Solution (ECE 4320 Final)

(Q-1) $A_0 = g_{m1} g_{m2} R_1 \cdot R_2 = 2 \times 4 \times 100 \times 10 = 8 \times 10^3 \rightarrow 78\,dB$

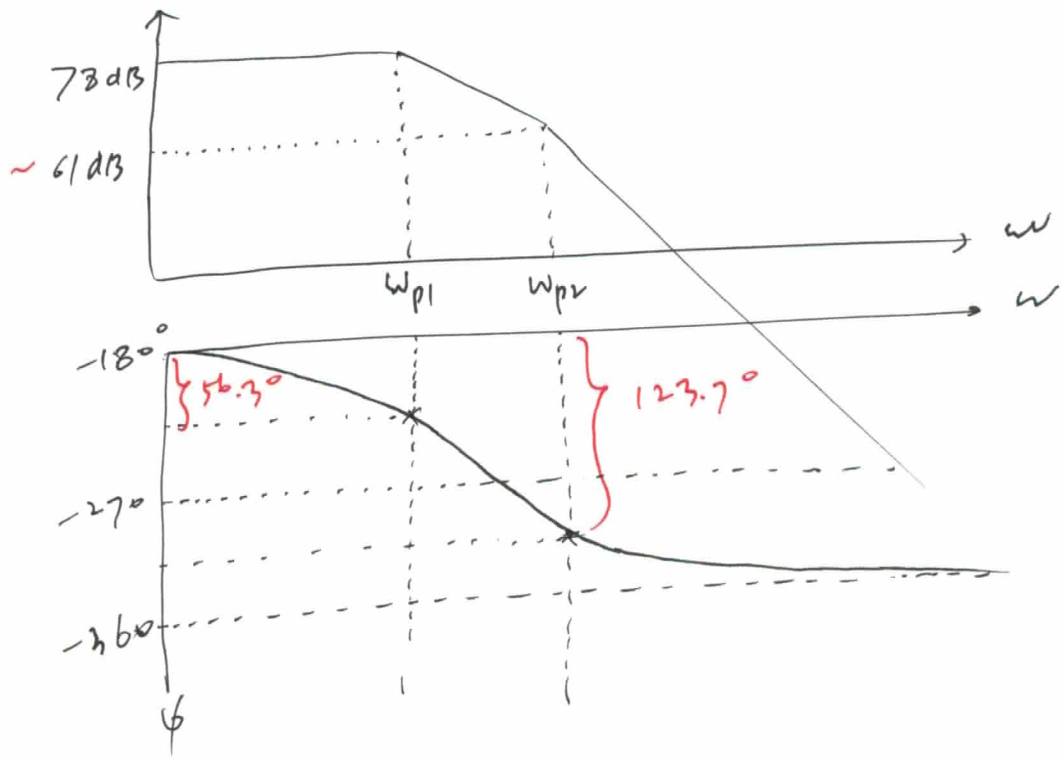
$\omega_{p1} = \frac{1}{R_1 C_1} = \frac{1}{100K\Omega \cdot 100fF} = 100M\ rps \rightarrow \frac{1}{2\pi} \times 100\ MHz$

$\omega_{p2} = \frac{1}{R_2 C_2} = \frac{1}{10K\Omega \cdot 200fF} = 500M\ rps \rightarrow \frac{1}{2\pi} \times 500\ MHz$

$$A(s) = -\frac{A_0}{(1+s/\omega_{p1})(1+s/\omega_{p2})}$$

$$A(j\omega_{p1}) = \frac{-A_0}{(1+j)(1+j0.2)} = \underbrace{-A_0 \times 0.7}_{(78-3)dB} \angle -56.3^\circ$$

$$A(j\omega_{p2}) = \frac{-A_0}{(1+j5)(1+j)} = \underbrace{-A_0 \times 0.14}_{78-17 = 61\,dB} \angle -123.7^\circ$$



(Q-2) $A(s) = -A_0 \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$

$\omega_0 = \sqrt{\omega_{p1} \cdot \omega_{p2}} = \sqrt{5} \times 10^2\ Mrps = \cancel{447.2}\ 223.6\ Mrps$

$Q = \frac{\omega_0}{\omega_{p1} + \omega_{p2}} = \frac{\sqrt{5} \times 100 \times 10^6}{6 \times 100 \times 10^6} = \frac{\sqrt{5}}{6} = 0.37$

Q-3) $A_f(s) = -A_{0f}\dfrac{\omega_{0f}^2}{s^2+\frac{\omega_{0f}}{Q_f}s+\omega_{0f}^2}$

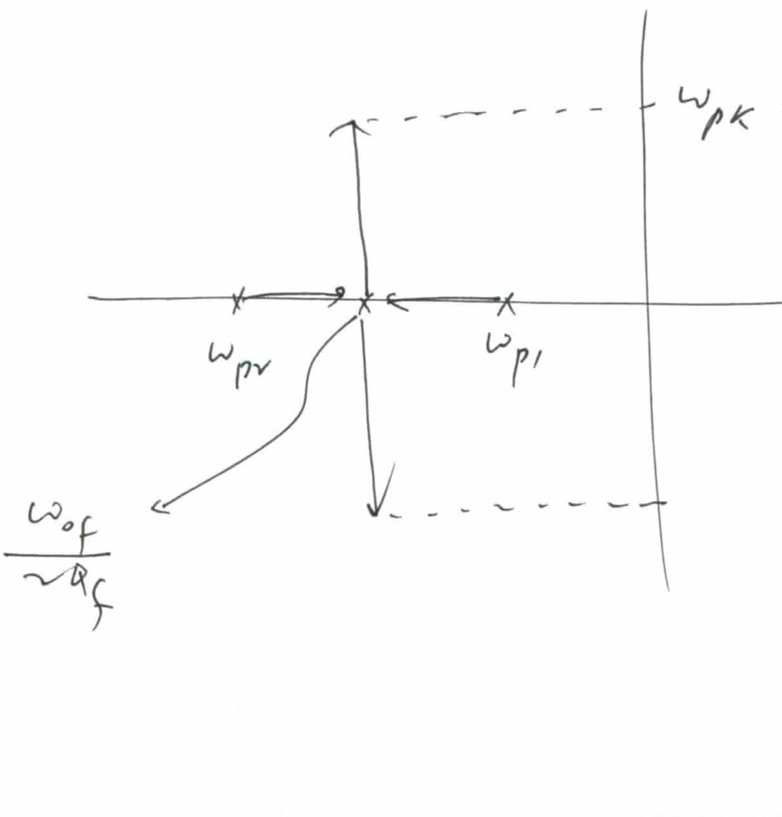
- $\omega_{0f} = \omega_0\sqrt{1+T} = \omega_0\sqrt{1+A} = \sqrt{A}\cdot\omega_0$
$= \sqrt{8\times10^3}\,\omega_0$
$= 89.4\,\omega_0 = 20\ \text{Grps}$

- $Q_f = Q\sqrt{1+T} = Q\sqrt{1+A} = \sqrt{A}\,Q$
$= 89.4\,Q = 33.1$

- $\omega_{pk} = \dfrac{\omega_{0f}}{2Q_f}\sqrt{4Q_f^2-1} \simeq \omega_{0f}$

- $|A_f(j\omega_{pk})| = |A_f(j\omega_{0f})| = \dfrac{A_0}{1+A_0}\cdot Q_f \simeq Q_f$
- $|A_f(0)| = \dfrac{A_0}{1+A_0} \simeq 1$

$\rightarrow \left|\dfrac{A_f(j\omega_{pk})}{A_f(0)}\right| = Q_f = 33.1 \longrightarrow 30.4\ \text{dB}$



$\omega_{pk} = \dfrac{\omega_{0f}}{2Q_f}\sqrt{4Q_f^2-1}$
$\simeq \omega_{0f}$

③

PM → $T(s) = A_0 \dfrac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$

$$T(j\omega_u) = A_0 \frac{\omega_0^2}{(\omega_0^2 - \omega_u^2) + j\frac{\omega_0\omega_u}{Q}}$$

$$|T(j\omega_u)| = \frac{A_0 \cdot \omega_0^2}{\sqrt{(\omega_0^2 - \omega_u^2)^2 + \frac{\omega_0^2\omega_u^2}{Q^2}}} = 1$$

$$\rightarrow A_0^2\omega_0^4 = (\omega_0^2 - \omega_u^2)^2 + \frac{\omega_0^2 \cdot \omega_u^2}{Q^2}$$

$$\rightarrow \omega_u^4 + \left(\frac{\omega_0^2}{Q^2} - 2\omega_0^2\right)\omega_u^2 + \omega_0^4 - A_0^2\omega_0^4 = 0$$

$$\rightarrow \omega_u = \omega_0\sqrt{1 + A_0} = \omega_{0f}$$

$$PM = 180^\circ - |\angle A(j\omega_u)| = 180^\circ - |\angle A(j\omega_{0f})|$$

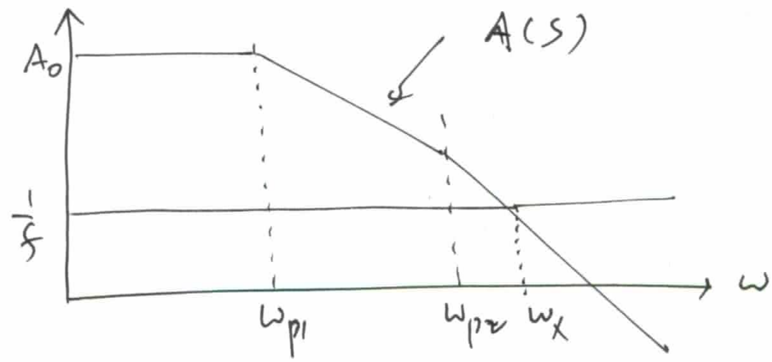
$$= 180^\circ - \left|\angle\left(\frac{-A_0}{\left(1 + j\frac{\omega_{0f}}{\omega_{p1}}\right)\left(1 + \frac{j\omega_{0f}}{\omega_{p2}}\right)}\right)\right|$$

$$= 180^\circ - \left|\angle\left(\frac{A_0}{\left(1 + j\frac{20\times10^9}{10^8}\right)\left(1 + j\frac{20\times10^9}{5\times10^8}\right)}\right)\right|$$

$$= 180^\circ - 178.3^\circ$$

$$= 1.7^\circ$$

Q-4)



For PM = 45°, $\left|A(j\omega_x)\right| = 135°$

$$\rightarrow \tan^{-1}\left(\frac{\omega_x}{\omega_{p1}}\right) + \tan^{-1}\left(\frac{\omega_x}{\omega_{p2}}\right) = 135°$$

$$\rightarrow \tan^{-1}\left(\frac{\frac{\omega_x}{\omega_{p1}} + \frac{\omega_x}{\omega_{p2}}}{1 - \frac{\omega_x^2}{\omega_{p1}\cdot\omega_{p2}}}\right) = 135°$$

$$\rightarrow \omega_x^2 - \omega_x(\omega_{p1} + \omega_{p2}) - \omega_{p1}\cdot\omega_{p2} = 0$$

$$\rightarrow \omega_x - \frac{\omega_0}{Q}\omega_x - \omega_0^2 = 0$$

$$\rightarrow \omega_x = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q}\sqrt{4Q^2+1}$$

$$= 6.78 \times 10^8 \text{ rps}$$

$$\left|A(j\omega_x)\right| = \left|\frac{A_0}{\left(1 + j\frac{\omega_x}{\omega_{p1}}\right)\left(1 + j\frac{\omega_x}{\omega_{p2}}\right)}\right| = A_0 \times 86.6 \times 10^{-3} = \frac{1}{f}$$

$$\rightarrow f = 1.43 \times 10^{-3}$$

$$\therefore \quad 0 \le f \le 1.43 \times 10^{-3}$$

Q-5) $f = 1.43\times10^{-3} \longrightarrow PM = 45°$

$$Q_f = Q\sqrt{1+A_0 f}$$
$$= 0.37\times\sqrt{1+8\times10^{3}\times1.43\times10^{-3}}$$
$$= 0.37\times\sqrt{1+8\times1.43}$$
$$= 1.3$$

$$\omega_{pk} = \frac{\omega_{of}}{2Q_f}\sqrt{4Q_f^2-1}$$
$$= \frac{\omega_o}{2Q}\sqrt{4Q_f^2-1}$$
$$= \frac{223.6\times10^6}{2\times0.37}\sqrt{4(1.3)^2-1}$$
$$= 7.25\times10^8 \text{ rps}$$

$$\left|\frac{A(j\omega_{pk})}{A(0)}\right| = \left|\frac{\omega_{of}^2}{s^2+\frac{\omega_{of}}{Q_f}s+\omega_{of}^2}\right|_{s=j\omega_{pk}}$$

$$= \frac{\omega_{of}^2}{\sqrt{\left(\omega_{of}^2-\omega_{pk}^2\right)^2+\left(\frac{\omega_{of}}{Q_f}\cdot\omega_{pk}\right)^2}}$$

$*$
$\omega_{of} = \omega_o\sqrt{1+Af} = 223.6\times10^6\times\sqrt{1+8\times1.43}$
$= 7.89\times10^8$ rps

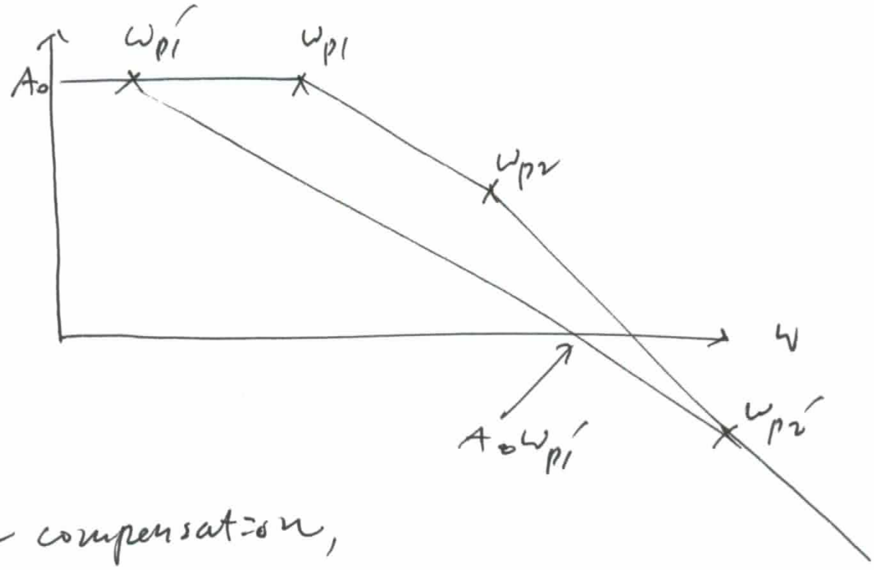
$\omega_{pk} = 7.25\times10^8$ rps

$Q_f = 1.3$

since $\omega_{pk} \approx \omega_{of}$

$$\therefore \left|\frac{w(j\omega_{pk})}{A(0)}\right| \approx Q_f = 1.3$$

(Q-6)



After compensation,

$$\omega_{p1}' = \frac{1}{R_1\{C_1 + C_c(g_{m2}R_2+1)\}}$$

$$\simeq \frac{1}{R_1 \cdot g_{m2}R_2 C_c}$$

$$\omega_{p2}' \approx \frac{g_{m2}}{C_1+C_2}$$

$$PM = 80^\circ \longrightarrow \tan^{-1}\left(\frac{A_0\omega_{p1}'}{\omega_{p2}'}\right) = 10^\circ$$

$$\therefore\ C_c = \frac{A_0}{R_1 \cdot g_{m2} \cdot R_2} \cdot \frac{C_1+C_2}{g_{m2}} \cdot \frac{1}{\tan(10^\circ)}$$

$$= \frac{200\,fF}{4\times4\times100\times10} \cdot \frac{8\times10^3}{\tan 10^\circ}$$

$$= 0.85\,pF$$

$$R_Z = \frac{1}{g_{m2}} = \frac{1}{4m} = 250\,\Omega$$

(Q-7) $\omega_{p1}' = \dfrac{1}{R_1 \cdot R_2 \cdot g_{m2} C_2} = \dfrac{1}{10k \cdot 100k \cdot 4m \cdot 0.85p}$

$= 2.94 \times 10^5$ rps

<u>First-order estimation</u>

- $\omega_{BW} = A_0 \omega_{p1}' = 8 \times 10^3 \times 2.94 \times 10^5$

  $= 2.35 \times 10^9$ rps

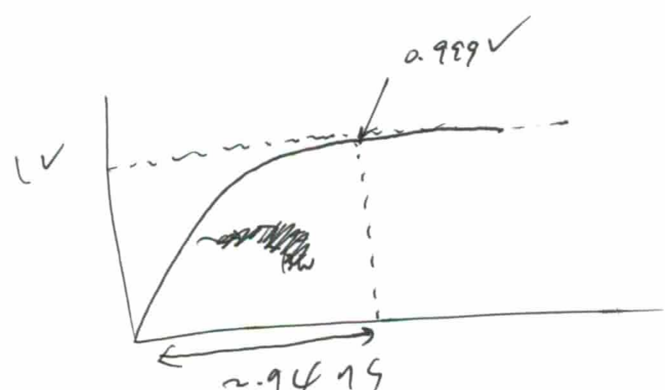
- $\tau_{BW} = \dfrac{1}{\omega_{BW}}$

0.1% settling → $1 - e^{\frac{t}{\tau_{BW}}} = 0.999$

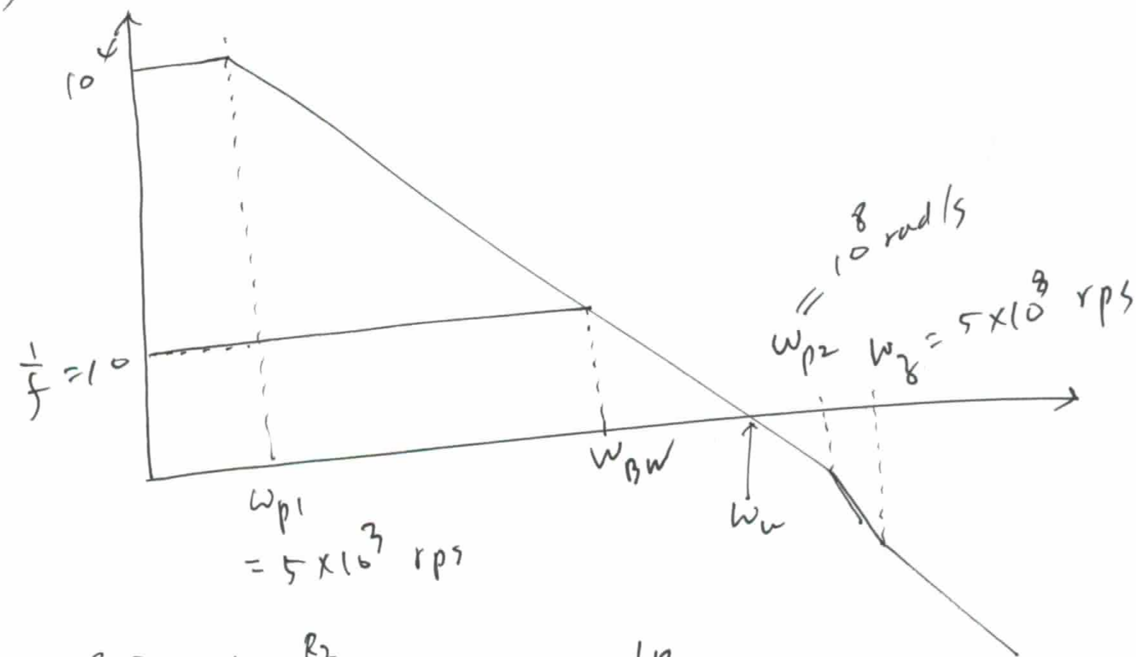
$t = -\tau_{BW} \cdot \ln 0.001$

$= -\dfrac{1}{2.35 \times 10^9} \times \ln 0.001$

$= 2.94$ ns



Q-8)



$$\text{gain} = 1 + \frac{R_2}{R_1} = 10 \rightarrow 20\,dB$$

$$= \frac{1}{f}$$

$$\therefore f = 0.1$$

$$\omega_u = 10^4 \times \omega_{p1} = 10 \times \omega_{BW}$$

$$\therefore \omega_{BW} = \frac{10^4}{10} \cdot \omega_{p1} = 10^3 \times 5\times10^3 = 5\times10^6 \text{ rps}$$

Q-9)

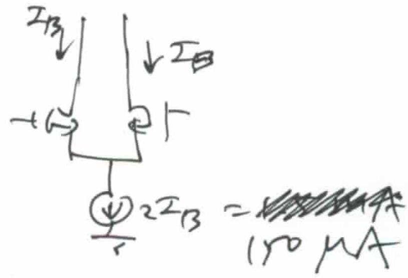
$$PM = 90^\circ - \tan^{-1}\frac{\omega_{BW}}{\omega_{p2}} + \tan^{-1}\frac{\omega_{BW}}{\omega_z}$$

$$= 90^\circ - \tan^{-1}\left(\frac{5\times10^6}{10^8}\right) + \tan^{-1}\left(\frac{5\times10^6}{5\times10^8}\right)$$

$$= 90^\circ - 2.86^\circ + 0.57^\circ$$

$$= 87.71^\circ$$

Q-10) $w_u = \frac{g_m}{C_c} = 5\times10^{7} = A_o \cdot \omega_{p1}$



$\rightarrow g_m = 5\times10^{7} \times 30\times10^{-12}$

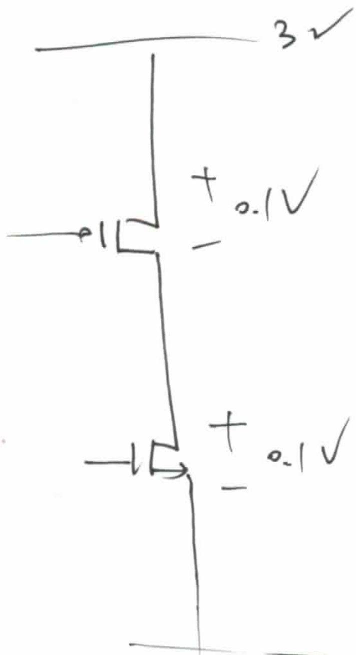
$= 15\times10^{-4} = 1.5\ m℧$

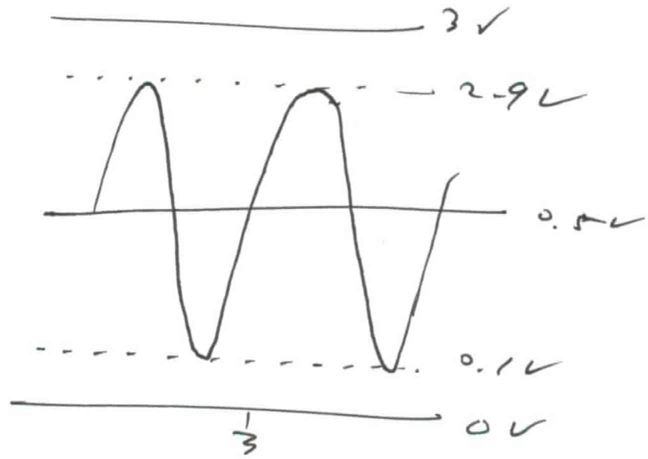
$= \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})$

$\rightarrow V_{GS}-V_{th} = \frac{1.5\ m℧}{150\mu\cdot\frac{50}{0.5}} = 100\,mV$

$\rightarrow I_B = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})^2 = \frac{1}{2}\cdot 150\mu\cdot\frac{50}{0.5}(100m)^2$

$= 75\,\mu A$





$V_{o,max} = 1.5 \pm 1.4\ V$

$\Delta V_{o,max} = 2.8\ V_{pp}$

$\rightarrow \Delta V_{i,max} = \frac{2.8\ V_{pp}}{10} = 0.28\ V_{pp}$

Q-11) $SR = \frac{2I_B}{C} = \frac{150\mu A}{30pF} = 5\times10^6 \ v/s$

10

$\frac{\Delta V_{i max}}{2} \cdot \omega_{i max} = SR = 5\times10^6 \ v/s$

$f_{max} = \frac{5\times10^6}{1.4} \times \frac{1}{2\pi} = 0.5684 \ MHz$

Q-12) $10 \ dB$ gain $= 3.162$

$\omega_{BW} = 5\times10^7 \times \frac{1}{3.162} = 1.58\times10^7 \ rps$

$\phi_M = 90 - \tan^{-1}\frac{\omega_{BW}}{\omega_{p2}} + \tan^{-1}\frac{\omega_{BW}}{\omega_z}$

$= 90 - \tan^{-1}\frac{1.58\times10^7}{10^8} + \tan^{-1}\frac{1.58\times10^7}{5\times10^8}$

$= 90^\circ - 9^\circ + 1.81$

$= 82.8^\circ$

Q-13) $\frac{V_T \ln n}{R_1} \cdot R_2 = \frac{26mV \cdot \ln 10}{1K\Omega} \cdot R = 0.3V$

$R = 0.3 \frac{1K\Omega}{26mV \cdot \ln 10} = 5.01 \ K\Omega$

Q-14) $\frac{V_T \ln n}{R_1} R + V_{BE} = V_0 \rightarrow \frac{\partial V_T}{\partial T}\frac{R}{R_1}\ln n + \frac{\partial V_{BE}}{\partial T} = 0$

$\rightarrow R = \frac{\partial V_{BE}}{-\partial T} \cdot \frac{R_1}{\ln n} \cdot \frac{q}{K}$

$= \frac{\partial V_{BE}}{-\partial T} \cdot \frac{R_1}{\ln n} \cdot \frac{T}{V_T}$

$R = 8.27 \ K\Omega \leftarrow = 1.65M \times \frac{1K\Omega}{\ln 10} \cdot \frac{300}{26M}$

Q-(5) $T = g_{m1} R_1 \cdot \frac{R_S}{R_F + R_S} = 100m \times 5K \times \frac{100}{500+100}$

$= 500 \times \frac{1}{6} = 83.33$

$Z_{in} = \frac{500 + \frac{1}{50m}}{1 + 83.33} = 6.16\Omega$

$Z_{out} = \frac{1k\Omega \parallel 600\Omega \parallel \frac{1}{50m}}{1 + 83.33}$

$\approx \frac{\frac{1}{50m}}{84.33} = 0.237\Omega$

$\frac{v_{o1}}{v_s} = \frac{-R_F}{R_S} = -5$

$\frac{V_{o2}}{V_s} = \cancel{\frac{v_{o1}}{v_s} \cdot \frac{v_{o2}}{v_{o1}}} = \cancel{5 \times g_{mp} \cdot R_L}$

$\cancel{= 5 \times 50m \times 1k\Omega}$

$\cancel{= 250}$

$= \frac{R_L}{R_S}$

$= \frac{1k\Omega}{100} = 10.$

Q-16)

$$T = g_m \cdot R_L \cdot \frac{R_s}{R_F + R_s}$$

$$= 100m * 2.5K \cdot \frac{100}{250+100}$$

$$= 71.43$$

$$Z_{in} = \frac{R_F + \frac{1}{g_m}}{1+T} = \frac{250+10}{71.43} = 3.64\,\Omega$$

$$Z_{out} = \frac{\frac{1}{g_m} \parallel (R_F + R_s)}{1+T} = \frac{10 \parallel (250+100)}{71.43} = 0.136\,\Omega$$

$$\frac{v_{o1}}{v_s} = -\frac{R_F}{R_s} = -2.5$$

$$\frac{v_{o2}}{v_s} = \frac{R_C}{-R_s} = -5.$$