connect the source to the substrate, thereby minimizing the voltage drop. Finally, note that, for opamps that drive purely capacitive loads, the buffer stage should not be included, and for this case, the output stage is clearly not a consideration.

Noise is another important consideration when choosing which input stage to use. Perhaps the major noise source of MOS opamps is due to 1/f noise caused by carriers randomly entering and leaving traps introduced by defects near the semiconductor surface. This 1/f noise source can be especially troublesome unless special circuit design techniques are used.[5] Typically, p-channel transistors have less 1/f noise than n-channel transistors since their majority carriers (holes) have less potential to be trapped in surface states. Thus, having a first-stage with p-channel inputs minimizes the output noise due to the 1/f noise. The same is not true when thermal noise is considered. When thermal noise is referred to the input of the opamp, it is minimized by using input transistors that have large transconductances (Chapter 4, Section 4.4), which unfortunately degrades the slew rate. However, when thermal noise is a major consideration, then a more modern architecture, such as a folded-cascode opamp (discussed in Chapter 6), is normally used.

In summary, when using a two-stage opamp, a p-channel input transistor for the first stage is almost always the best choice because it optimizes slew rate, unity-gain frequency, and minimizes 1/f noise, with the major disadvantage being an increase in wideband thermal noise.

5.2 FEEDBACK AND OPAMP COMPENSATION

This section discusses using opamps in closed-loop configurations and how to compensate an opamp to ensure that the closed-loop configuration is not only stable, but also has good settling characteristics. Although the two-stage opamp is used as an example, almost all the material discussed here applies to most other opamps as well.

Optimum compensation of opamps is typically considered to be one of the most difficult parts of the opamp design procedure. However, if the systematic approach taken here is used, then a straightforward procedure can be used that almost always results in a near-optimum compensation network.

Before discussing compensation, some properties of feedback and closed-loop amplifiers will first be reviewed.

First-Order Model of Closed-Loop Amplifier

A simple first-order model for the transfer function of a dominant-pole compensated opamp, $A(s)$, is given by

$$A(s) = \frac{A_0}{(1 + s/\omega_{p1})} \tag{5.31}$$

5. Some useful circuit techniques to reduce the effects of 1/f noise are correlated double sampling and chopper stabilization.

where A_0 is the dc gain of the opamp and ω_{p1} is the (real-axis) dominant pole. At this point, it should be mentioned that all the poles and zeros in this chapter occur on the real axis and are represented by the notations ω_p and ω_z, respectively.[6]

Recall that the definition of the unity-gain frequency of an opamp, ω_{ta}, is the frequency at which $|A(j\omega_{ta})| = 1$. We then have the following approximation, since $\omega_{ta} \gg \omega_{p1}$:

$$|A(j\omega_{ta})| = 1 \cong \frac{A_0}{\omega_{ta}/\omega_{p1}} \tag{5.32}$$

Thus, we have the following important relationship for this first-order model:

$$\omega_{ta} \cong A_0\omega_{p1} \tag{5.33}$$

From here on, we will define ω_{ta} to be exactly equal to $A_0\omega_{p1}$, which is approximately equal to the unity-gain frequency of the opamp (assuming a first-order model for the opamp).[7] Substituting (5.33) into (5.31) for the case in which $\omega_{p1} \ll \omega \ll \omega_{ta}$, we have at midband frequencies

$$A(s) \cong \frac{\omega_{ta}}{s} \tag{5.34}$$

This approximate relationship is often used to analyze a closed-loop circuit for the effects of the opamp's finite bandwidth at midband frequencies.

An opamp with feedback can be modelled by the block diagram shown in Fig. 5.5 [Sedra, 1991]. Here, the feedforward amplifier, $A(s)$, models the open-loop response of the opamp. The feedback term, β, represents the feedback factor, which is assumed to be frequency independent; this is typically the case for amplifiers, but may not be the case for applications such as damped integrators. It can be shown using signal-flow graph analysis that the closed-loop gain, $A_{CL}(s)$, for this model is given by

$$A_{CL}(s) = \frac{A(s)}{1 + \beta A(s)} \tag{5.35}$$

At midband frequencies, the transfer function of the closed-loop amplifier may be found by substituting (5.34) into (5.35), resulting in

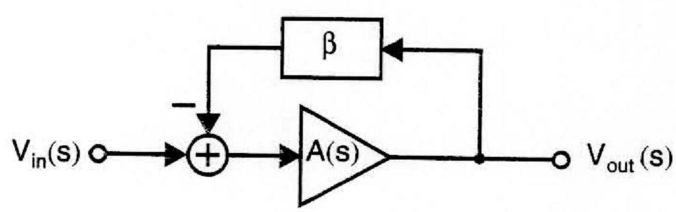

Fig. 5.5 A block diagram of a feedback circuit.

6. To be more precise, ω_p and ω_z are the inverses of the coefficients multiplying the s terms in the denominators and numerators, respectively. The actual poles and zeros are the negatives of these terms.

7. The unity-gain frequency of an optimally compensated opamp can be very different from ω_{ta} due to high-frequency poles and zeros that are ignored in the first-order model, especially when the closed-loop configuration has large gain. However, at midband frequencies (5.34) is still valid.

$$A_{CL}(s) \approx \frac{\omega_{ta}}{\beta\omega_{ta} + s} = \frac{1}{\beta} \frac{1}{(1 + s/\beta\omega_{ta})} \tag{5.36}$$

Thus, the closed-loop amplifier has a closed-loop gain at low frequencies approximately equal to $1/\beta$ [8] and it has a –3-dB frequency given by

$$\omega_{-3\text{ dB}} \cong \beta\omega_{ta} \tag{5.37}$$

It will be shown shortly that, for optimally compensated opamps, $\beta\omega_{ta} \cong \omega_t$, where ω_t is the unity-gain frequency of the open-loop transfer function. It will also be shown that, for optimally compensated amplifiers, ω_t is independent of the feedback factor, β, and dependent only on high-frequency poles and zeros. *Thus, the –3-dB frequency of the closed-loop gain of an optimally compensated amplifier is approximately equal to the unity-gain frequency of the loop gain, independent of the actual closed-loop gain.* However, the component sizes of the compensation network and ω_{ta} will be shown to depend on the desired closed-loop gain for an optimally compensated amplifier.

Linear Settling Time

The settling-time performance of integrated amplifiers is often an important design parameter. For example, in switched-capacitor circuits, the charge from one or more capacitors must be mostly transferred to a feedback capacitor within about half a clock period. This charge transfer is closely related to the opamp's step response. As a result, the *settling time* is defined to be the time it takes for an opamp to reach a specified percentage of its final value when a step input is applied.

This settling time consists of two distinct segments—linear and nonlinear settling time segments. The linear settling-time portion is due to the finite unity-gain frequency of the opamp, and thus it sets a minimum value for the overall settling time independent of the step size of the opamp's output. In contrast, the nonlinear settling time is due to slew-rate limiting, and thus this portion is strongly dependent on the output's step size. For example, for small step sizes in the output signal's level, the opamp may not reach a slew-rate limit at all, resulting in a nonlinear settling time of zero. Here, we discuss only the linear settling time by modelling the opamp as ideal but with a finite unity-gain frequency. Another simplification is to use the first-order opamp model, which has a 90-degree phase margin. Such a simplification results in a simple settling behavior that can be easily analyzed. Thus, the results here are used only to estimate the necessary unity-gain frequency for a circuit to settle within the linear settling-time segment. Simulations should be used in the latter parts of a design to determine more accurate settling-time estimates.

From (5.36), we recognize that the time constant of the closed-loop amplifier, τ, is given by

8. For inverting configurations, (5.35) should be modified to be $A_{CL}(s) = KA(s)/[1 + \beta A(s)]$, and the closed-loop gain at low frequencies is given by K/β. But these facts have no effect on the discussions to follow regarding optimum compensation because we are primarily interested in the loop gain, which is still given by $\beta A(s)$.

$$\tau = \frac{1}{\omega_{-3\,\mathrm{dB}}} = \frac{1}{\beta\omega_{ta}} \tag{5.38}$$

The important result here is that the –3-dB frequency determines the settling-time response for a step input. Recall that the transient response of any first-order circuit is given by

$$x(t) = x(\infty) - [x(\infty) - x(t)]e^{-t/\tau} \tag{5.39}$$

where τ is the time constant of the circuit. For the closed-loop amplifier, the step response is found using (5.39) to be given by

$$V_{out}(t) = V_{step}(1 - e^{-t/\tau}) \tag{5.40}$$

Here, V_{step} is the size of the voltage step. With this exponential relationship, the time required for a first-order circuit to settle to within a specified value can be found. For example, if 1 percent accuracy is required, then one must allow $e^{-t/\tau}$ to reach 0.01, which is achieved at a time of 4.6τ. For settling to within a 0.1 percent accuracy, the settling time needed becomes approximately 7τ. Also, note that just after the step input, the slope of the output will be at its maximum, given by

$$\left.\frac{d}{dt}V_{out}(t)\right|_{t=0} = \frac{V_{step}}{\tau} \tag{5.41}$$

If the slew rate of the opamp is larger than this value, no slew-rate limiting would occur.

EXAMPLE 5.5

One phase of a switched-capacitor circuit is shown in Fig. 5.6, where the input signal can be modelled as a voltage step. If 0.1 percent accuracy is needed in the linear settling-time portion corresponding to 0.1 μs, find the required unity-gain frequency in terms of the capacitance values, C_1 and C_2. For $C_2 = 10C_1$, what is the necessary unity-gain frequency of the opamp? What unity-gain frequency is needed in the case in which $C_2 = 0.2C_1$?

Solution

We first note that a capacitive feedback network is used rather than a resistive one. A difficulty with this network in a nonswitched circuit is that no bias current

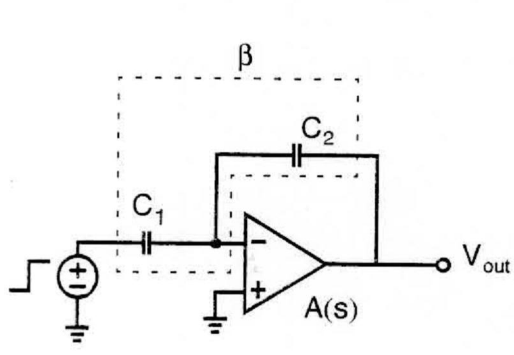

Fig. 5.6 One phase of a switched-capacitor circuit.

flows into the negative opamp input terminal. However, in a switched circuit using a CMOS opamp, this connection shown occurs only for a short time and does not cause any problems. Second, note that the feedback factor is simply a constant value since it is determined by a capacitive divider consisting of C_1 and C_2.

The feedback factor, β, is given by

$$\beta = \frac{1/(sC_1)}{1/(sC_1) + 1/(sC_2)} = \frac{C_2}{C_1 + C_2} \tag{5.42}$$

Now for 7τ settling within 0.1 μs, we see that τ must be less than 14.2 ns, and, since $\tau = 1/(\beta\omega_{ta})$, we see that

$$\omega_{ta} \geq \left(\frac{C_1 + C_2}{C_2}\right)\left(\frac{1}{14.2 \text{ ns}}\right) \tag{5.43}$$

For the case in which $C_2 = 10C_1$, a unity-gain frequency of 12.3 MHz is needed, whereas in the case of $C_2 = 0.2C_1$, f_{ta} should be larger than 66.8 MHz.

Opamp Compensation

When one is compensating an opamp, the first-order model given by (5.31) is insufficient. It ignores high-frequency poles and zeros, which cause possible instability, even though they may be at frequencies greater than ω_t. It is important to *accurately* model the open-loop transfer function at higher frequencies where the loop-gain is approximately unity. Fortunately, when all poles and zeros are on the real axis, they can be modelled reasonably well by a single additional pole. Specifically, we can model $A(s)$ by

$$A(s) = \frac{A_0}{(1 + s/\omega_{p1})(1 + s/\omega_{eq})} \tag{5.44}$$

where ω_{p1} is the first dominant-pole frequency and ω_{eq} is the pole frequency that models higher-frequency poles. The relationship to approximate ω_{eq} given a set of real-axis poles, ω_{pi}, and zeros, ω_{zi}, is given by

$$\frac{1}{\omega_{eq}} \cong \sum_{i=2}^{m} \frac{1}{\omega_{pi}} - \sum_{i=1}^{n} \frac{1}{\omega_{zi}} \tag{5.45}$$

It should be noted here that the approximation in (5.45) is different than that given in [Sedra, 1991] since we are mostly interested in the phase shifts, rather than the attenuation, due to higher-frequency poles and zeros. In practice, ω_{eq} is found from simulation as the frequency at which the transfer function has a –135° phase shift (–90° due to the dominant pole and another –45° due to the higher-frequency poles and zeros).

At frequencies much greater than the dominant pole frequency, $\omega >> \omega_{p1}$, we see that $1 + j\omega/\omega_{p1} \cong j\omega/\omega_{p1}$, and so (5.44) can be accurately approximated by

$$A(s) \cong \frac{\omega_{ta}}{s(1 + s/\omega_{eq})} \tag{5.46}$$

Note that this approximation result is especially valid at the unity-gain frequency of the loop (which we are presently interested in) since, from (5.33), $\omega_t/\omega_{p1} \cong (1/\beta)A_0$, which is almost certainly much greater than 1.

When the opamp is included in a circuit that has a feedback factor, β, then, at frequencies around ω_{ta}, the loop gain, $LG(s)$, is given by

$$LG(s) = \beta A(s) = \frac{\beta\omega_{ta}}{s(1 + s/\omega_{eq})} \tag{5.47}$$

The unity-gain frequency, ω_t, of the loop gain, $LG(s)$, can now be found by setting the magnitude of (5.47) equal to unity after substituting $s = j\omega_t$. Once this is done and the equation is rearranged, one can write,

$$\beta\frac{\omega_{ta}}{\omega_{eq}} = \frac{\omega_t}{\omega_{eq}}\sqrt{1 + \left(\frac{\omega_t}{\omega_{eq}}\right)^2} \tag{5.48}$$

This equation will be used shortly to relate a specified phase margin to the Q factor of the closed-loop configuration.

Note also that from (5.48) we have

$$\omega_{ta} = \frac{\omega_t}{\beta}\sqrt{1 + \left(\frac{\omega_t}{\omega_{eq}}\right)^2} \cong \frac{\omega_t}{\beta} \tag{5.49}$$

for the special case in which the unity-gain frequency is much less than the equivalent nondominant pole frequency (i.e., $\omega_t << \omega_{eq}$).

The phase margin, PM, is an often-used measure for how far an opamp with feedback is from becoming unstable. PM is defined as the difference between the loop-gain phase shift and $-180°$. From (5.47), the phase shift, $\angle LG(j\omega)$, is found as

$$\angle LG(j\omega) = -90° - \tan^{-1}(\omega/\omega_{eq}) \tag{5.50}$$

This equation implies that at the unity-gain frequency, $s = j\omega_t$, we have

$$PM = \angle LG(j\omega_t) - (-180°) = 90° - \tan^{-1}(\omega_t/\omega_{eq}) \tag{5.51}$$

and, therefore,

$$\omega_t/\omega_{eq} = \tan(90° - PM) \tag{5.52}$$

which implies that

$$\omega_t = \tan(90° - PM)\omega_{eq} \tag{5.53}$$

Equation (5.52) can be used to derive ω_t/ω_{eq} for a specified phase margin. *Note that the unity-gain frequency of the loop is independent of the feedback factor,* β*, and therefore, in the case of an optimally compensated amplifier, it is independent of the closed-loop gain as well.*

EXAMPLE 5.6

A closed-loop amplifier is compensated to have a 75° phase margin for $\beta = 1$. What is ω_t if $f_{eq} = \omega_{eq}/(2\pi) = 50\ \text{MHz}$? What is ω_{ta}?

Solution

Using (5.53), we have $\omega_t = 0.268\omega_{eq}$, which implies that the loop-gain unity-gain frequency, f_t, is given by $f_t = \omega_t/2\pi = 13.4\ \text{MHz}$. Using (5.49), we also have, for $\beta = 1$, $\omega_{ta} = \omega_t\sqrt{1 + 0.268^2} = 1.035\omega_t$, which implies that $f_{ta} = \omega_{ta}/2\pi = 13.9\ \text{MHz}$.

When considering optimum compensation, (5.34) is not accurate enough and one must use the more accurate relationship for $A(s)$ given in (5.46). Assuming β is frequency independent, substituting (5.44) into (5.35), and rearranging gives

$$A_{CL}(s) = \frac{A_{CL0}}{1 + \dfrac{s(1/\omega_{p1} + 1/\omega_{eq})}{1 + \beta A_0} + \dfrac{s^2}{(1 + \beta A_0)(\omega_{p1}\omega_{eq})}} \tag{5.54}$$

where

$$A_{CL0} = \frac{A_0}{1 + \beta A_0} \cong \frac{1}{\beta} \tag{5.55}$$

Thus, the closed-loop response near the unity-gain frequency, ω_t, is closely approximated by a second-order transfer function. It should be noted here that this approximation is inaccurate in the case where the high-frequency poles and zeros are quite close to ω_t. However, in this case, it would be extremely difficult to adequately compensate the opamp, and thus, this case is ignored.

The result of (5.54) can be equated to the general equation for a second-order all-pole transfer function, written as

$$H_2(s) = \frac{K\omega_0^2}{s^2 + \left(\dfrac{\omega_0}{Q}\right)s + \omega_0^2} = \frac{K}{1 + \dfrac{s}{\omega_0 Q} + \dfrac{s^2}{\omega_0^2}} \tag{5.56}$$

Recall that parameter ω_0 is called the resonant frequency and parameter Q is called the Q factor[9] [Sedra, 1991]. It is well known that, if $Q = \sqrt{1/2} \cong 0.707$, then the magnitude of the transfer function will have the widest passband without any peaking. Furthermore, for $Q = \sqrt{1/2}$, the –3-dB frequency is equal to ω_0. When the step response is investigated, restrictions on the Q factor can also be found to guar-

9. The Q factor is 1/2 times the inverse of the damping factor. The damping factor is an alternative method of indicating the pole locations in second-order transfer functions.

antee no peaking. Specifically, for there to be no peaking in the step response, it is necessary that both poles be real, which is equivalent to the requirement that $Q \le 0.5$. In the case where $Q > 0.5$, the percentage overshoot of the output voltage is given by

$$\%\ \text{overshoot} = 100e^{\frac{-\pi}{\sqrt{4Q^2-1}}} \tag{5.57}$$

Equating (5.54) with (5.56) and solving for ω_0 and Q results in

$$\omega_0 = \sqrt{(1+\beta A_0)(\omega_{p1}\omega_{eq})} \cong \sqrt{\beta\omega_{ta}\omega_{eq}} \tag{5.58}$$

and

$$Q = \frac{\sqrt{(1+\beta A_0)/\omega_{p1}\omega_{eq}}}{1/\omega_{p1} + 1/\omega_{eq}} \cong \sqrt{\frac{\beta A_0 \omega_{p1}}{\omega_{eq}}} = \sqrt{\frac{\beta\omega_{ta}}{\omega_{eq}}} \tag{5.59}$$

where the approximation of Q is valid since $\beta A_0 >> 1$ and $\omega_{p1} << \omega_{eq}$.

It is now possible to relate a specified phase margin to the Q factor. Equation (5.52) can be used to find ω_t/ω_{eq}. This result can be substituted into (5.48) to find $\beta(\omega_{ta}/\omega_{eq})$, which can then be substituted into (5.59) to find the equivalent Q factor. Finally, (5.57) can be used to find the corresponding percentage overshoot for a step input. This procedure gives us the information in Table 5.1.

Table 5.1 leads to some interesting observations. First, a frequency response with $Q \cong \sqrt{1/2}$ roughly corresponds to a phase margin of 65°. Therefore, one should design for a phase margin of at least 65°, given both process and temperature changes. Second, if one wants to ensure that there is no overshoot for a step input, then the phase margin should be at least 75°, again, given both process and temperature variations. Normally, values of 80° to 85° should be the nominal phase margin to account for these variations. These phase margins are much larger than what was traditionally thought to be necessary.

Finally, it is worth mentioning here that, when the feedback network is frequency independent and less than unity, (i.e., when $\beta \le 1$), the worst-case phase margin occurs for $\beta = 1$. Thus, for a general-purpose opamp where $0 < \beta \le 1$, if the opamp is compensated for $\beta = 1$, it is guaranteed to be stable for all other β, although it will not be optimally compensated and will be slower than necessary.

Table 5.1 The relationship between PM, ω_t/ω_{eq}, Q factor, and percentage overshoot

PM (Phase margin)	ω_t/ω_{eq}	Q factor	Percentage overshoot for a step input
55°	0.700	0.925	13.3%
60°	0.580	0.817	8.7%
65°	0.470	0.717	4.7%
70°	0.360	0.622	1.4%
75°	0.270	0.527	0.008%