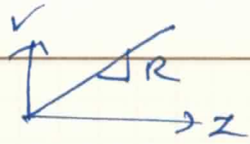


→ real impedance = resistance

$Z = R = \frac{V}{I}$ (Ω)

$\Rightarrow V = IR$

In IC-process R has ±20% variation.

→ real impedane means "same phase" between V an I.

→ "same phase" means real power dissipation

$$P_{diss} = V \cdot I = I^2 R = \frac{V^2}{R} \quad (W)$$

Inside resistor, when appered external voltage,
~~electron gets an energy~~
the voltage will create an E-field inside the resistor.

⇒ electron gets an energy from the E-field and moves toward the reverse direction of E-field.

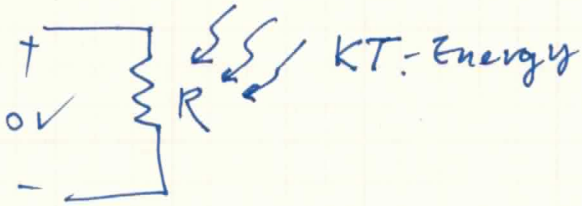
⇒ ("electron drifts" with a speed of

⇒ electron drift velocity)

⇒ During this ~~elec~~ movement,
electron keeps colliding with lattice atoms.
And generates heat.

⇒ Namely, electrons dissipates energy,
and due to this energy dissipation,
voltage should be droped across the resistor.

Q) What happens if not applying voltage?



Even if there is zero applied voltage,

→ ambient temperature around R is not absolutey zero.

→ If ambient temperature is T, then
each electron will get thermal energy

$$E_{thermal} = \frac{3}{2} KT$$

→ This thermal energy excites electron and ~~changed~~ can be transformed into kinetic energy.

$m_0 = 9.11 \times 10^{-31}$ Kg

$$E_{thermal} = \frac{3}{2} KT = E_{kinetic} = \frac{1}{2} m^* V^2$$, m^* = effective mass of electron.

$$v = \sqrt{\frac{3KT}{m^*}} \sim 10^5 \, m/s = 100 \, Km/s \approx 62 \, mile/s$$

→ this kinetic energy causes random movement of electron

→ This causes random noise current inside the resistor, ~~output~~ and therefore, random noise voltage across the resistor.

⇒ "Thermal random noise voltage"

$$R \rightarrow V_{n,rms} = \sqrt{4KTR\Delta f}$$

ex) $T = 300K$, $R = 1K\Omega$, $\Delta f = 1Hz$ $\Big\}$ $V_{n,rms} = 4 \, nV_{rms}/\sqrt{Hz}$

$= 4 \, \mu V_{rms}$ @ $\Delta f = 1MHz$

$KT = 26 \, meV$ @ $T = 300K$

$= 26 \times 10^{-3} \times 1.6 \times 10^{-19}$ J·s

K = Boltzmann's constant

$= 1.38 \times 10^{-23}$ J/K.

(*) How to quantify random noise in time domain ?

$$\overline{v_n} = \lim_{T\to\infty} \frac{1}{T}\int_0^T v_n(t)\,dt = 0$$

⇒ Time average of a random noise is zero, therefore we need a different way of describing the noise quantity.

$$\overline{v_n^2} = \lim_{T\to\infty} \frac{1}{T}\int_0^T v_n(t)^2\,dt$$

Take a time-average of a square-value

Take square-root

$$\overline{v_{n,rms}} = \sqrt{\overline{v_n^2}} = \sqrt{\lim_{T\to\infty} \frac{1}{T}\int_0^T v_n(t)^2\,dt}$$

⇒ This is called RMS-value of $v_n(t)$ and used for describing noise quantity.

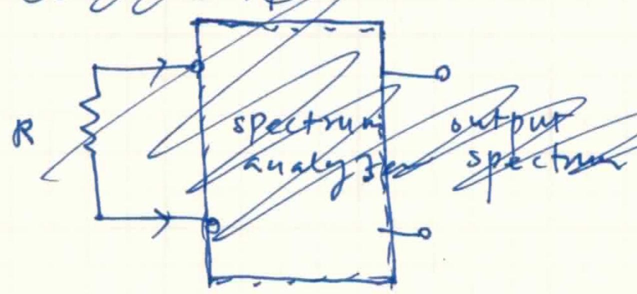
⇒ noise power $= \left(\overline{v_{n,rms}}\right)^2 = \overline{v_n^2}$

~~⇒ noise power is the square of $v_{n,rms}$.~~

~~noise power $= \left(\overline{v_{n,rms}}\right)^2 = \overline{v_n^2}$~~

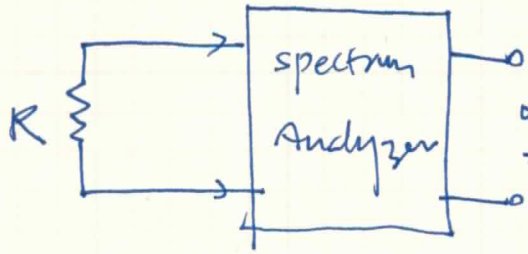
~~white~~

~~(*) Noise Spectrum~~



~~⇒ $V_n(f) = \dfrac{2hf \cdot R}{e^{\frac{hf}{kT}}}$~~

* How to quantify random noise in freq-domain?



output $= P_n(f) =$
$=$ power spectrum of noise in R

$$P_n(f) = \frac{2hf}{e^{\frac{hf}{KT}} - 1} \times R \,,$$

$h =$ Plank's constant
$= 6.63 \times 10^{-34}$ J.s
$= 4.14 \times 10^{-15}$ eV-s

$KT = 26\,meV$ @ $T = 300K$

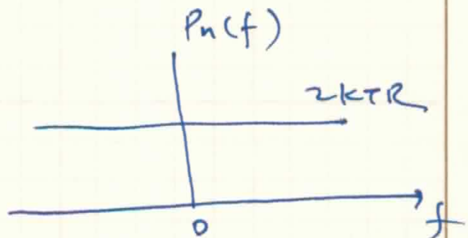
if $f \ll \frac{KT}{h} = \frac{26\,meV}{4.14\,feV.s} = 6.3\,THz$

$$e^{\frac{hf}{KT}} \simeq 1 + \frac{hf}{KT}$$

apply

$\Rightarrow P_n(f) = 2KTR$

this is DSB-SPECTRUM.



$\Rightarrow P_n(f) = 4KTR$

this is SSB-SPECTRUM.

Parseval's theorem

Time-domain power $=$ ~~sp~~ freq-domain power

$$\boxed{\therefore \overline{V_N^2} = 4KTR\Delta f}$$

$$\hookrightarrow \overline{V_n} = \sqrt{4KTR\Delta f}$$

⊛ Resistor Thermal Noise Statistics

R (resistor) $\; + \; v_n(t) \; -$

→ random signal
→ Can't be determined instantaneously
→ can only be described in statistical behavior
→ $V_n(t)$ follows Gaussian distribution.

The probability of $V \le v_n(t) \le V + dV$

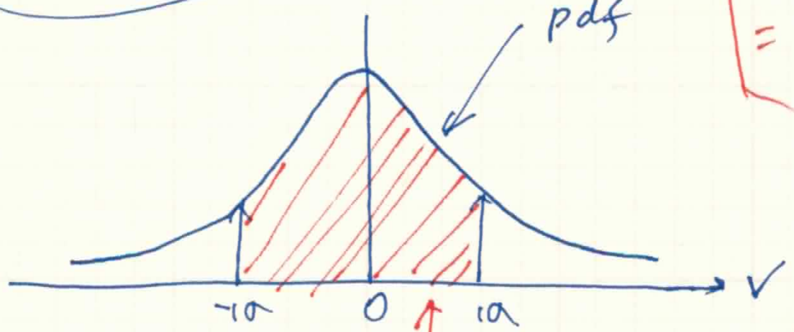
$$\Rightarrow p(V, V+dV) = \frac{1}{V_{n,rms} \cdot \sqrt{2\pi}} \cdot e^{-\frac{V^2}{2V_{n,rms}^2}} \, dV$$

$$\Rightarrow \text{Pdf} = \frac{1}{V_{n,rms} \cdot \sqrt{2\pi}} \, e^{-\frac{V^2}{2V_{rms}^2}}$$

$$\frac{p(V, V+dV)}{dV}$$

Gaussian pdf(x)

$$= \frac{1}{\sigma\sqrt{2\pi}} \, e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}$$



$$\Rightarrow \boxed{1\sigma = V_{n,rms}}$$

68.2%

Gaussian
$\pm 1\sigma \rightarrow 68.2\%$
$\pm 2\sigma \rightarrow 95.4\%$
$\pm 3\sigma \rightarrow 99.6\%$

ex

$v_n(t)$ $R = 1k\Omega \rightarrow$ @ 300K $\;\; V_{n,rms} = 4nV/\sqrt{Hz}$

if you measure $V_n(t)$ 1000 times with 1Hz BW

$-4nV \le V_n(t) \le +4nV \Rightarrow 682$ times

$-8nV \le V_n(t) \le 8nV \Rightarrow 952$ times

$-12nV \le V_n(t) \le 12nV \Rightarrow 996$ times

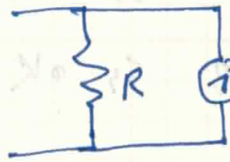
⊛ Noise Modeling

Noisy R

Thevenin Form $\longrightarrow$

$\overline{V_n} = \sqrt{4kTR\Delta f}$

Norton Form $\longrightarrow$



$\overline{i_n} = \sqrt{\dfrac{4kT\Delta f}{R}} = \dfrac{\overline{V_n}}{R}$

ex

$\Rightarrow$

Not correct

$\cancel{\Rightarrow}$

Addition of noise amount

$$\overline{V_{n1}} + \overline{V_{n2}} \xrightarrow[\text{power}]{} (\overline{V_{n1}} + \overline{V_{n2}})(\overline{V_{n1}} + \overline{V_{n2}})^*$$

$$= \overline{V_{n1}}^2 + \overline{V_{n2}}^2 + \overline{V_{n1}} \cdot \overline{V_{n2}}^* + \overline{V_{n1}}^* \cdot \overline{V_{n2}}$$

$$= \overline{V_{n1}}^2 + \overline{V_{n2}}^2 + \cancel{2\overline{V_{n1}} \cdot \overline{V_{n2}}} \rightarrow 0$$

If $V_{n1}(t)$ and $V_{n2}(t)$ have no correlation.

$$= \overline{V_{n1}}^2 + \overline{V_{n2}}^2$$

Noise amount has to be added in power domain or to be added in resistance

How about { subtraction? multiplication division

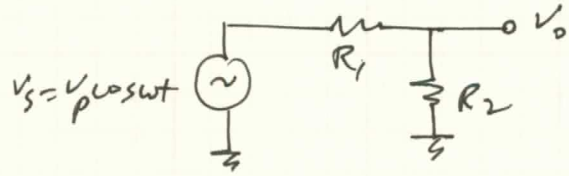
rms voltage $\longrightarrow$

$$\sqrt{\overline{V_{n1}}^2 + \overline{V_{n2}}^2} = 4\sqrt{1+2}$$

$$= 4\sqrt{3}\ \text{nVrms}/\sqrt{\text{Hz}}$$

$\longleftarrow$ This is the noise from $R = 3k\Omega$

(*) SNR = Signal to Noise power ratio



i) $V_{os} = V_s \dfrac{R_2}{R_1+R_2} \rightarrow V_{os,rms} = \dfrac{V_p}{\sqrt{2}} \dfrac{R_2}{R_1+R_2}$

$\rightarrow \overline{V_{os}^2} = V_{os,rms}^2 = \frac{1}{2} V_p^2 \left(\dfrac{R_2}{R_1+R_2}\right)^2$

ii) $\overline{V_{on}} = \sqrt{4KT(R_1 // R_2)\Delta f} \rightarrow \overline{V_{on}^2} = 4KT(R_1 // R_2)\Delta f$

$S/N = \dfrac{V_{os,rms}^2}{\overline{V_{on}^2}} = \left(\dfrac{V_{os,rms}}{\overline{V_{on}}}\right)^2$

$\Rightarrow 20\log\left(\dfrac{V_{os,rms}}{\overline{V_{on}}}\right)$ (dB)

if $R_1 = R_2 = 2K\Omega$, $T = 300K$, $\Delta f = 1MHz$

$\rightarrow V_{os,rms} = \dfrac{V_p}{2\sqrt{2}}$

$\rightarrow \overline{V_{on}} = 4\mu V$ ← remember rms noise voltage is just a statistical value.

$\Rightarrow SNR = 20\log\left(\dfrac{\frac{V_p}{2\sqrt{2}}}{4\mu V}\right)$

Q) How big V_p should be?

if $V_p = 2\sqrt{2} \times 4\mu V \xrightarrow{0dB\ SNR}$ 68% Yield

$V_p = 2 \times 2\sqrt{2} \times 4\mu V \xrightarrow{6dB\ SNR}$ 95% Yield

Most communication systems are in this range $10dB \le SNR \le 20dB$ →

$\boxed{\begin{array}{l} V_p = 3 \times 2\sqrt{2} \times 4\mu V \xrightarrow{10dB\ SNR} 99\% \text{ Yield} \\ V_p = 10 \times 2\sqrt{2} \times 4\mu V \xrightarrow{20dB\ SNR} 100\% \text{ Yield} \end{array}}$

Noise factor

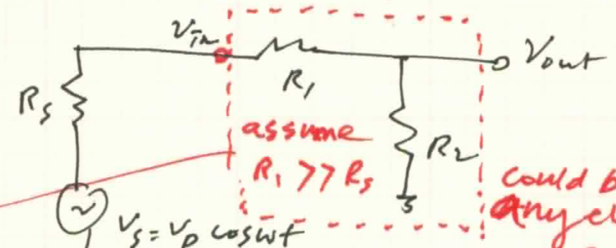
$$\rightarrow F = \frac{(SNR)_{input}}{(SNR)_{output}} = \frac{S_i/N_i}{S_o/N_o} = \frac{N_o}{\frac{S_o}{S_i} \cdot N_i} = \frac{N_o}{G_p \cdot N_i}$$

G_p: power gain

$$\rightarrow NF = 10 \log(F)$$

Q) What does this means?
Let's take an example.

NOTE: Any signal source has a source impedance, typically source resistance → generate noise.



assume $R_1 \gg R_s$

$R_1 + R_s \simeq R_1$

could be any electronic system with gain of G. $\left(G = \frac{R_2}{R_1+R_2}\right)$

① $(SNR)_{input} = S_i/N_i$

i) $v_{is} = v_s \dfrac{R_1+R_2}{R_1+R_2+R_s} \rightarrow \overline{v_{is}^2} = V_{is,rms}^2 = \frac{1}{2} V_p^2 \left(\dfrac{R_1+R_2}{R_1+R_2+R_s}\right)^2$

ii) $\overline{v_{in}} = \overline{v_{Rs}} \dfrac{R_1+R_2}{R_1+R_2+R_s} \rightarrow \overline{v_{in}^2} = \overline{v_{Rs}^2} \left(\dfrac{R_1+R_2}{R_1+R_2+R_s}\right)^2$

$$\Rightarrow S_i/N_i = \frac{\frac{1}{2}V_p}{\overline{V_{Rs}^2}} = \frac{V_{s,rms}^2}{\overline{V_{Rs}^2}}$$

② $(SNR)_{output} = S_o/N_o$

i) $v_{os} = v_{is} \dfrac{R_2}{R_1+R_2} \rightarrow \overline{v_{os}^2} = v_{is,rms}^2 \times \left(\dfrac{R_2}{R_1+R_2}\right)^2 = v_{is,rms}^2 \times G^2$

output noise due to input source → ii) $\left(\overline{v_{on}}\right)_{R_s} = \overline{v_{in}} \dfrac{R_2}{R_1+R_2} \rightarrow \left(\overline{v_{on}^2}\right)_{R_s} = \overline{v_{in}^2} \times G^2$

output noise due to the system itself → iii) $\left(\overline{v_{on}}\right)_{R_1 \parallel R_2} \approx \sqrt{4KT(R_1 \parallel R_2)\Delta f} \rightarrow \left(\overline{v_{on}^2}\right)_{R_1 \parallel R_2} = 4KT(R_1 \parallel R_2)\Delta f$

$\Rightarrow \quad S_o = \overline{V_{os}^2} = V_{is,rms}^2 \times G^2$

$N_o = (N_o)_{\text{due to input noise}} + (N_o)_{\text{due to system noise}}$

$= \overline{V_{i,n}^2} \times G^2 + 4KT(R_1 // R_2)\Delta f$

$$\Rightarrow \quad F = \frac{(SNR)_{input}}{(SNR)_{output}} = \frac{S_i/N_i}{S_o/N_o} = \boxed{\frac{N_o}{G_p \cdot N_i}} = \boxed{\frac{\text{Total Noise power}}{\text{Noise power due to source noise only}}}$$

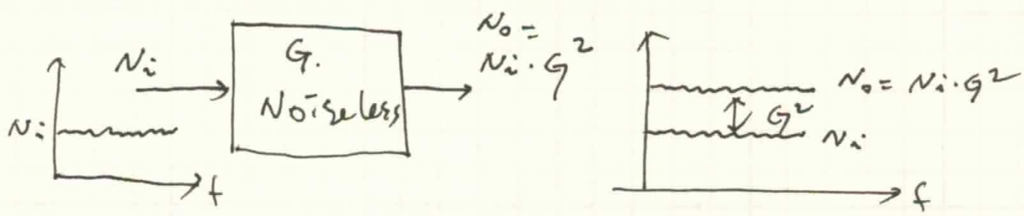
$$= \frac{\overline{V_{i,n}^2} \cdot G^2 + 4KT(R_1 // R_2)\Delta f}{\overline{V_{i,n}^2} \cdot G^2}$$

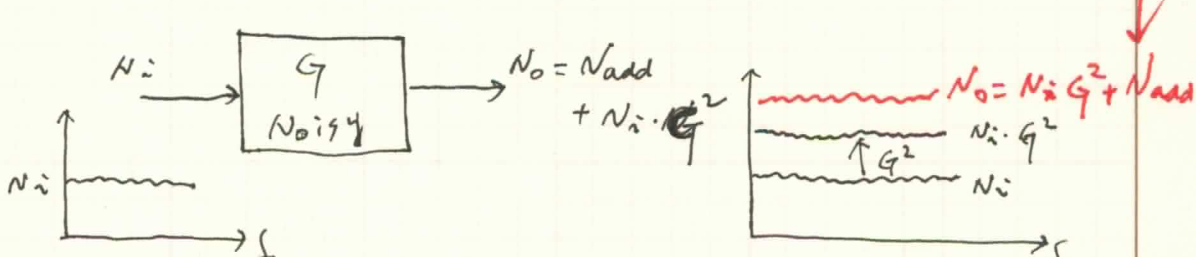
$$= 1 + \frac{4KT(R_1 // R_2)\Delta f}{\overline{V_{i,n}^2} \cdot G^2} \approx 1 + \frac{4KT \frac{R_1 R_2}{R_1 + R_2}\Delta f}{4KTR_s\Delta f \cdot \left(\frac{R_2}{R_s + R_2}\right)^2}$$

$$\approx 1 + \frac{R_1(R_1 + R_2)}{R_s R_2}$$

$$= \boxed{1 + \frac{\text{output noise due to system noise}}{\text{output noise due to input source noise}}}$$

Total output noise
= output noise due to input source noise
+ output noise due to system noise





NOTE: Most electrical circuits have noise, N_{add}, and noise factor can not be 1.

$$\Rightarrow \boxed{F = \frac{N_o}{N_i \cdot G^2} = \frac{N_i G^2 + N_{add}}{N_i \cdot G^2} = 1 + \frac{N_{add}}{N_i \cdot G^2}}$$

if $N_{add} = 0$ (noiseless) $\rightarrow F = 1$

$N_{add} = N_i G^2 \rightarrow F = 2$

$N_o = F \cdot (N_i G^2) = 2 \times (N_i G^2)$ ← (output noise is increased by a fator of 2)

$N_{add} = 2 \cdot N_i G^2 \rightarrow F = 3$

$N_o = 3 \times (N_i G^2)$ ← (output noise is increased by a factor of 3)

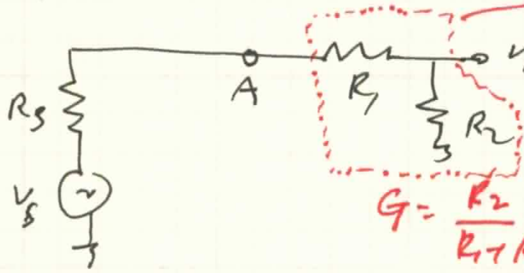
$N_{add} = M \cdot N_i G^2 \rightarrow F = 1 + M$

$N_o = \cancel{M} \times (N_i G^2)$ (M+1) ← (output noise is increased by a factor of ~~M~~ (M+1))

NOTE: ① Noise factor, F, is the measure of extra noise added by a system.

② F is not dependent of V_s, but dependent of source noise, device (system) noise and gain.

⊛ Input referred noise



assume $R_1 \gg R_s$
$\Rightarrow R_1 + R_s \simeq R_1$

← Noisy device

$G = \frac{R_2}{R_1 + R_2}$

$\Rightarrow$ output noise due to device (system) itself

$$(\overline{V_{on}})_{R_1 /\!/ R_2} = \sqrt{4KT(R_1 /\!/ R_2)\Delta f}$$

$\Rightarrow$ Input referred noise voltage

: refer $(\overline{V_{on}})_{R_1 /\!/ R_2}$ to input side (node-A) by dividing $(\overline{V_{on}})_{R_1 /\!/ R_2}$ by gain G.

$$(\overline{V_{on}})_{input} = \frac{(\overline{V_{on}})_{R_1 /\!/ R_2}}{G} = \sqrt{4KT \frac{R_1 R_2}{R_1 + R_2} \Delta f \times \frac{R_1 + R_2}{R_2}}$$

$$= \sqrt{4KT \frac{R_1}{R_2}(R_1 + R_2)\Delta f}$$

$\Rightarrow$ Input referred noise power

$$(\overline{V_{on}^2})_{input} = \frac{(\overline{V_{on}^2})_{R_1 /\!/ R_2}}{G^2} = \frac{4KT(R_1 /\!/ R_2)\Delta f}{G^2}$$

$$F = \frac{N_o}{G_p \cdot N_i} = \frac{G_p \cdot N_i + N_{add}}{G_p \cdot N_i} = 1 + \frac{N_{add}}{G_p \cdot N_i}$$

$$= 1 + \frac{\frac{N_{add}}{G_p}}{N_i}$$

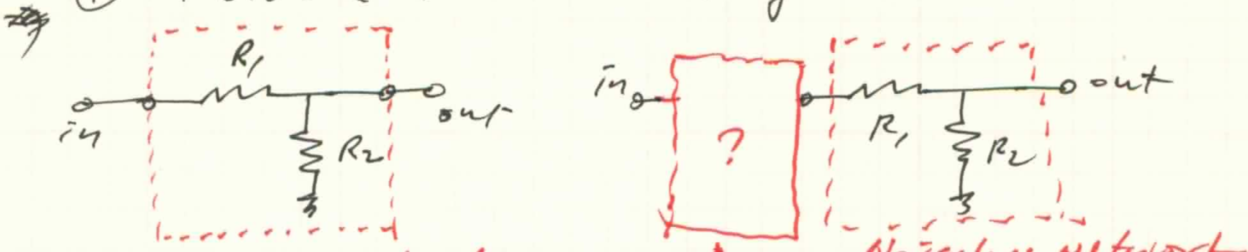
input referred noise power

$$= 1 + \frac{\frac{4KT(R_1 /\!/ R_2)\Delta f}{G^2}}{\overline{V_{i,n}^2}}$$

NOTE: ① F can be calculated at the output side

② F can be calculated at input side by referring device noise to input side.

⊛ Network Noise Modeling.



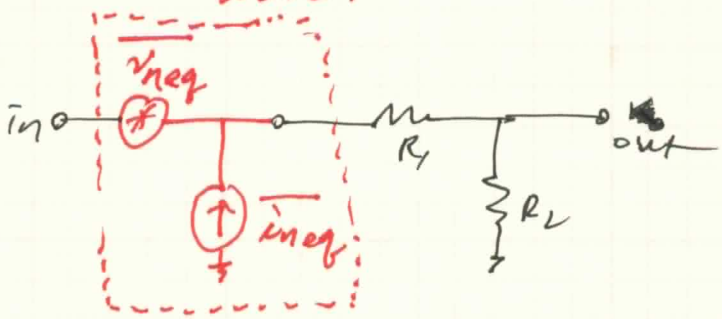
Noisy Network

original network

Noiseless network

equivalent noise generators. (combination of voltage & current sources)

Motivation: We want to represent Noisy network equivalently using Noiseless network and equivalent noise sources.



Noise equivalent network

equivalent noise source should like this. (Why?)

① $\overline{v_{neq}}$ calculation

- Short input to ground

⇒ original network → $\overline{v_{on}} = \sqrt{4kT(R_1 // R_2)\Delta f}$ — Ⓐ

⇒ equivalent network → $\overline{v_{on}} = \overline{v_{neq}} \times \dfrac{R_2}{R_1+R_2}$ — Ⓑ

Ⓐ = Ⓑ → $\overline{v_{neq}} = \sqrt{4kT(R_1 // R_2)\Delta f} \times \dfrac{R_1+R_2}{R_2}$

$$= \sqrt{4kT \frac{R_1(R_1+R_2)}{R_2} \Delta f}$$

② $\overline{i_{neq}}$ calculation

open input port

⇒ original network → $\overline{v_{on}} = \sqrt{4kTR_2\Delta f}$ — Ⓐ

⇒ equivalent network → $\overline{v_{on}} = \overline{i_{neq}} \times R_2$ — Ⓑ

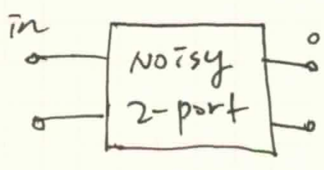
Ⓐ = Ⓑ → $\overline{i_{neq}} = \sqrt{4kTR_2\Delta f} \cdot \frac{1}{R_2}$

$= \sqrt{\frac{4kT\Delta f}{R_2}}$

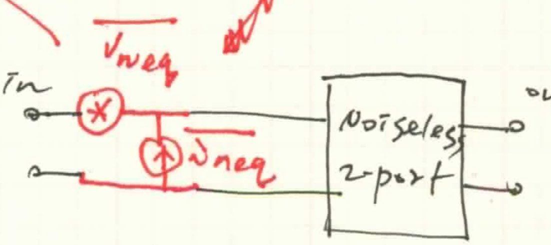
① For typical RF circuit ⇒ $\overline{v_{neq}}$ and $\overline{i_{neq}}$ are correlated.

② For typical low freq. analog CMOS circuit ⇒ $\overline{v_{neq}}$ and $\overline{i_{neq}}$ are not correlated.

~~usually $\overline{v_{neq}}$ and $\overline{i_{neq}}$ are correlated But for most typical cases of analog~~



<original>



< equivalent >

① $\overline{v_{neq}}$ calculation

⇒ short input node to ground

⇒ calculate output noises for original and equivalent circuits.

⇒ make these two noise amount equal

⇒ $\overline{v_{neq}}$ can be calculated

② $\overline{i_{neq}}$ calculation

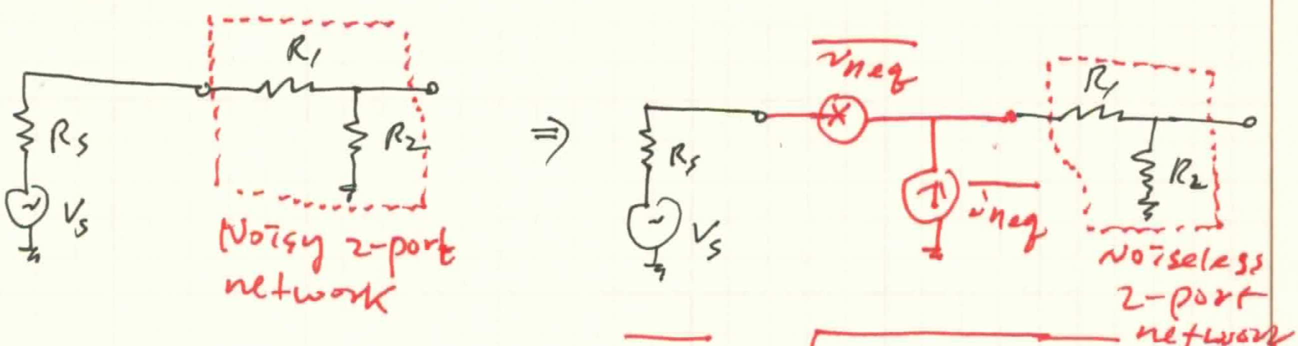
⇒ open input node

⇒ Calculate output noises for original and equivalent circuits

⇒ make these two noise amount equal

⇒ $\overline{i_{neq}}$ can be calculated

example



$$\overline{v_{neq}} = \sqrt{4KT\frac{R_1}{R_2}(R_1+R_2)\Delta f}$$

$$\overline{i_{neq}} = \sqrt{\frac{4KT\Delta f}{R_2}}$$

$\boxed{\text{Assume } R_s \ll R_1}$

① input referred noise voltage

$$\overline{v_{n,in}} = \overline{v_{neq}} + \overline{i_{neq}}\cdot R_{eq} \quad , \quad R_{eq} = R_s \,//\, (R_1+R_2)$$

$$\overline{v_{n,in}^2} = \left(\overline{v_{neq}} + \overline{i_{neq}}\cdot R_{eq}\right)\left(\overline{v_{neq}^*} + \overline{i_{neq}^*}\cdot R_{eq}\right)$$

$$= \overline{v_{neq}^2} + \overline{i_{neq}^2}\cdot R_{eq}^2 +$$

$$+ \overline{v_{neq}}\cdot\overline{i_{neq}^*}\cdot R_{eq} + \overline{i_{neq}}\cdot R_{eq}\cdot\overline{v_{neq}^*}$$

$$= \overline{v_{neq}^2} + \overline{i_{neq}^2}\cdot R_{eq}^2 + 2\overline{v_{neq}}\cdot\overline{i_{neq}}\cdot R_{eq}$$

$\because R_s \ll R_1$

$$\approx \overline{v_{neq}^2}$$

$\neq 0$ (there is correlation.)

$$\therefore \overline{v_{n,in}} = \overline{v_{neq}}$$

$$= \sqrt{4KT\frac{R_1}{R_2}(R_1+R_2)\Delta f}$$

→ this is same as previous calc ~~[illegible]~~

③ Noise factor

$$= \frac{\text{Total Noise power}}{\text{Noise power due to source noise only}}$$

$$= \frac{\overline{v_{neq}^2} + \overline{V_{R_S}^2}}{\overline{V_{R_S}^2}}$$

$$= 1 + \frac{\overline{v_{neq}^2}}{\overline{V_{R_S}^2}}$$

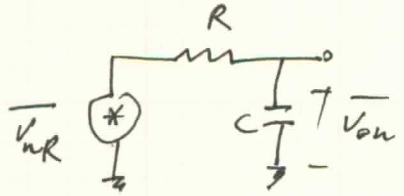
$$= 1 + \frac{4KT\frac{R_1}{R_2}(R_1+R_2)\Delta f}{4KT R_S \Delta f}$$

$$= 1 + \frac{\frac{R_1}{R_2}(R_1+R_2)}{R_S}$$

$$= 1 + \frac{R_1(R_1+R_2)}{R_S \cdot R_2}.$$

→ This is same as previous calc.

⊛ Noise Bandwidth & $\frac{KT}{C}$ - noise



$$\overline{V_{nR}^2} = 4KTR\Delta f$$

$$\overline{V_{on}} = \overline{V_{nR}} \cdot \left| \frac{1}{1+j\omega CR} \right|$$

$$\Rightarrow \overline{V_{on}^2} = 4KTR\Delta f \frac{1}{1+\omega^2 C^2 R^2}$$

$\Rightarrow$ Total Noise power

$$= \int_0^\infty \frac{4KTR}{1+(2\pi f)^2 C^2 R^2} df$$

$$(2\pi f)^2 C^2 R^2 = X^2$$

$$2(2\pi)^2 f C^2 R^2 \cancel{/df} = 2X\, dX$$

$$df = \frac{X}{4\pi^2 f C^2 R^2} dX$$

$$= \frac{2\pi f \cdot CR}{4\pi^2 f C^2 R^2} dX$$

$$= \frac{1}{2\pi} \frac{1}{RC} dX$$

$$= \frac{4KT}{2\pi \cdot C} \int_0^\infty \frac{1}{1+X^2} dX$$

$$= \frac{2KT}{\pi \cdot C} \times \frac{\pi}{2}$$

$$= \boxed{\frac{KT}{C}}$$

← total Noise power is only dependent on C, not R

$$\Rightarrow \overline{V_{n,C}} = \sqrt{\frac{KT}{C}}$$

(ex) $C = 1pF \rightarrow \overline{V_{n,C}} = 63.2\mu V$

How many electron in C?

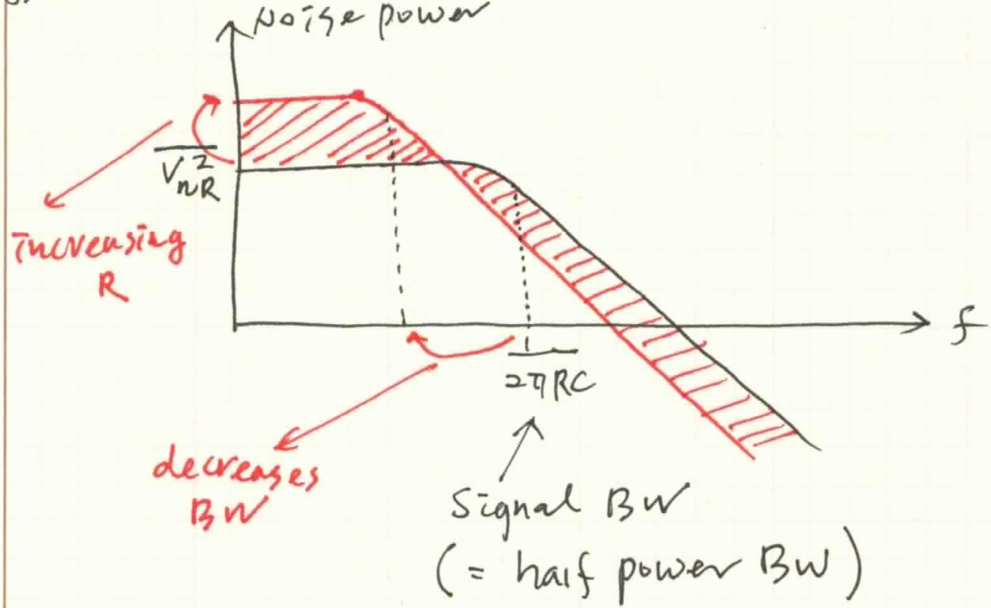
$$Q_n = C\overline{V_{n,C}} = \sqrt{CKT}$$

$$\# \text{ of noise electron} = \frac{Q_n}{q} = \frac{\sqrt{CKT}}{q}$$

$$= 402 \times 10^6 \times \sqrt{C}$$

$= 402$ electrons (if) $C = 1pF$

① case-I: Increasing R



when increasing R → ~~...~~ equivalent noise voltage increases

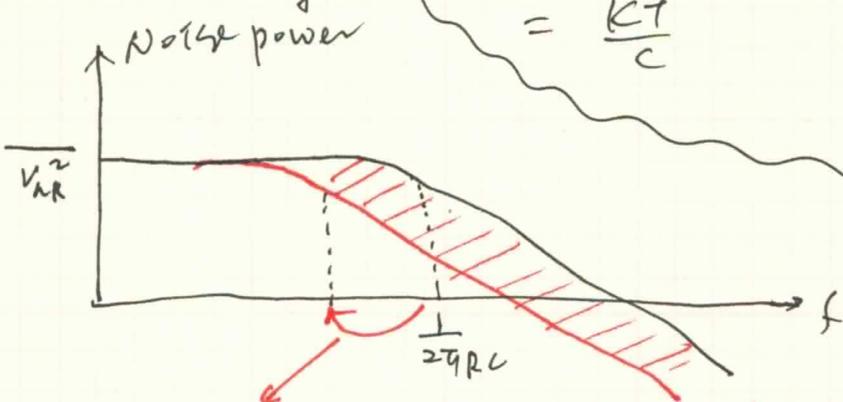
→ But, BW decreases

→ total integrated area = constant

→ Noise power is same.

$= \frac{KT}{C}$

② case-II: increasing C

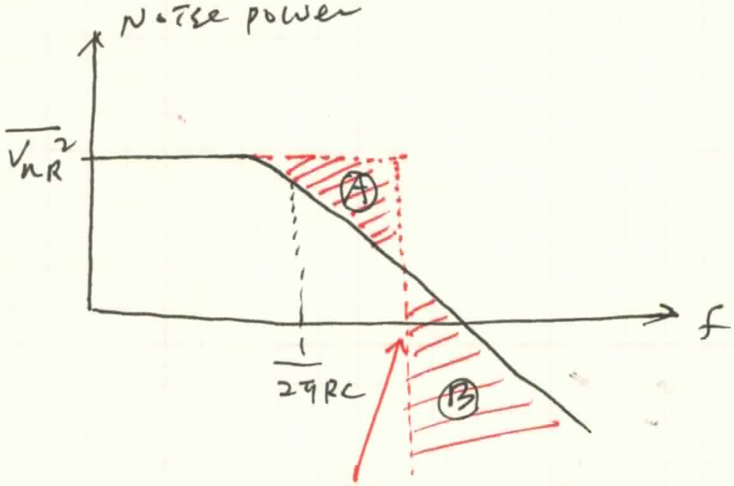


increasing C will ~~...~~ decrease BW

→ decreases Area

→ decreases integrated noise power

Noise Basics



NBW = noise bandwidth

Assume an ideal brick wall type low-pass filter which has a cut-off freq where area-A and area-B is same

$\Rightarrow$ same integrated noise power between R-C filter and brick-wall filter.

$$\Rightarrow \boxed{\frac{KT}{C} = 4KTR \times NBW \qquad \therefore NBW = \frac{1}{4RC}}$$

(*) NBW is $\frac{\pi}{2}$ time larger than signal bandwidth.

$$NBW = \frac{\pi}{2} \times f_{BW}$$

signal BW $= \frac{1}{2\pi}\frac{1}{RC}$