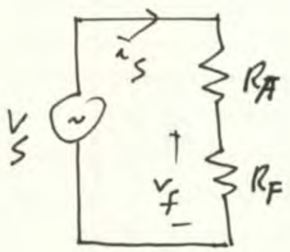
⊛ 2-resistor feedback (series)



i) Without R_F, i_s will be developed as

$$i_s = \frac{V_s}{R_A}.$$

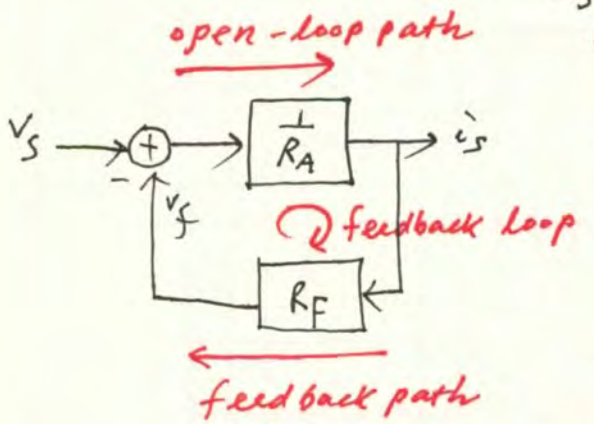
ii) When R_F included,
R_F will detect i_s and develop V_f.
And the V_f will be subtracted from V_s.
The resultant $i_s = \dfrac{V_s - V_f}{R_A}$. — Ⓐ

$\Rightarrow$ $V_f = i_s R_F$ apply to Ⓐ

$\Rightarrow$ $i_s R_A = V_s - V_f = V_s - i_s R_F$

$$\therefore i_s = \frac{V_s}{R_A + R_F} = \frac{V_s}{R_A} \frac{1}{\left(1 + \frac{R_F}{R_A}\right)}$$

equivalent feedback system



open loop gain, $A_{op} = \frac{1}{R_A}$
feedback factor, $f = R_F$
loop gain, $T = A_{op} \cdot f = \frac{R_F}{R_A}$
closed-loop gain $(A_s) = \frac{i_s}{V_s}$

$$= \frac{A_{op}}{1+T} = \frac{A_{op}}{1 + A_{op} f}$$

$$= \frac{\frac{1}{R_A}}{1 + \frac{R_F}{R_A}} = \frac{1}{R_A + R_F}$$

$\Rightarrow$ Without R_F, $Z_{in} = R_A$

With R_F, $Z_{in} = R_A + R_F = R_A\left(1 + \frac{R_F}{R_A}\right)$

$= R_A(1+T)$

Z_{in} increases by a factor of $(1+T)$. (Why?)

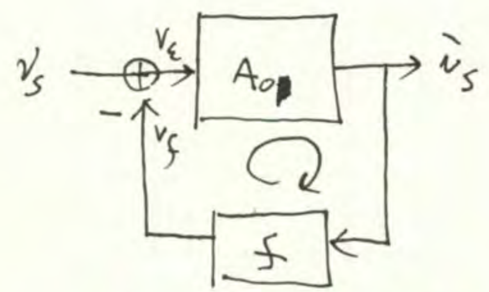
$\Rightarrow$ without R_F, $Z_{out} = R_A$

with R_F, $Z_{out} = R_A + R_F = R_A\left(1 + \frac{R_F}{R_A}\right)$

$= R_A(1+T)$

Z_{out} increases by a factor of $(1+T)$. (Why?)

⟨ Generalization ⟩



$v_s - v_f = v_\varepsilon = \frac{1}{1+T} \times v_s$

NOTE: input is Voltage
output is Current

① Loop gain $= T = A_{op} \cdot f$

② closed loop gain $= A_f = \dfrac{A_{op}}{1+T} = \dfrac{A_{op}}{1+A_{op}f}$

$= \dfrac{1}{f}\left(\dfrac{1}{1+\frac{1}{A_{op}f}}\right) = \dfrac{1}{f}\left(\dfrac{1}{1+\frac{1}{T}}\right)$

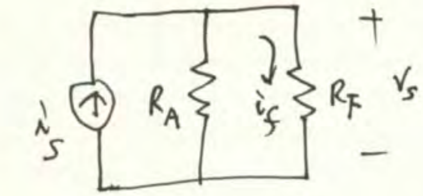
1'st-order approximation: $\approx \frac{1}{f}\left(1 - \frac{1}{T}\right)$ if $T \gg 1$

2nd-order approximation: $\approx \frac{1}{f}$ if $T \gg 1$

③ $Z_{inf} = Z_{in}(1+T)$ ← makes more ideal voltage driving.

④ $Z_{out} = Z_{out}(1+T)$

(*) 2-resistor feedback (parallel)



i) Without R_F, V_S will be developed as

$$V_S = i_S R_A.$$

ii) when R_F paralleled,

R_F will detect V_S and generate i_f.

And the i_f will be subtracted from i_S.

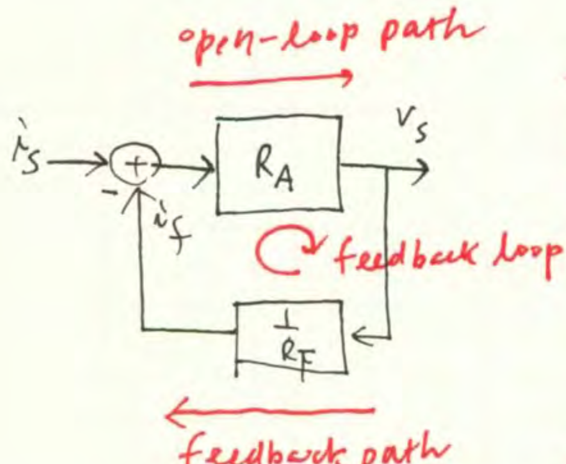
The resultant $V_S = (i_S - i_f) R_A$ — (A)

$\Rightarrow i_f = \dfrac{V_S}{R_F}$ apply to (A)

$$\Rightarrow V_S = \left(i_S - \frac{V_S}{R_F}\right) R_A$$

$$\therefore V_S = \frac{i_S R_A}{1 + \frac{R_A}{R_F}}$$

equivalent feedback system.



open loop gain, $A_o = R_A$

feed back factor, $f = \dfrac{1}{R_F}$

loop gain, $T = A_o f$

$= R_A / R_F$

Closed loop gain (A_f)

$$= \frac{V_S}{i_S} = \frac{A_o}{1+T} = \frac{R_A}{1 + \frac{R_A}{R_F}}$$

$$= \frac{R_A \cdot R_F}{R_A + R_F} = R_A \parallel R_F$$

⇒ without R_F, $Z_{in} = R_A$

with R_F, $Z_{in} = R_A \parallel R_F = \dfrac{R_A \cdot R_F}{R_A + R_F} = \dfrac{R_A}{1 + \frac{R_A}{R_F}}$

$= \dfrac{R_A}{1+T}$

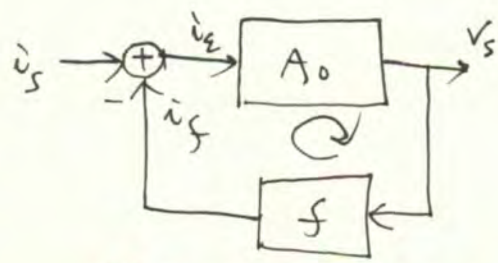
Z_{in} decreases by a factor of $(1+T)$. (Why?)

Z_{in} decreases by a factor of $(1+T)$. (Why?)

⇒ without R_F, $Z_{out} = R_A$

with R_F, $Z_{out} = R_A \parallel R_F = \dfrac{R_A}{1 + \frac{R_A}{R_F}} = \dfrac{R_A}{1+T}$

< generalization >



$$i_s - i_f = i_\varepsilon = \frac{1}{1+T} \times i_s$$

NOTE: Input is current
output is voltage

① loop gain, $T = A_o f$

② Closed loop gain, $A_f = \dfrac{A_o}{1+T} = \dfrac{A_o}{1+A_o f}$

$$= \frac{1}{f}\left(\frac{1}{1+\frac{1}{A_o f}}\right) = \frac{1}{f}\left(\frac{1}{1+\frac{1}{T}}\right)$$

$$\simeq \frac{1}{f}\left(1 - \frac{1}{T}\right) \quad \text{if } T \gg 1$$

$$\simeq \frac{1}{f}$$

③ $Z_{inf} = Z_{in} \dfrac{1}{1+T}$ ← makes more ideal current driving.

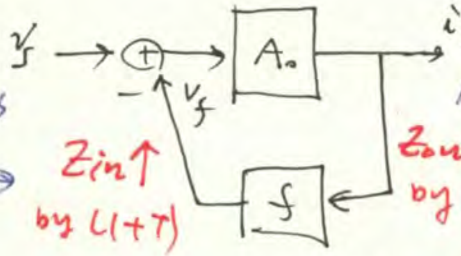
④ $Z_{outf} = Z_{out} \dfrac{1}{1+T}$

Feedback Basics

(*) feedback configurations

this makes better current mode driving to next stage.

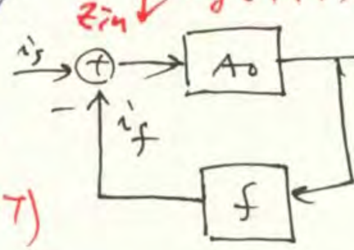
This makes better current mode driving for previous stage



This makes better voltage driving for previous stage.

Zin ↑ by (1+T)

Zout ↑ by (1+T)



Zin ↓ by (1+T)

Zout ↓ by (1+T)

This makes better voltage mode driving to next stage.

⇒ These are two basic configurations

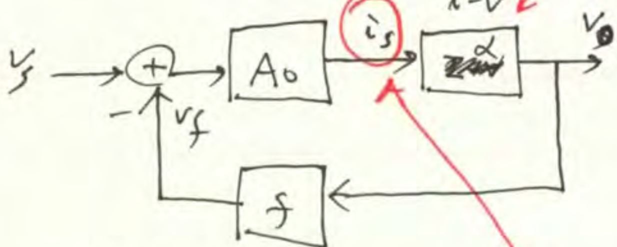
⇒ driving source should be either voltage or current.

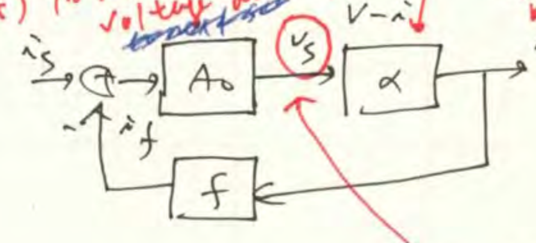
⇒ if driving source is voltage, immediate output response is current, which is first case.

⇒ if driving source is current, immediate output response is voltage, which is the second case.

why do we need this? A) To make better voltage driving to next stage

why do we need this? A) To make better current driving to next stage





⇒ i-V converter can be used to convert i_s to V_o.

⇒ But, still the point of feedback operation is to decrease i_s.

⇒ decreasing i_s results in decreasing v_o.

⇒ Zin ↑ ×(1+T)
Zout ↓ ×(1+T)

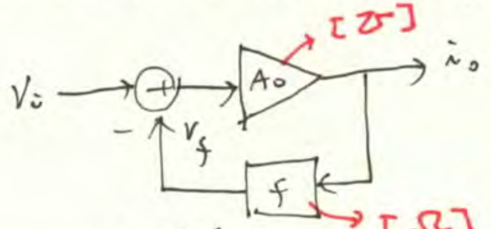
⇒ V-i converter can be used to convert v_s to i_o

⇒ But, still the point of feedback operation is to decrease V_s.

⇒ decreasing v_s results in decreasing i_o.

⇒ Zin ↓ ×(1+T)
Zout ↑ ×(1+T)

① Transconductance amplifier



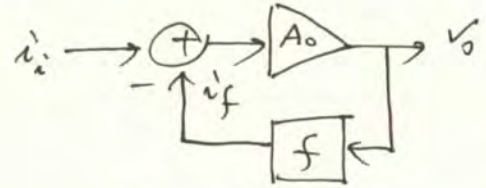
V I-feedback
= series-series feedback

$T = A_o f$

$Z_{inf} = Z_{in}(1+T)$ ← Better voltage driving from previous stage

$Z_{outf} = Z_{out}(1+T)$ ← Better current driving to next stage.

② Transimpedance amplifier



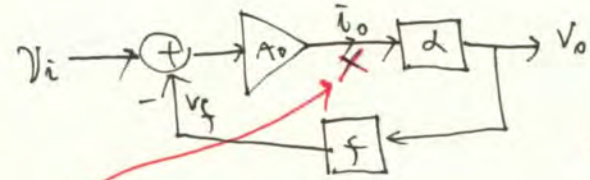
IV-feedback
= shunt-shunt feedback

$T = A_o f$

$Z_{inf} = \frac{Z_{in}}{1+T}$ ← Better current driving from previous stage

$Z_{outf} = \frac{Z_{out}}{1+T}$ ← Better voltage driving to next stage

③ Voltage gain amplifier



V-V-feedback
= V²-feedback
= series-shunt feedback

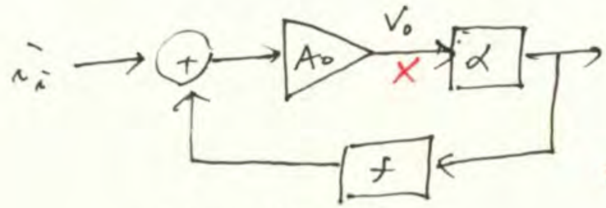
$T = A_o \alpha f$

$Z_{inf} = Z_{in}(1+T)$ ← Better voltage driving from previous stage

$Z_{outf} = \frac{Z_{out}}{1+T}$ ← Better ~~current~~ voltage driving to next stage

$Z_{Xf} = Z_X(1+T)$ ← internal node impedance ↑

④ Current gain amplifier



I-I-feedback
= I²-feedback
= shunt-series feedback.

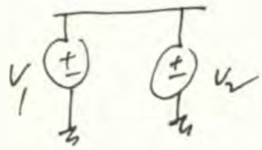
$T = A_o \alpha f$

$Z_{inf} = \frac{Z_{in}}{1+T}$ ← Better current driving from previous stage

$Z_{outf} = Z_{out}(1+T)$ ← Better current driving to next stage

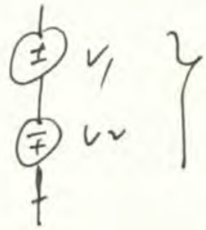
$Z_{Xf} = \frac{Z_X}{1+T}$ ← Internal node impedance ↓

* voltage addition/subtraction ⇒ series network.



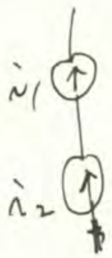
(X)

$= v_1 + v_2$

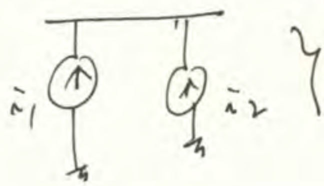


$= v_1 - v_2$

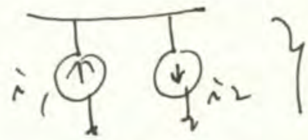
* Current addition/subtraction = parallel (shunt) network



(X)



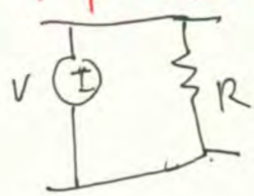
$i_1 + i_2$



$i_1 - i_2$

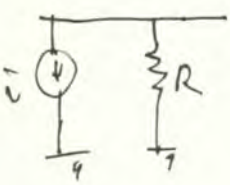
* voltage sensing ⇒ parallel network (shunt network)

(X)



(o)

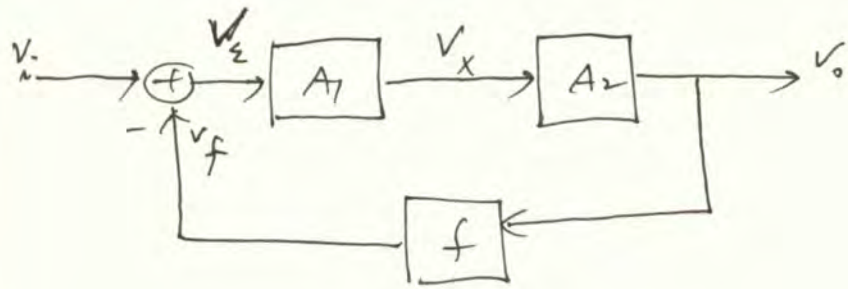
* Current sensing ⇒ series network



(X)

(o)

Remember this!



- loop gain, $T = A_1 A_2 f$

- $A_f = \dfrac{V_o}{V_i} = \dfrac{A_1 A_2}{1+T}$

$\div A_2$

- $\dfrac{V_x}{V_i} = \dfrac{A_1}{1+T}$

$\div (A_2 A_1)$

$\div A_1$

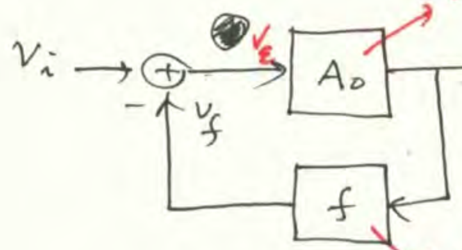
- $\dfrac{V_\varepsilon}{V_i} = \dfrac{1}{1+T}$

We will study more complex form of feedback in active filter design section.

study "Mason's Gain Formula".

(*) V-I feedback (series-series feedback)

unit: [℧] → conductance (admittance)
⇒ Transconductance amplifier



unit: [Ω] → resistance (impedance)

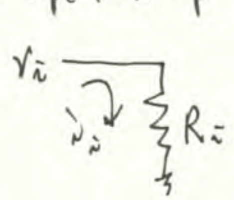
- $T = A_o f$
- $A_f = \frac{i_o}{v_i} = \frac{A_o}{1+T} = \frac{A_o}{1+A_o f} \simeq \frac{1}{f}\left(1 - \frac{1}{A_o f}\right) \simeq \frac{1}{f}$

  → $i_o = \frac{A_o}{1+T} v_i$

  → $v_\varepsilon = \frac{1}{1+T} v_i$ (if perfect feedback $v_\varepsilon = 0$, $v_i = v_f$)

① Z_{in} & Z_{inf}

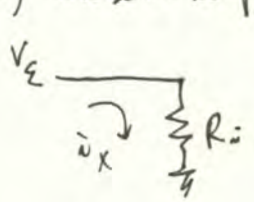
i) open-loop



$i_i = \frac{v_i}{R_i}$

$Z_{in} = \frac{v_i}{i_i} = R_i$

ii) closed-loop



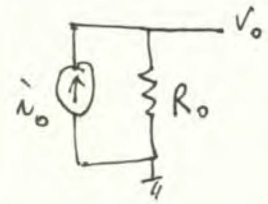
$i_x = \frac{v_\varepsilon}{R_i} = \frac{v_i}{(1+T)R_i}$

$Z_{inf} = \frac{v_i}{i_x} = (1+T)R_i$

To compare input impedance, the same reference of v_i is chosen.

② Z_{out} & Z_{outf}

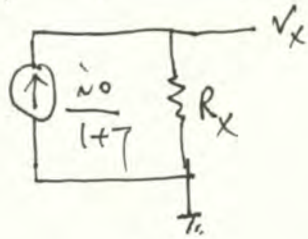
i) open-loop



$V_o = i_o R_o$

$Z_{out} = \frac{V_o}{i_o} = R_o$

ii) closed-loop



$V_X = \frac{i_o}{1+T} R_X = \frac{V_o}{(1+T) R_o} \cdot R_X$

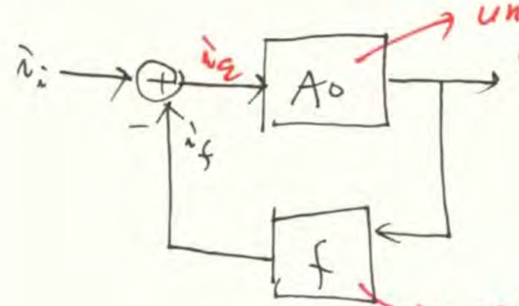
$\Rightarrow \quad R_X = (1+T) R_o \frac{V_X}{V_o}$

To compare these two impedances, we need to set $V_o = V_X$ (same reference).

$\therefore \quad R_X = (1+T) R_o$

$\Rightarrow Z_{out} = R_X = (1+T) R_o$

⊛ I-V feedback (shunt-shunt feedback)



unit: [Ω] → resistance (impedance) ⇒ Trans impedance amplifier

unit: [℧] → conductance (admittance)

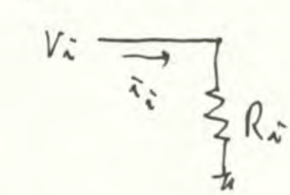
- $T = A_o f$
- $A_f = \frac{v_o}{i_i} = \frac{A_o}{1+T} = \frac{A_o}{1+A_o f} \simeq \frac{1}{f}\left(1 - \frac{1}{A_o f}\right) \simeq \frac{1}{f}$

→ $v_o = \frac{A_o}{1+T} i_i$

→ $i_\epsilon = \frac{1}{1+T} i_i$ (if perfect feedback $i_\epsilon = 0$, $i_i = i_f$)

① Z_{in} & Z_{inf}

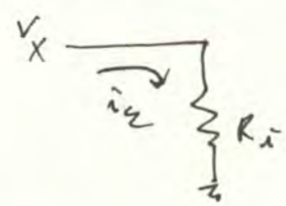
i) open-loop



$V_i = i_i R_i$

$Z_{in} = \frac{V_i}{i_i} = R_i$

ii) closed-loop



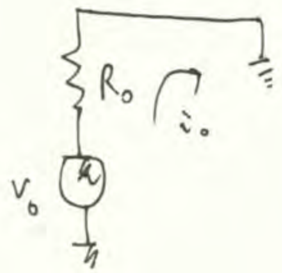
$V_X = i_\epsilon R_i = \frac{i_i}{1+T} R_i$

$Z_{inf} = \frac{V_X}{i_i} = \frac{R_i}{1+T}$

To compare input impedance, the same reference of i_i is chosen.

② Z_{out} & Z_{outf}

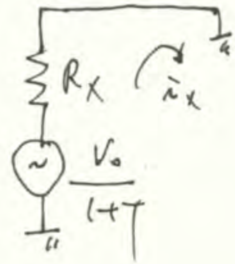
i) open-loop



$$i_o = \frac{V_o}{R_o}$$

$$Z_{out} = \frac{V_o}{i_o} = R_o$$

ii) closed-loop



$$i_x = \frac{V_o}{(1+T)}\frac{1}{R_x} = \frac{i_o R_o}{1+T}\frac{1}{R_x}$$

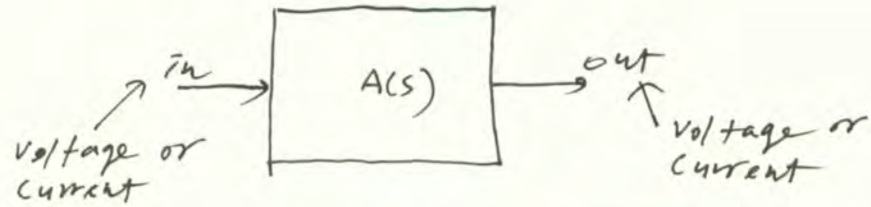
$$\Rightarrow R_x = \frac{R_o}{1+T}\frac{i_o}{i_x}$$

To compare these two impedances, we need to set $i_x = i_o$ (same reference)

$$\therefore R_x = \frac{R_o}{1+T}$$

$$\Rightarrow Z_{out} = R_x = \frac{R_o}{1+T}$$

⊛ one-pole system

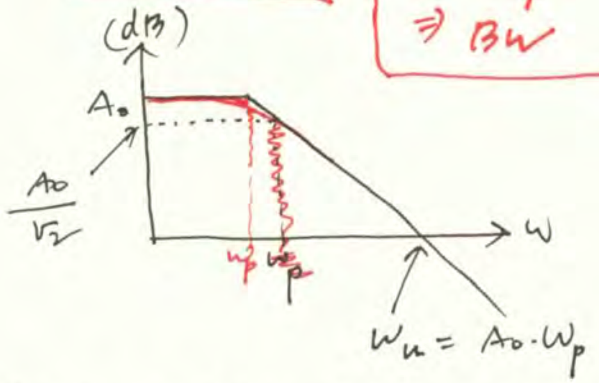


$$\Rightarrow A(s) = \boxed{\frac{A_0}{1+s\tau_p}} = \boxed{\frac{A_0}{1+s/\omega_p}} = \frac{A_0\omega_p}{s+\omega_p} = \boxed{\frac{\omega_u}{s+\omega_p}}$$

τ_p: Time constant

pole frequency $\omega_p = \frac{1}{\tau_p}$ ⇒ BW

Gain × BW = GBW = ω_u



⇒ These equations give a good intuition for understanding a system in view of Gain, BW and frequency response.

⇒ Another useful expression

$$A(s) = \frac{\omega_u}{s+\omega_p} = \boxed{\frac{\omega_u}{s+\frac{\omega}{Q}}}, \quad \boxed{Q = \frac{\omega}{\omega_p} = \omega\tau_p}$$

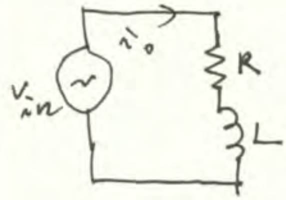
pole quality factor

This equation gives a good intuition for understanding a system in view of energy.

⇓

Q is a metric to express a stored energy in a system, which is relative amount to dissipated energy by the system.

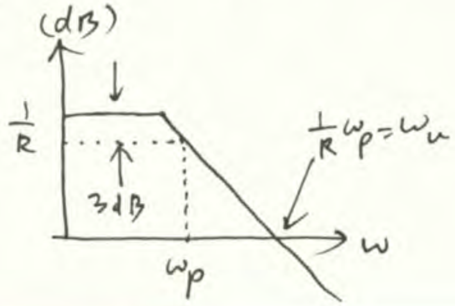
examples



$$A(s) = \frac{i_o}{v_{in}} = \frac{1}{R + sL} = \frac{1}{R}\cdot\frac{1}{(1 + sL/R)}$$

what's the meaning of τ_p?

$$= \frac{1}{R}\,\frac{1}{1+s\tau_p}, \quad \boxed{\tau_p = \frac{L}{R}}$$



$$= \frac{1}{R}\,\frac{1}{1+s/\omega_p}, \quad \boxed{\omega_p = \frac{1}{\tau_p}}$$

$$= \frac{\frac{1}{R}\cdot\omega_p}{s+\omega_p} = \frac{\omega_u}{s+\omega_p}$$

(*) remember

$$\boxed{Q = \frac{\text{Peak Energy stored in L or C}}{\text{Energy dissipated by R per unit radian}}}$$

$$= \frac{\frac{1}{2}L\hat{i}_o^2}{\frac{1}{2}\hat{i}_o^2 R \times T \times \frac{1}{2\pi}}$$

$$= \frac{\frac{1}{2}L\hat{i}_o^2}{\frac{1}{2}\hat{i}_o^2 R \times \frac{1}{w}}$$

$$\boxed{= w\frac{L}{R} = w\tau_p = \frac{w}{w_p}}$$

$Q\uparrow \rightsquigarrow \tau_p\uparrow$
$\rightsquigarrow$ larger inertia energy
$\rightsquigarrow$ slow response time
$\rightarrow$ slow speed.

$Q=1$
$\Rightarrow$ energy balance between stored & dissipated energy

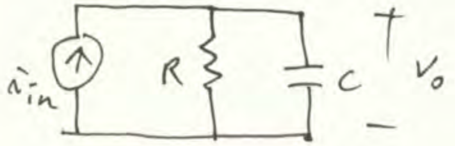
$\Rightarrow$ higher $Q \rightarrow$ larger stored energy than dissipated energy by R

$Q=\frac{1}{2}$
$\Rightarrow$ stored E is 50% of dissipated E.

$\Rightarrow$ (if) $R = 0 \rightarrow Q = \infty \rightarrow$ pure L
$\rightarrow$ pure energy storage element.

$$\boxed{\therefore\ A(s) = \frac{1}{R}\,\frac{1}{1+s\tau_p} = \frac{1}{R}\,\frac{1}{1+s/w_p} = \frac{w_u}{s+\frac{w}{Q}}.}$$

examples

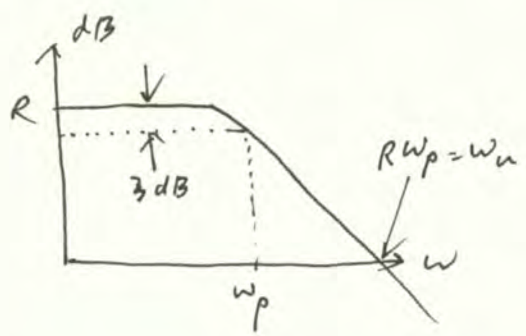


$$A(s) = \frac{v_o}{i_{in}} = \frac{1}{\frac{1}{R} + sC} = \frac{R}{1+sCR}$$

$$= R\,\frac{1}{1+s\tau_p}, \quad \boxed{\tau_p = RC}$$

$$= R\,\frac{1}{1+s/\omega_p}, \quad \boxed{\omega_p = \frac{1}{\tau_p}}$$

$$= \frac{R\omega_p}{s+\omega_p} = \frac{\omega_u}{s+\omega_p}$$



$$Q = \frac{\text{Peak Energy stored in } L \text{ or } C}{\text{Energy dissipated by } R \text{ per unit radian}}$$

$$= \frac{\frac{1}{2}CV_o^2}{\frac{1}{2}\frac{V_o^2}{R} \times T \times \frac{1}{2\pi}}$$

$$= \frac{\frac{1}{2}CV_o^2}{\frac{1}{2}\frac{V_o^2}{R}\cdot\frac{1}{W}}$$

$$= WCR = W\tau_p = \frac{W}{W_p}$$

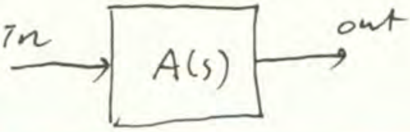
$Q\uparrow \rightsquigarrow \tau_p\uparrow$
$\rightsquigarrow$ large inertia energy
$\rightsquigarrow$ slow response time
$\rightsquigarrow$ slow speed.

$\Rightarrow$ higher Q $\rightarrow$ higher stored energy than dissipated energy by R

$\Rightarrow$ ex $R=\infty \rightarrow Q=\infty \longrightarrow$ Pure C
$\longrightarrow$ pure energy storage element

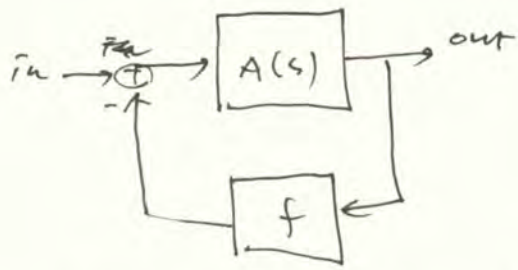
$$\boxed{\therefore A(s) = \frac{R}{1+s\tau_p} = \frac{R}{1+s/\omega_p} = \frac{W_u}{s+\frac{W}{Q}}}$$

* one-pole system with feedback



understand meaning of each expression.

$$\Rightarrow \boxed{A(s) = \frac{A_o}{1+s\tau_p} = \frac{A_o}{1+s/\omega_p} = \frac{\omega_u}{s+\omega_p} = \frac{\omega_u}{s+\frac{\omega}{Q}}}$$



$$\Rightarrow A_f(s) = \frac{A(s)}{1+T} = \frac{A(s)}{1+A(s)f} = \frac{\frac{A_o}{1+s\tau_p}}{1+\frac{A_o}{1+s\tau_p}f}$$

$$= \frac{A_o}{(1+A_o f)+s\tau_p}$$

$$= \frac{A_o}{1+A_o f}\,\frac{1}{1+s\frac{\tau_p}{1+A_o f}}$$

$$= \frac{A_o}{1+T_o}\,\frac{1}{1+s\left(\frac{\tau_p}{1+T_o}\right)} \leftarrow T_o = A_o f \text{ (DC loop gain)}$$

Time constant decreases by a factor of $(1+T_o)$.
⇒ Can you explain this in more intuitive way?

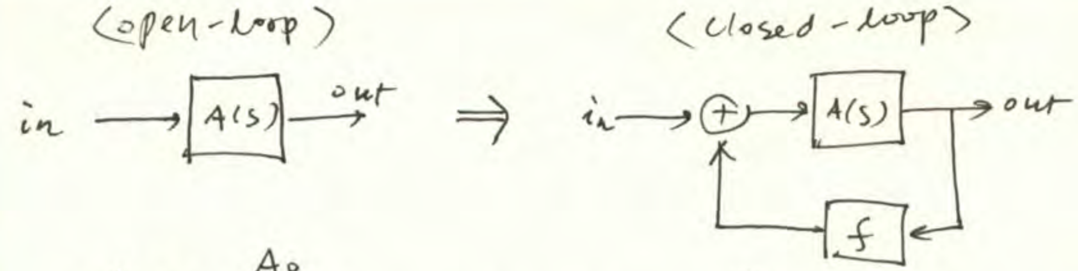
*

Negative feedbacks

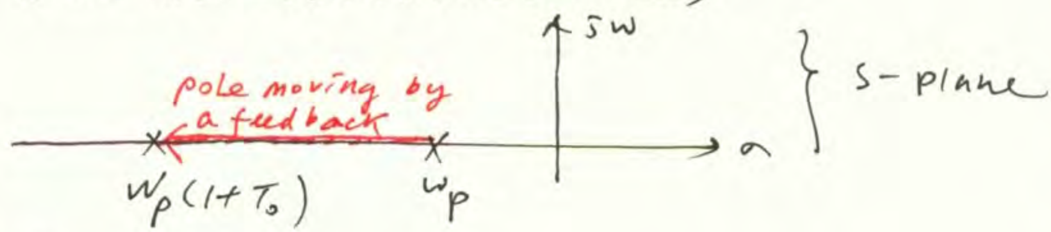
1. decreased τ_p by a factor of $(1+T_o)$
2. increase ω_p by a factor of $(1+T_o)$
3. decrease Q by a factor of $(1+T_o)$

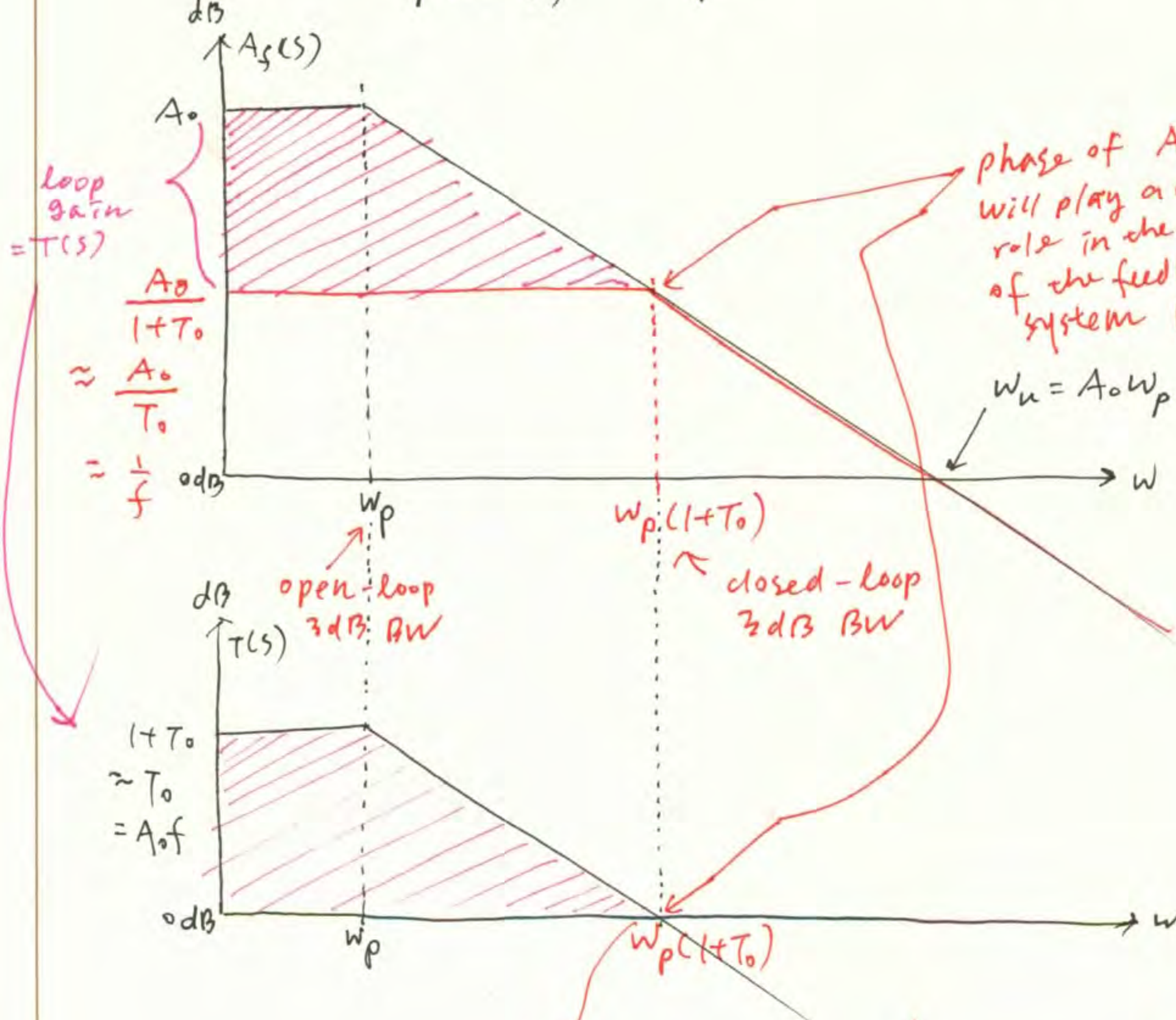
$$\therefore A_f(s) = \frac{A_o}{1+T_o}\,\frac{1}{1+s\frac{\tau_p}{1+T_o}} = \frac{A_o}{1+T_o}\,\frac{1}{1+\frac{s}{\omega_p(1+T_o)}} = \frac{\overset{\text{constant}}{\omega_u = A_o\omega_p}}{s+\frac{\omega}{\frac{Q}{1+T_o}}}$$

$\tau_p \downarrow \times (1+T_o)$ $\quad$ $\omega_p \uparrow \times (1+T_o)$ $\quad$ $Q \downarrow \times (1+T_o)$



$$A(s) = \frac{A_o}{1+s/\omega_p} \Rightarrow A_f(s) = \frac{A_o}{1+T_o}\,\frac{1}{1+s/\omega_p(1+T_o)}$$





$$|T(s)| = 1 \;,\quad \angle T(s)\Big|_{@\,\omega_p(1+T_o)} = ?$$

① $A(s) = \frac{A_o}{1+s/\omega_p} \longrightarrow$ $|A(s)| = \frac{A_o}{\sqrt{1+\frac{\omega^2}{\omega_p^2}}}$

$\angle A(s) = -\tan^{-1}\left(\frac{\omega}{\omega_p}\right)$

$\rightarrow$ @ $\omega = \omega_p \longrightarrow \angle A(s) = -45^\circ$

@ $\omega = \infty \longrightarrow \angle A(s) = -90^\circ$

② $T(s) = A(s) f = \frac{A_o f}{1+s/\omega_p} \rightarrow$ $|T(s)| = |A(s)| f$

→ Just a scaled version of A(s)

$\angle T(s) = \angle A(s) = -\tan^{-1}\left(\frac{\omega}{\omega_p}\right)$

→ the same as that of A(s).

③ $A_f(s) = \frac{A(s)}{1+T(s)} \rightarrow$ if $|T(s)| = 1 \rightarrow 0\,dB$

and $\angle T(s) = -180^\circ$

→ This means

$T(s) = -1$

$A_f(s) = \infty$

→ unstable.

Phase Margin (ϕ_M)

⇒ A measure of a stability in feedback systeems.

⇒ How much far away of $\angle T(s)$ from -180° @ $\omega = \omega_p(1+T_o)$

⇒ $\phi_M = 180^\circ - \left|\angle T(s)\right|_{@\,\omega = \omega_p(1+T_o)}$

$= 180^\circ - \tan^{-1}\left(\frac{\omega_p(1+T_o)}{\omega_p}\right)$

$= 180^\circ - \tan^{-1}(1+T_o)$

$\approx 180^\circ - \tan^{-1}(T_o)$

$= 180^\circ - \tan^{-1}(A_o f)$

Most electronic circuits & systems $\phi_M \geq 60^\circ$ (practically $\phi_M \geq 90^\circ$)

→ for one-pole system, $\phi_M \geq 90^\circ$

$$A_f(s) = \frac{A(s)}{1+T(s)}$$

if $|T(s)|$ is smaller than 1 when $\angle T(s) = -180°$, $A_f(s)$ will also be stable.

⇓

Gain Margin (G_M)

⇒ Alternative way of measuring stability of a feedback system (But not popular way)

⇒ By how much $|T(s)|$ is smaller than 1 when $\angle T(s) = -180°$

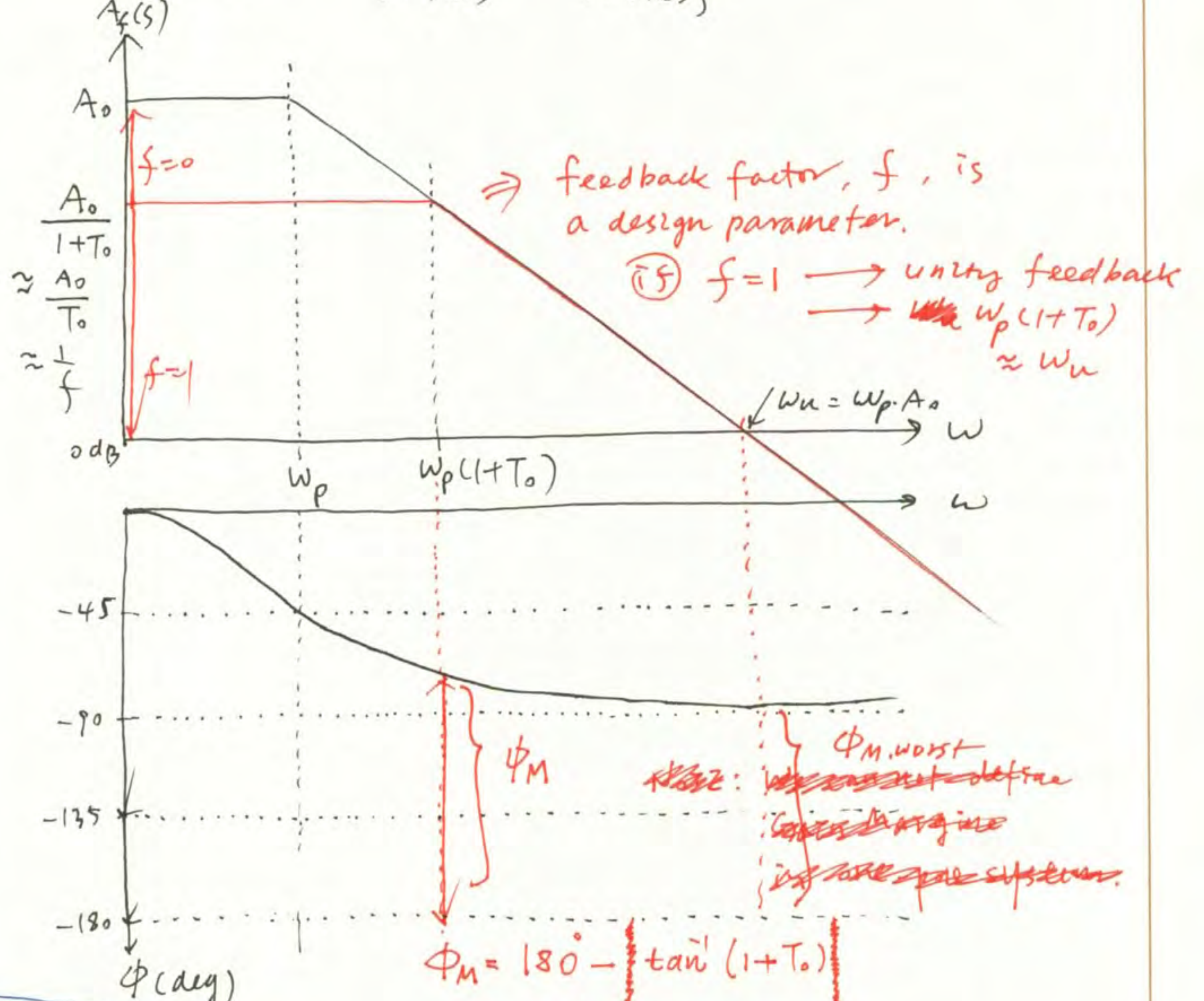
⇒ $$G_M = \frac{1}{|T(s)|\Big|_{\text{when } \angle T(s) = -180°}}$$

⇒ usually dB-scale.

example

one-pole system with feedback

$$A_f(s) = \frac{A(s)}{1+T(s)} = \frac{A(s)}{1+A(s)f}$$



$$\phi_M = 180^\circ - \left|\tan^{-1}(1+T_o)\right|$$
$$\approx 180^\circ - \tan^{-1}(T_o)$$
$$\approx 180^\circ - \tan^{-1}(A_o f)$$

NOTE: We can't define Gain Margin in one-pole system

⇒ Worst case ϕ_M is when $f=1$

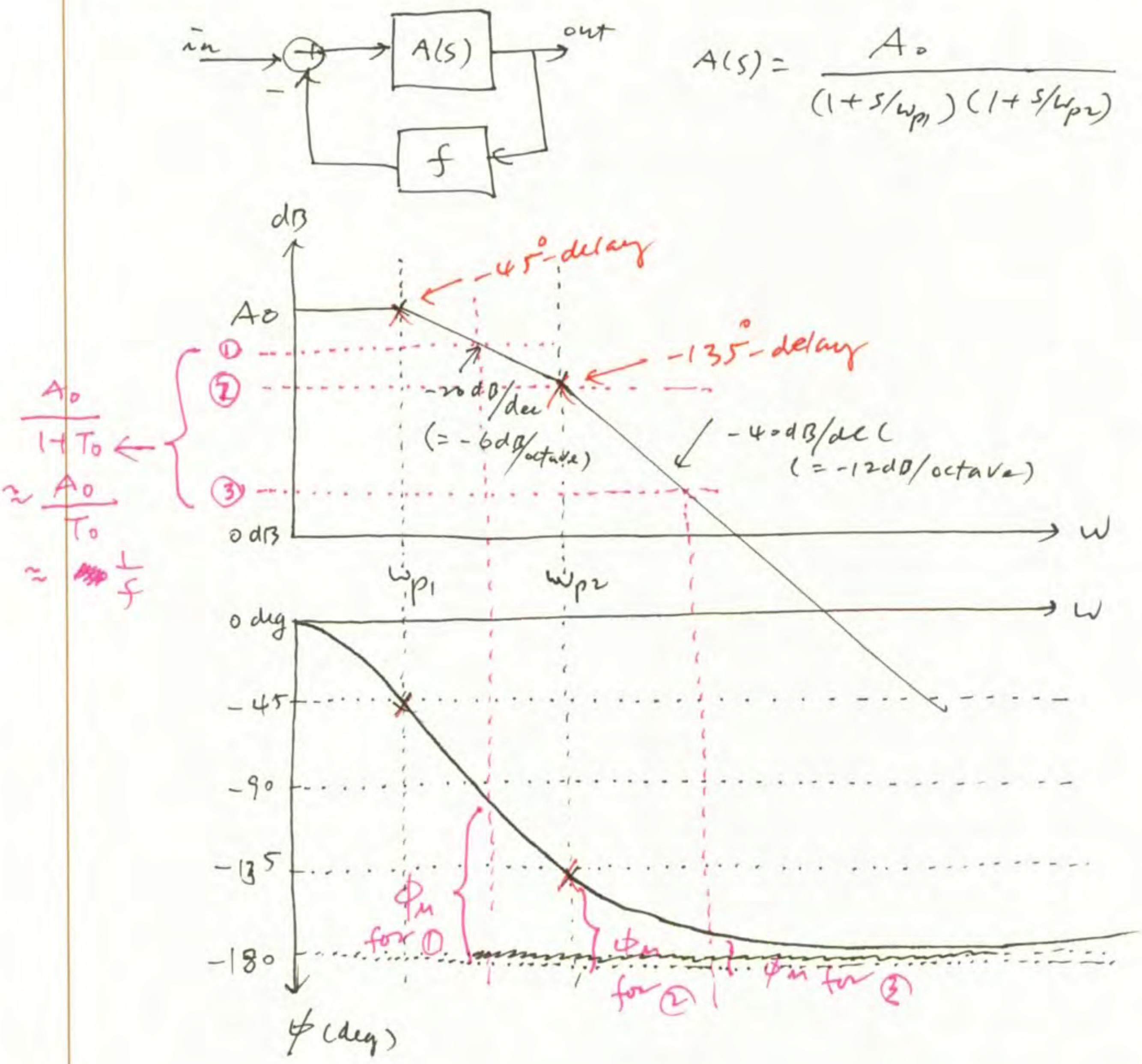
⇒ $\phi_{M,worst} = 180^\circ - \tan^{-1}\left(\frac{\omega_u}{\omega_p}\right) = 180^\circ - \tan^{-1}(A_o)$

⇒ $\phi_{M,worst} > 90^\circ$

⇒ Absolutely stable for any feedback factor of f

example — uncompensated two-pole system.

~~Example~~ Two-pole system with feedback



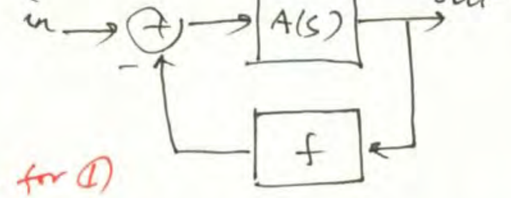
⇒ examples for three different cases of f

⇒ $\phi_M > 0$ (In a strict sense 2-pole system is stable for any value of f. But usually we have extra parasitic poles which causes $\phi_M < 0$)

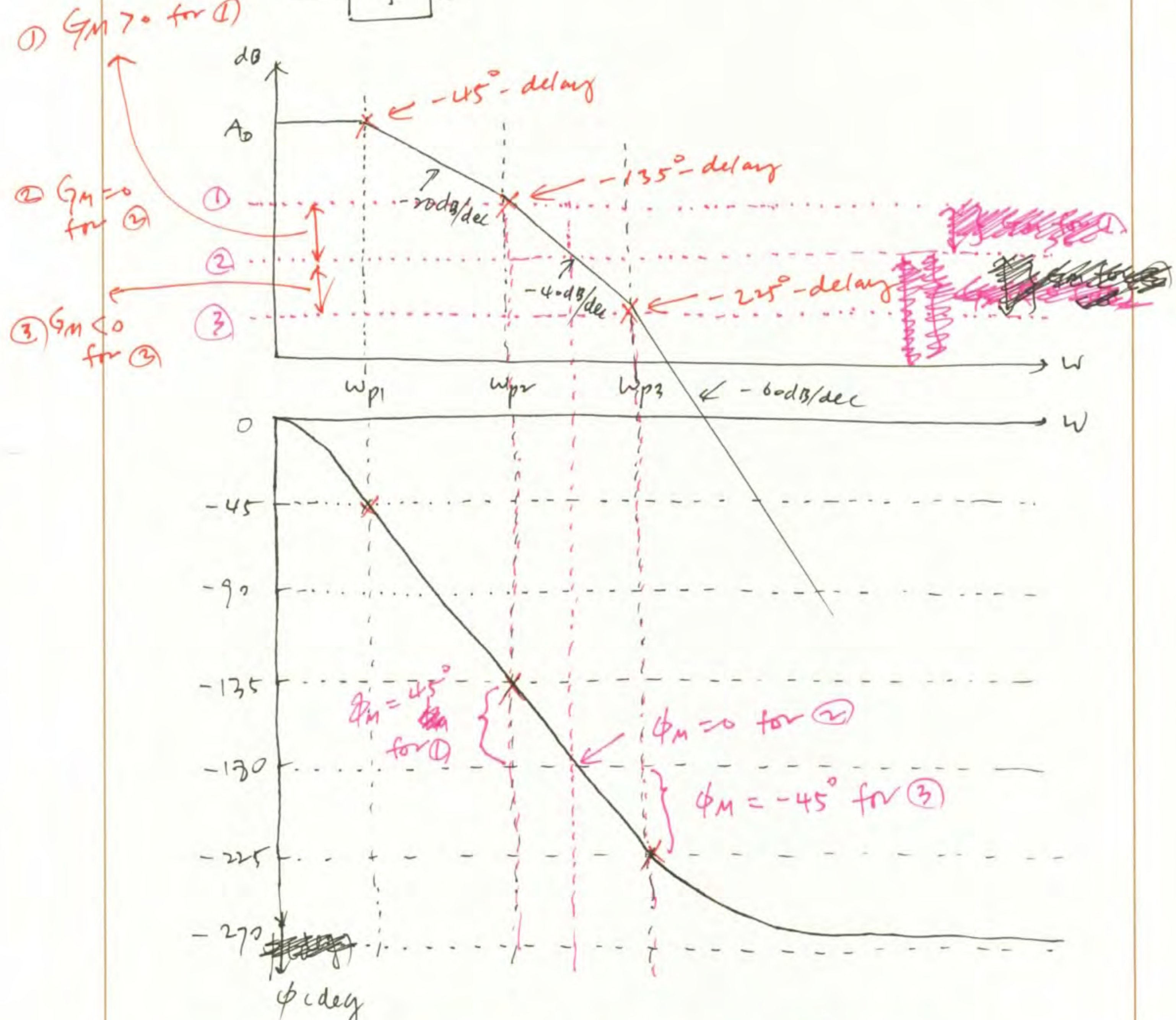
⇒ We cannot define Gain Margin in 2-pole system.

example

three-pole system with feedback



$$A(s) = \frac{A_0}{(1+s/\omega_{p1})(1+s/\omega_{p2})(1+s/\omega_{p3})}$$



⇒ 2-pole, 3-pole systems are not common.

⇒ They need frequency compensation.

① pole-splitting

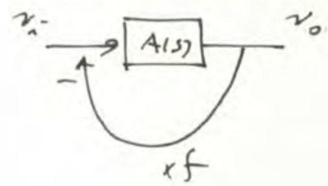
② zero-insertion ($\underbrace{\text{2-pole 1-zero}}_{}$, $\underbrace{\text{3-pole 2-zero}}_{}$)

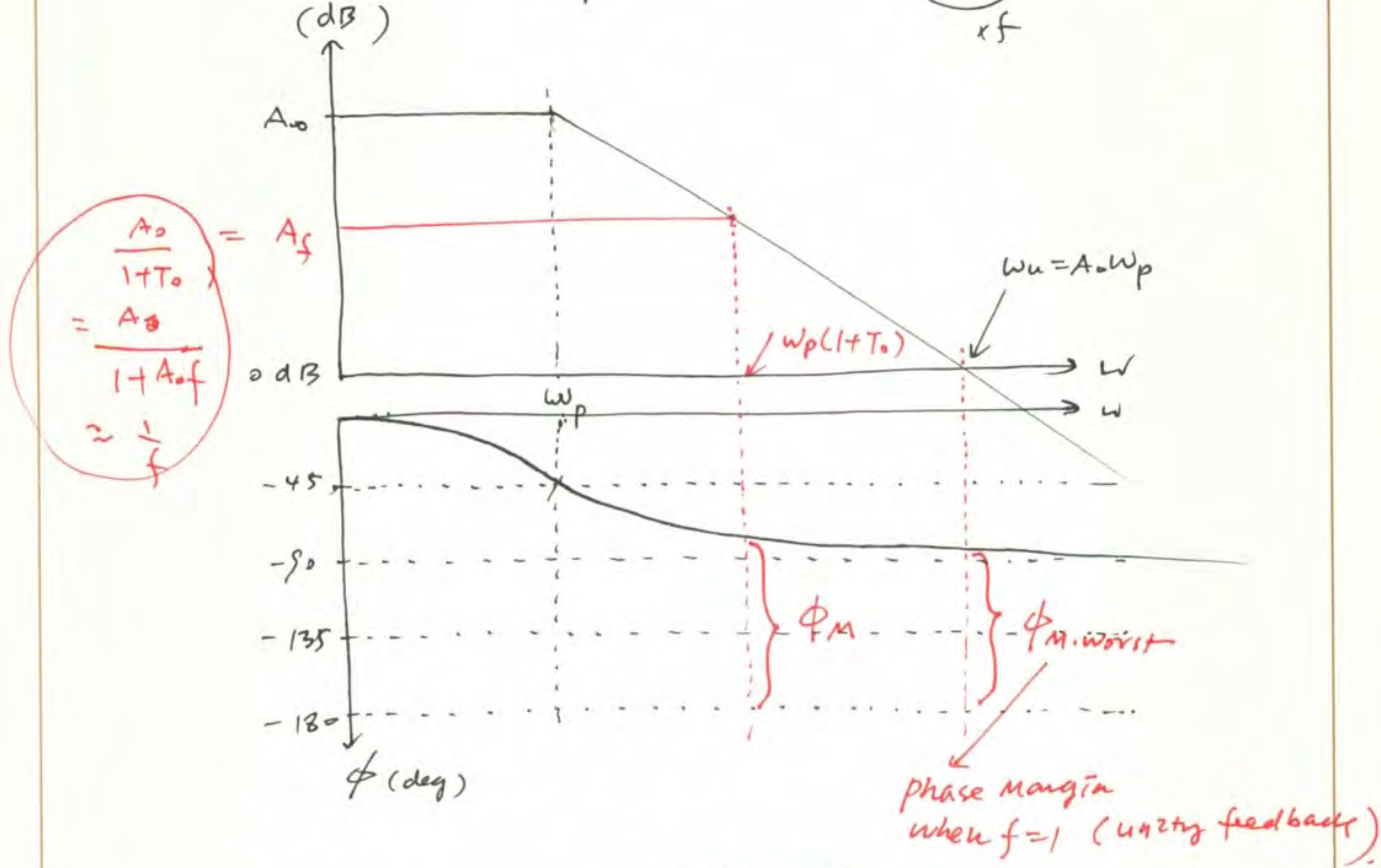
(effectively 1-pole system over each)

example (ϕ_M)

① one-pole system

$$A(s) = \frac{A_0}{1+s/\omega_p} \longrightarrow$$





$$\phi_M = 180^\circ - \tan\left(\frac{\omega_p(1+T_0)}{\omega_p}\right)$$
$$= 180^\circ - \tan^{-1}(1+T_0)$$
$$\simeq 180^\circ - \tan^{-1}(T_0)$$
$$= 180^\circ - \tan^{-1}(A_0 f)$$

$$\phi_{M.worst} = 180^\circ - \tan^{-1}\left(\frac{\omega_u}{\omega_p}\right)$$
$$= 180^\circ - \tan^{-1}(A_0)$$

$\boxed{\text{Gain} \times \text{BW} = \text{constant}}$

This can be applied to only one-pole system.

⊛ For one-pole systems, $\phi_M \geq 90^\circ$.

(ex single-stage amplifiers, single-stage op-amp.
Delay-locked loop (DLL))

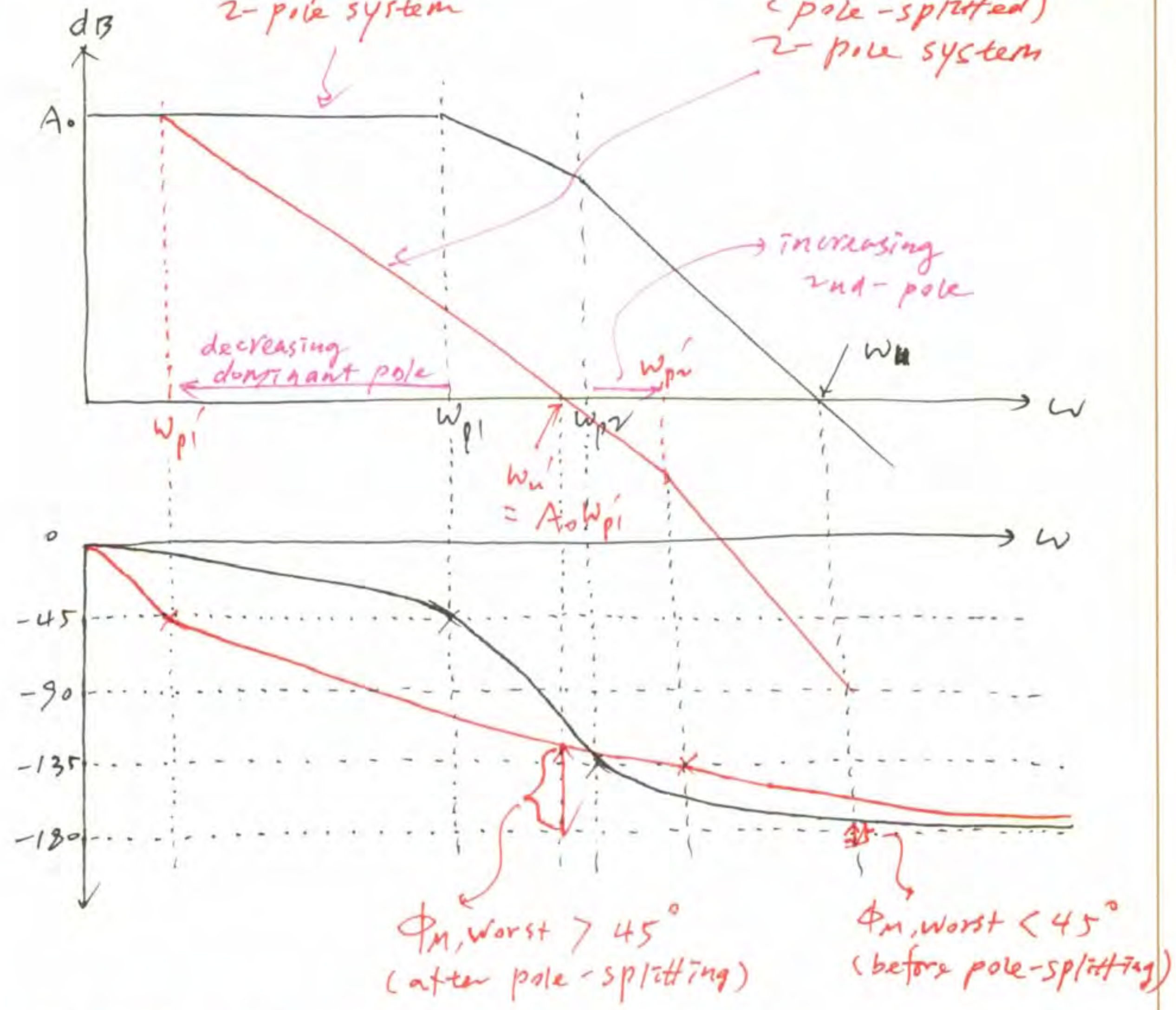
example (ϕ_M)

② Two-pole system (pole-splitting)

$$A(s) = \frac{A_o}{(1+s/\omega_{p1})(1+s/\omega_{p2})} \longrightarrow A(s) = \frac{A_o}{(1+s/\omega_{p1}')(1+s/\omega_{p2}')}$$

uncompensated 2-pole system

frequency-compensated (pole-splitted) 2-pole system

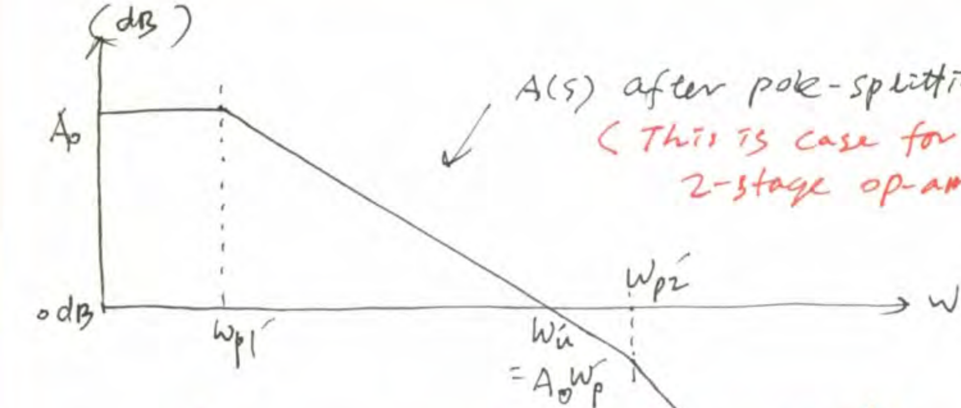


i) Before pole-splitting

$$\phi_{M,worst} = 180° - \left(90° + \tan^{-1}\frac{\omega_u}{\omega_{p2}}\right), \quad \omega_u > \omega_{p2}$$
$$< 45°$$

ii) After pole-splitting

$$\phi_{M,worst} = 180° - \left(90° + \tan^{-1}\frac{\omega_u'}{\omega_{p2}'}\right), \quad \omega_u' < \omega_{p2}'$$
$$> 45°$$



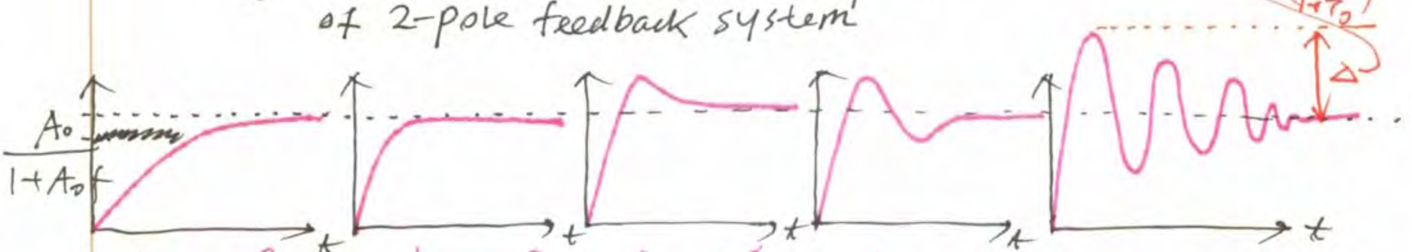
A(s) after pole-splitting
(This is case for most 2-stage op-amp designs)

$$\Phi_{M,worst} = 180^\circ - \left(90^\circ + \tan^{-1}\frac{\omega_u'}{\omega_{p2}'}\right)$$
$$= 90^\circ - \tan^{-1}\left(\frac{\omega_u'}{\omega_{p2}'}\right)$$

if
- $\omega_{p2}' \geq 10\cdot\omega_u' \longrightarrow \Phi_{M,worst} \cong 84.3^\circ$
- $\omega_{p2}' \simeq 2\cdot\omega_u' \longrightarrow \Phi_{M,worst} = 63.4^\circ$
- $\omega_{p2}' = \omega_u' \longrightarrow \Phi_{M,worst} = 45^\circ$

$$\%\text{-overshooting} = \frac{\Delta}{\left(\frac{A_0}{1+T_0}\right)} \times 100$$

⊛ Time-domain unit-step response of 2-pole feedback system



$\Phi_M > 80^\circ$ $\quad$ $\Phi_M \simeq 75\sim80^\circ$ $\quad$ $\Phi_M \simeq 60\sim65^\circ$ $\quad$ $\Phi_M = 45\sim50^\circ$ $\quad$ $\Phi_M = 10^\circ$

These are acceptable for most cases

This is acceptable for some cases. (PLL system)

Not ok.

Φ_M	ω_{p2}'/ω_u'	%-overshooting	Q
10°	0.18	76 %	5.7
45°	1	20 %	1.2
50°	1.2	16.3 %	1
55°	1.4	13.3 %	0.92
60°	1.7	8.7 %	0.82
65°	2.1	4.7 %	0.72
70°	2.7	1.4 %	0.62
75°	3.7	0.008 %	0.52
80°	5.7	No overshooting	0.42

⊛⊛⊛

$Q = 0.5 = \frac{1}{2} \longrightarrow \Phi_M \simeq 75^\circ$
⇒ fastest response without overshooting

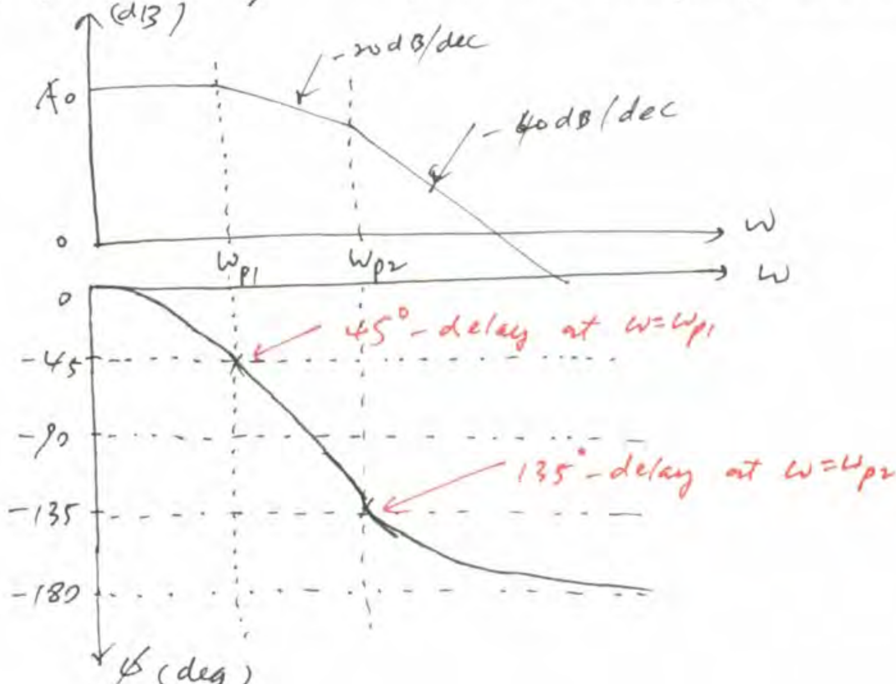
$Q = \frac{1}{\sqrt{2}} = 0.707 \longrightarrow \Phi_M \simeq 65^\circ$
⇒ maximally flat response in freq.-domain

* Two-pole system

in → $A(s)$ → out

$$\Rightarrow A(s) = \frac{A_o}{(1+s/\omega_{p1})(1+s/\omega_{p2})} = \frac{A_o}{(1+s\tau_{p1})(1+s\tau_{p2})}$$

⇒ provides good information in freq-domain



⇒ Another form

$$A(s) = \frac{A_o}{(1+s/\omega_{p1})(1+s/\omega_{p2})} = \frac{A_o}{1+s\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)+s^2\frac{1}{\omega_{p1}\cdot\omega_{p2}}}$$

$$= \frac{A_o\,\omega_{p1}\cdot\omega_{p2}}{s^2+s\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)\omega_{p1}\cdot\omega_{p2}+\omega_{p1}\cdot\omega_{p2}}$$

$$= A_o\frac{\omega_o^2}{s^2+\frac{\omega_o}{Q}s+\omega_o^2}$$

⇒ characteristic eq.
= Biquad eq.
= Quadratic eq.

natural freq. of a system ← $\omega_o = \sqrt{\omega_{p1}\cdot\omega_{p2}}$

pole quality factor ← $Q = \frac{\sqrt{\omega_{p1}\cdot\omega_{p2}}}{\omega_{p1}+\omega_{p2}}$

$$A(s) = \frac{A_0}{(1+s/\omega_{p1})(1+s/\omega_{p2})} = A_0 \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

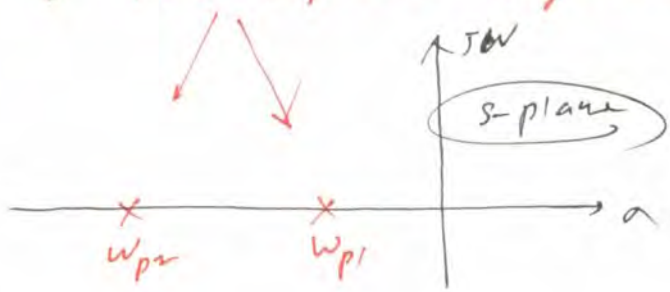
$$\boxed{\begin{aligned} &* \ \omega_0 = \sqrt{\omega_{p1}\cdot\omega_{p2}} \\ &* \ Q = \frac{\sqrt{\omega_{p1}\cdot\omega_{p2}}}{\omega_{p1}+\omega_{p2}} \\ &* \ \omega_{p1} = -\frac{\omega_0}{2Q} + \frac{\omega_0}{2Q}\sqrt{1-4Q^2} \\ &* \ \omega_{p2} = -\frac{\omega_0}{2Q} - \frac{\omega_0}{2Q}\sqrt{1-4Q^2} \end{aligned}}$$

(*) In general, open-loop amplifiers,

$0 \le Q < \frac{1}{2}$

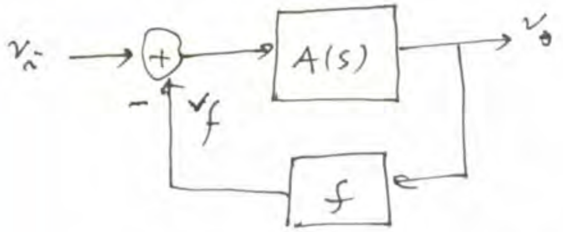
⇒ Stored energy is ~~less than~~ ~~half~~ less than 50% of dissipated energy.

⇒ Two separate negative ~~real~~ real poles



Q) What happen when applying feedback to the 2nd-order system?

(*) Two-pole system with feedback



$$A_f(s) = \frac{v_o}{v_i} = \frac{A(s)}{1+T(s)} = \frac{A(s)}{1+A(s)f}$$

$$= \frac{\dfrac{A_o}{(1+s/\omega_{p1})(1+s/\omega_{p2})}}{1 + \dfrac{A_o}{(1+s/\omega_{p1})(1+s/\omega_{p2})} f}$$

$$= \frac{\dfrac{A_o \cdot \omega_o^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}}{1 + \dfrac{A_o \cdot \omega_o^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2} f}$$

$$= \frac{A_o \omega_o^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2 + A_o f \cdot \omega_o^2}$$

$$= \frac{A_o \omega_o^2}{s^2 + \frac{\omega_o}{Q}s + (1+A_o f)\omega_o^2}$$

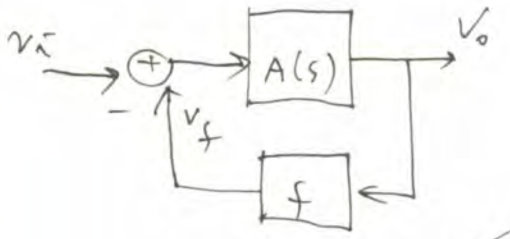
$$= \frac{A_o}{1+A_o f} \cdot \frac{(1+A_o f)\,\omega_o^2}{s^2 + \frac{\omega_o}{Q}s + (1+A_o f)\omega_o^2}$$

change this to quadratic form.

$$= \frac{A_o}{1+A_o f} \cdot \frac{\omega_{of}^2}{s^2 + \frac{\omega_{of}}{Q_f}s + \omega_{of}^2}$$

$$\omega_{of} = \omega_o\sqrt{1+A_o f} = \omega_o\sqrt{1+T_o}$$

$$Q_f = Q\sqrt{1+A_o f} = Q\sqrt{1+T_o}$$



$$A(s) = \frac{A_0}{(1+s/\omega_{p1})(1+s/\omega_{p2})}$$

$$= \frac{A_0 \cdot \omega_o^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}$$

$$A_f(s) = \left(\frac{A_0}{1+A_0 f}\right) \times \left(\frac{\omega_{of}^2}{s^2 + \frac{\omega_{of}}{Q_f}s + \omega_{of}^2}\right)$$

* $\omega_{of} = \omega_o \sqrt{1+A_o f} = \omega_o \sqrt{1+T_o}$
* $Q_f = Q\sqrt{1+A_o f} = Q\sqrt{1+T_o}$
* $\omega_{p1f} = -\frac{\omega_{of}}{2Q_f} + \frac{\omega_{of}}{2Q_f}\sqrt{1-4Q_f^2}$
* $\omega_{p2f} = -\frac{\omega_{of}}{2Q_f} - \frac{\omega_{of}}{2Q_f}\sqrt{1-4Q_f^2}$

Net results of feedback to second-order system.

① ω_o increases by a factor of $\sqrt{1+T_o}$

② Q increases by a factor of $\sqrt{1+T_o}$
(This is the opposite of one-pole system. In one-pole system, feedback decreases Q.)

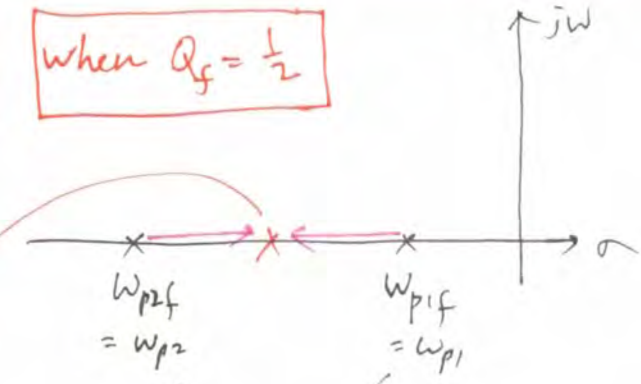
⇒ Q_f can be larger than $\frac{1}{2}$.

⇒ This means accumulated system energy can be larger than 50% of dissipated energy

⇒ ω_{p1f} & ω_{p2f} become complex poles.

(Q) what's the meaning of a complex pole?

① $\boxed{\text{when } Q_f = \frac{1}{2}}$



$$\omega_{p1f} = -\frac{\omega_{of}}{2Q_f} + \frac{\omega_{of}}{2Q_f}\sqrt{1-4Q_f^2}$$

$$\omega_{p2f} = \frac{\omega_{of}}{-2Q_f} - \frac{\omega_{of}}{2Q_f}\sqrt{1-4Q_f^2}$$

$$\omega_{of} = \omega_o\sqrt{1+T_o}$$

$$Q_f = Q\sqrt{1+T_o}$$

if $T_o = 0$ (No feedback)

⇒ 2 negative real poles

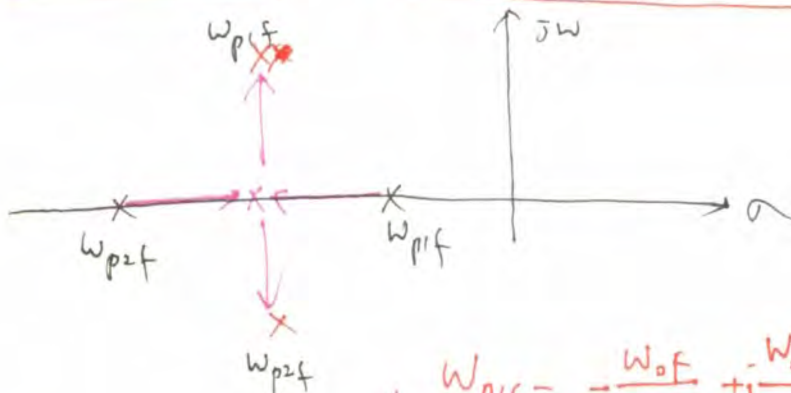
⇒ ∴ $Q < \frac{1}{2}$

if $T_o \uparrow \rightarrow Q_f = Q\sqrt{1+T_o} \uparrow$

when $Q_f = \frac{1}{2}$ (accumulated system energy = 50% of dissipated energy)

$$\omega_{p1f} = \omega_{p2f} = \frac{\omega_{of}}{-2Q_f} = -\frac{\sqrt{\omega_{p1}\cdot\omega_{p2}}}{2\,\frac{\sqrt{\omega_{p1}\cdot\omega_{p2}}}{\omega_{p1}+\omega_{p2}}} = -\frac{1}{2}(\omega_{p1}+\omega_{p2})$$

② $\boxed{\text{when } Q_f > \frac{1}{2}}$



$$\omega_{p1f} = -\frac{\omega_{of}}{2Q_f} + j\frac{\omega_{of}}{2Q_f}\sqrt{4Q_f^2-1} = -\frac{\omega_o}{2Q} + j\frac{\omega_o}{2Q}\sqrt{4Q^2(1+T_o)-1}$$

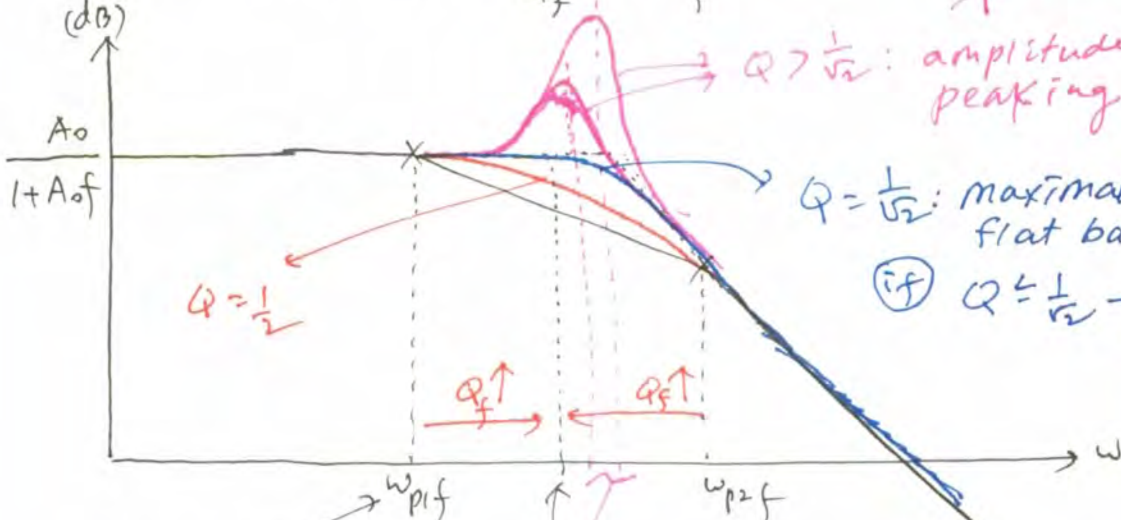
$$\omega_{p2f} = -\frac{\omega_o}{2Q} - j\frac{\omega_o}{2Q}\sqrt{4Q^2(1+T_o)-1}$$

poles becomes complex poles

Feedback Basics

$$A(s) = \frac{A_o}{1+A_of} \frac{\omega_{of}^2}{s^2 + \frac{\omega_{of}}{Q_f}s + \omega_{of}^2}$$

looks like ↑ freq. filter.
frequency selectivity ↑



$$= \frac{\omega_{of}}{2Q_f} - \frac{\omega_{of}}{2Q_f}\sqrt{1-4Q_f^2}$$
$$= \frac{\omega_o}{2Q} - \frac{\omega_o}{2Q}\sqrt{1-4Q^2(1+T_o)}$$
$(Q < \frac{1}{2})$

$$= \frac{\omega_{p1f}+\omega_{p2f}}{2}$$
$$= \frac{\omega_{p1}+\omega_{p2}}{2}$$
$$= \frac{\omega_o}{2Q} = \boxed{\omega_o}$$
$(\because Q = \frac{1}{2})$

$$= \frac{\omega_{of}}{2Q_f} + \frac{\omega_{of}}{2Q_f}\sqrt{1-4Q_f^2}$$
$$= \frac{\omega_o}{2Q} + \frac{\omega_o}{2Q}\sqrt{1-4Q^2(1+T_o)}$$
$(Q < \frac{1}{2})$

peaking frequency

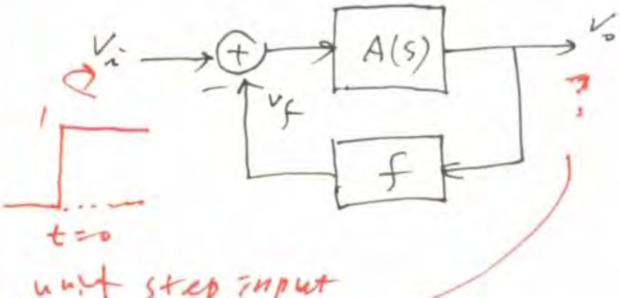
$$\omega_{peak} = |\text{imaginary freq.}|$$
$$= \left| j\frac{\omega_{of}}{2Q_f}\sqrt{4Q_f^2-1} \right|$$
$$= \frac{\omega_o}{2Q}\sqrt{4Q^2(1+T_o)-1}$$
$$\approx \frac{\omega_o}{2Q}\sqrt{4Q^2(1+T_o)}$$
$$\approx \omega_o\sqrt{T_o}$$

In frequency domain

- Meaning of imaginary freq.
  ⇒ Amplitude peaking at the frequency

(ex)

Q_f	Peaking
$Q_f = 1$	1.25 dB
$Q_f = 1.15$	2 dB
$Q_f = 1.3$	3 dB
$Q_f = 1.5$	4 dB
$Q_f = 1.7$	5 dB
$Q_f = 2$	6.3 dB

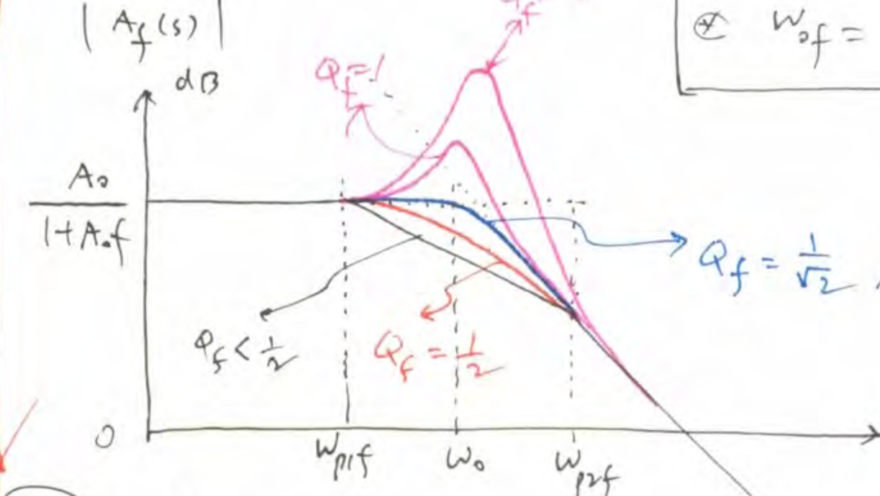


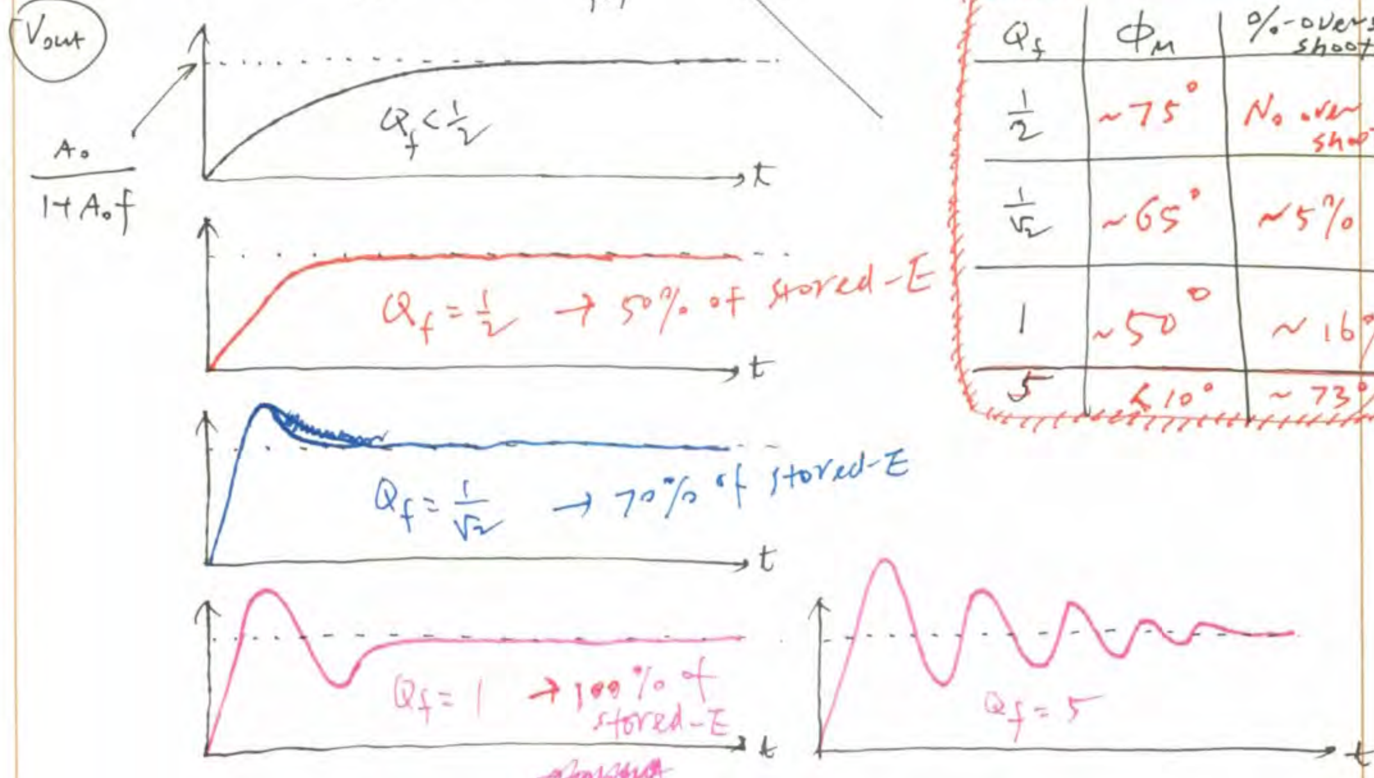
$$\circledast\ A(s) = \frac{A_0}{(1+s/\omega_{p1})(1+s/\omega_{p2})} = \frac{A_0 \cdot \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\circledast\ A_f(s) = \frac{A_0}{1+T_0}\,\frac{\omega_{0f}^2}{s^2 + \frac{\omega_{0f}}{Q_f}s + \omega_{0f}^2}$$

$$\circledast\ Q_f = Q\sqrt{1+T_0} = Q\sqrt{1+A_0 f}$$

$$\circledast\ \omega_{0f} = \omega_0\sqrt{1+T_0} = \omega_0\sqrt{1+A_0 f}$$





Q_f	Φ_M	%-overshooting
$\frac{1}{2}$	~75°	No overshooting
$\frac{1}{\sqrt{2}}$	~65°	~5%
1	~50°	~16%
5	< 10°	~73%