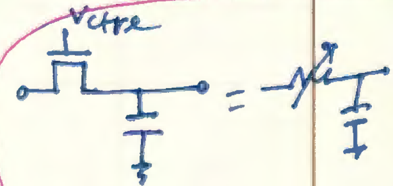
$\left. L: \text{length} \right)$

⇒ $g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \mu_n C_{ox} \frac{L}{W} V_{DS}$

⇒ I_{DS}-V_{DS} : linear-relationship

⇒ linear region

⇒ $R_{DS} = \frac{V_{DS}}{I_{DS}} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th})}$



⇒ NOTE: ① The electron channel is a linear resistor.

② R_{DS} can be controlled by V_{GS}

⇒ voltage controlled resistor.

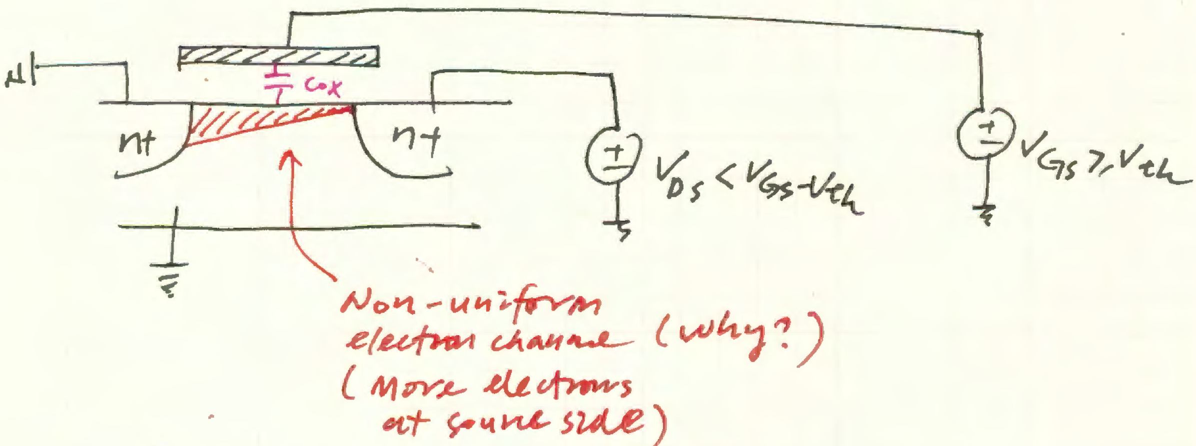
③ $V_{GS} \geq V_{th}$, $V_{DS} < V_{GS} - V_{th}$

⇒ triode region

$\Rightarrow V_D - V_S < V_G - V_S - V_{th}$

$\Rightarrow V_{th} < V_G - V_D$

$\Rightarrow \boxed{V_{th} < V_{GD}}$



⇒ channel charge approximation

① channel charge at source side is determined by $(V_{GS} - V_{th})$.

$$\therefore Q_{ch1} = -\frac{C_{ox}}{2}(V_{GS} - V_{th})$$

② channel charge at drain side is determined by $(V_{GD} - V_{th})$

$$\therefore Q_{ch2} = -\frac{C_{ox}}{2}(V_{GD} - V_{th})$$

③ total channel charge per unit ~~[illegible]~~ area

$$Q_{ch} = Q_{ch1} + Q_{ch2}$$

$$= -\frac{C_{ox}}{2}\left(V_{GS} + V_{GD} - 2V_{th}\right)$$

$$= -\frac{C_{ox}}{2}\left(V_{GS} + V_{GS} + V_{SD} - 2V_{th}\right)$$

$$= -\frac{C_{ox}}{2}\left(2V_{GS} - 2V_{th} - V_{DS}\right)$$

$$= -C_{ox}\left(V_{GS} - V_{th} - \frac{1}{2}V_{DS}\right)$$

④ Total channel charge per unit length

$$Q_{ch}' = -C_{ox}W\left(V_{GS} - V_{th} - \frac{1}{2}V_{DS}\right)$$

NMOS Device

$$\Rightarrow I_{DS} = -Q_{ch}' \cdot v_{drift}$$

$$= -Q_{ch}' \cdot \mu_n E$$

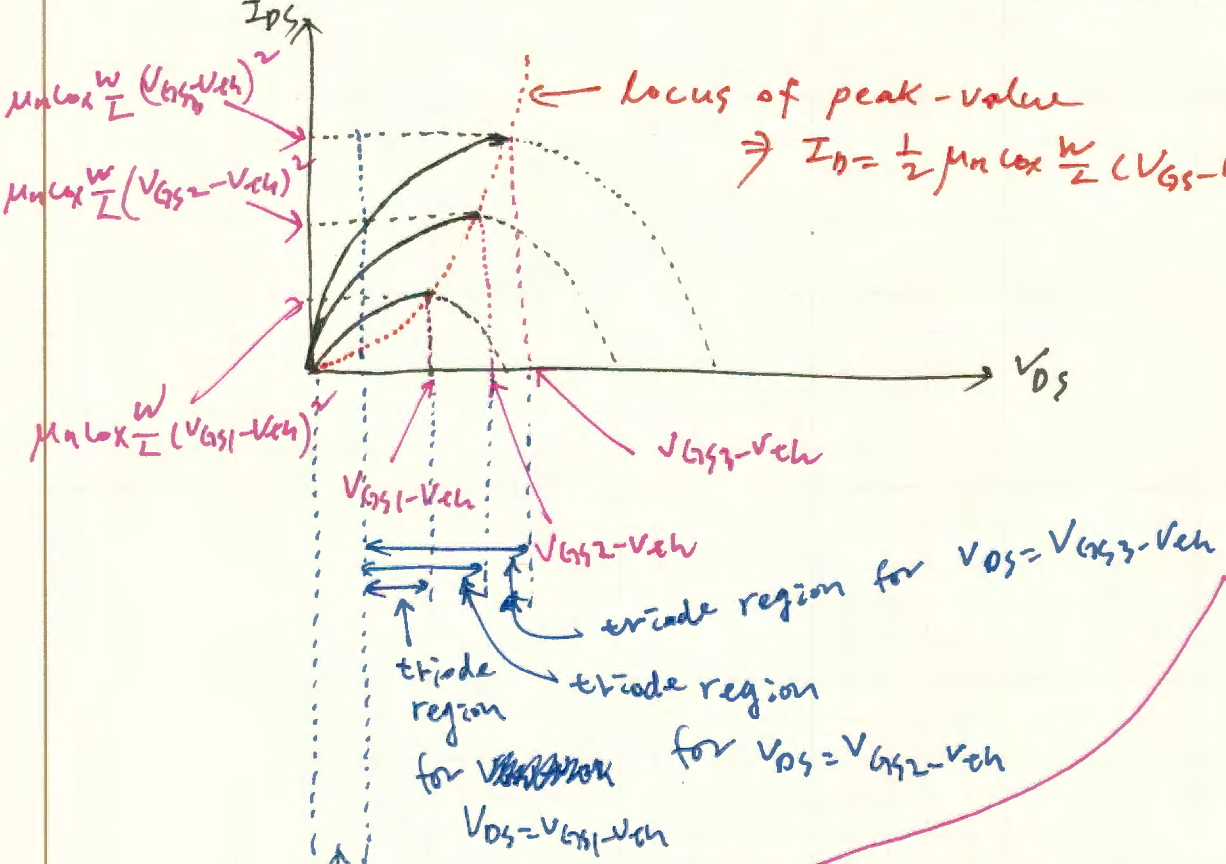
$$= -Q_{ch}' \cdot \mu_n \frac{V_{DS}}{L}$$

$$= \mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_{th} - \frac{1}{2} V_{DS}\right) V_{DS}$$

$$\boxed{= \mu_n C_{ox} \frac{W}{L} \left((V_{GS} - V_{th}) V_{DS} - \frac{1}{2} V_{DS}^2 \right)}$$

cf $V_{DS} \ll V_{GS} - V_{th} \rightarrow I_{DS} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th}) V_{DS}$

$\rightarrow$ same as linear mode



Linear region

there is no clear line between linear region and triode region.

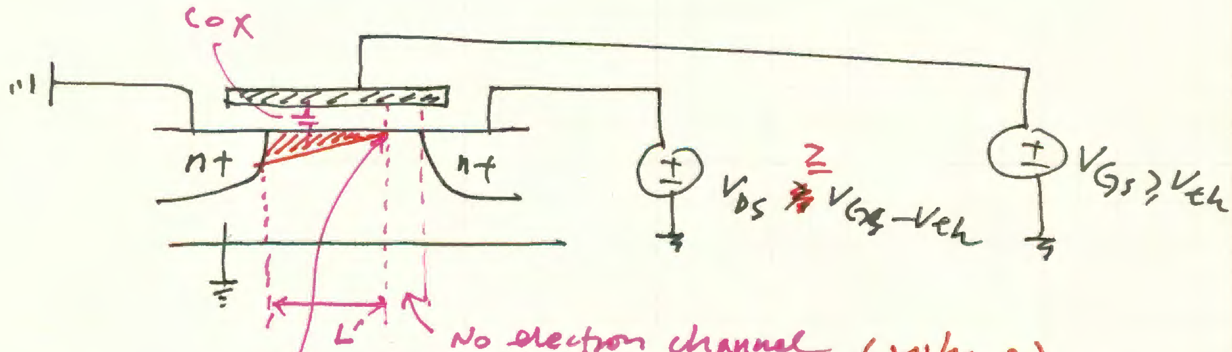
("DEEP Triode" region = Linear region)

④ $V_{GS} \geq V_{th}$, $\quad V_{DS} \overset{\geq}{\cancel{>}} V_{GS}-V_{th}$

$\Rightarrow V_D - V_S \overset{\geq}{\cancel{>}} V_G - V_S - V_{th}$

$\Rightarrow V_{th} \overset{\geq}{\cancel{>}} V_G - V_D$

$\Rightarrow \boxed{V_{th} \overset{\geq}{\cancel{>}} V_{GD}}$



No electron channel (why?)
= "pinch-off"
= depletion region

Voltage at this tail point ⇒ $(V_{GS}-V_{th})$

what happen to ~~traveling~~ electrons traveling in this region?

⇒ channel charge approximation

① Channel charge at source side is determined by $(V_{GS}-V_{th})$.

$$\therefore Q_{ch1} = -\frac{C_{ox}}{2}(V_{GS}-V_{th})$$

② Channel charge at Drain side is negligible

③ total channel charge per unit area

$$Q_{ch} = Q_{ch1} = -\frac{C_{ox}}{2}(V_{GS}-V_{th})$$

④ total channel charge per unit length

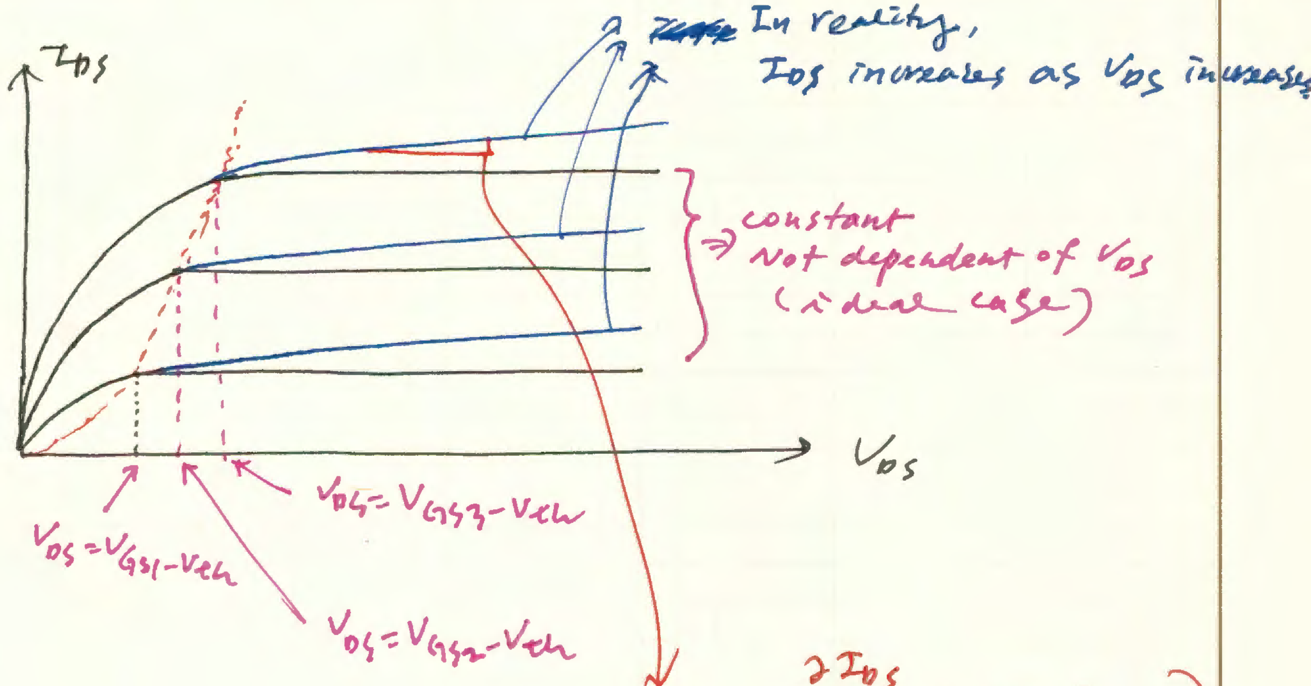
$$Q'_{ch} = -\frac{C_{ox}}{2}\cdot W(V_{GS}-V_{th})$$

$$\Rightarrow I_{DS} = -Q'_{ch}\cdot v_{drift} = -Q'_{ch}\cdot\mu_n E = -Q'_{ch}\,\mu_n\frac{V_{GS}-V_{th}}{L'}$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L'}(V_{GS}-V_{th})^2$$

$$\approx \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})^2$$

approximation $L \approx L'$



$$\text{slop} = \frac{\partial I_{DS}}{\partial V_{DS}} = g_{ds}$$

$$\Rightarrow \frac{1}{\text{slop}} = \frac{1}{g_{ds}} = r_o$$

⇒ output resistance

(*) As V_{DS} increases,
effective electron ~~traveling~~ drift distance, L',
will be dreases
⇒ effective channel length decreases
⇒ channel-length modulation
⇒ As $V_{DS} \uparrow \sim I_{DS} \uparrow$

(*)
$$\boxed{I_{DS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})^2(1+\lambda V_{DS})}$$

λ: channel-length modulation factor

$$\Rightarrow \frac{1}{\lambda I_{DS}} = r_o$$

$$\Rightarrow \lambda = \frac{1}{r_o I_{DS}}$$

* I-V characteristics

① linear region ($V_{DS} \ll V_{GS} - V_{th}$)

$$I_{DS} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th}) V_{DS}$$

② triode region ($V_{DS} < V_{GS} - V_{th}$)

$$I_{DS} = \mu_n C_{ox} \frac{W}{L} \left((V_{GS} - V_{th}) V_{DS} - \frac{1}{2} V_{DS}^2 \right)$$

③ saturation region ($V_{DS} \geq V_{GS} - V_{th}$)

$$I_{DS} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th})^2$$

④ saturation region with channel-length modulation

$$I_{DS} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th})^2 (1 + \lambda V_{DS})$$

* transconductance

$$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th})$$

$$= \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_{DS}}$$

$$= \frac{2 I_{DS}}{V_{GS} - V_{th}}$$

* $V_{DS,sat} = V_{GS} - V_{th}$ ← This is minimum V_{DS} for NMOS to operate in saturation region.

$$= \sqrt{\frac{2 I_D}{\mu_n C_{ox} \frac{W}{L}}}$$

$\Rightarrow V_{GS} = V_{DS,sat} + V_{th}$

V_{GS} can go higher than $V_{DS,sat}$ only by V_{th}.