

# Refractive Index Measurement with a THz Triangulator and Radar

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**Abstract**— We have built a triangulator with radar capability based on pulsed THz radiation. In this paper we show experimental results and study the technical specifications of the instrument with particular emphasis on its potential of measuring the refractive index.

**Index Terms**— Millimeter wave radar, refractive index, terahertz, time domain reflectometry

## I. INTRODUCTION

ONE of the most promising properties of THz radiation is its potential to penetrate a large variety of materials opening up applications in security and quality control. Often it will not be sufficient to find out the shape of a sample but it is desired to obtain information about the material of the object under test. Such information may come from spectral features, if such features exist in the investigated frequency region, otherwise it may come from the refractive index.

In most laboratory experiments with pulsed THz radiation, the refractive index of a sample can be calculated from the pulse delay time since the sample thickness is known. In a real-world environment, e.g., for security applications, the sample thickness is usually unknown. In such cases one may think of calculating the reflection coefficient of a sample from the amplitude of the reflected THz radiation but this quantity depends not only on the refractive index and extinction coefficient but also on factors as surface roughness, inclination etc., it can hardly be measured precisely enough for a determination of the refractive index.

To complement the delay-time measurement, additional information can come from a triangulator. Triangulation is a method of distance measurement where the direction of incidence differs from the direction of detection. When an object is irradiated, and the scattered or reflected radiation is imaged on the sensor (which is at least one-dimensional), the position of the signal on the sensor reveals the 3D coordinates of the irradiated spot on the object based on the sampling geometry. In the case of THz radiation, when the incident beam penetrates a sample, all surfaces or interfaces can create

signals on the sensor revealing their geometric positions. At the same time the optical path length can be measured based on the pulse delay time, and the refractive index can be retrieved from the difference between the optical and geometric path lengths.

We have built a THz triangulator with delay-time (radar) capability for a variety of purposes including refractive index measurements. We show results and study the technical specifications of the instrument

## II. MATH

The sample geometry for calculating the refractive index  $n$  from a THz triangulation and radar experiment is shown in Fig. 1. THz radiation enters from the left at the angle  $\alpha$ . At the first surface the beam is partly reflected and partly refracted at the angle  $\beta$  with

$$\sin \alpha = n \sin \beta. \quad (1)$$

From Fig. 1 one reads

$$b = 2d \tan \beta \cos \alpha \quad (2)$$

where  $d$  is the sample thickness and  $b$  the distance between the reflected and refracted beams, and

$$\begin{aligned} c \Delta t &= \frac{2d}{\cos \beta} (n - \sin \beta \sin \alpha) \\ &= 2nd \cos \beta \end{aligned} \quad (3)$$

where  $\Delta t$  is the time difference between the beams and  $c$  the vacuum velocity of light.

Dividing (3) over (2) eliminates  $d$  resulting in

$$n^2 = \frac{c \Delta t}{2b} \sin 2\alpha + \sin^2 \alpha \quad (4)$$

and multiplying (3) with (2) eventually eliminates  $n$  so that

$$d^2 = \frac{b c \Delta t}{2 \sin 2\alpha}. \quad (5)$$

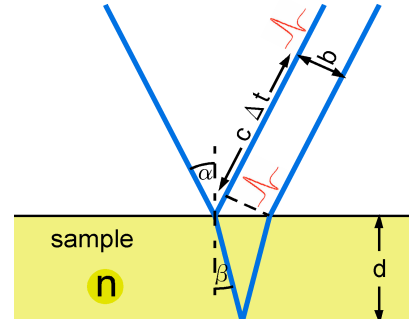


Fig. 1. A THz beam (from left) is reflected at the surfaces of a transparent sample. The refractive index  $n$  can be calculated from the position and the pulse delay time of the reflected beams.

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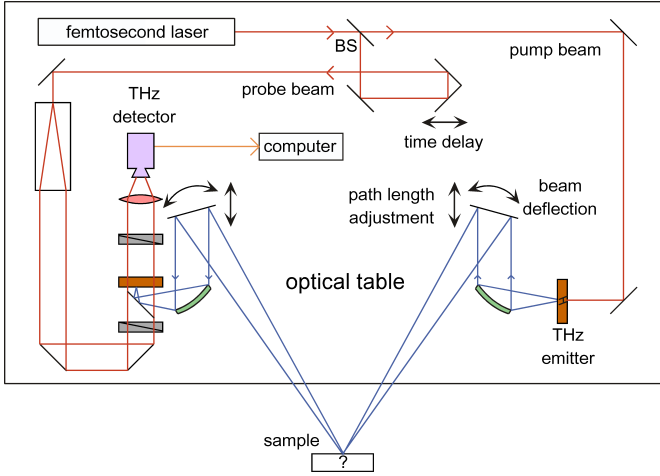


Fig. 2. Sketch of the THz triangulator and radar setup.

### III. EXPERIMENTAL

The experimental setup is shown in Fig. 2. It was originally based on a 3 W Maitai femtosecond laser which we preferred to a regenerative laser amplifier due to its reliability for a potential operation in an industrial environment. The laser was later replaced by a 100 mW IMRA femtosecond fiber laser. The fiber laser has less power but also less noise resulting in a THz signal with approximately the same signal-to-noise ratio. Semiconductor surfaces and photoconductive antennas have been used as THz emitters, the detector is an electro-optic ZnTe crystal in crossed-polarizer configuration.

The THz beam path includes a set of planar mirrors for directing the THz beam to the desired spot of a sample. These mirrors can be laterally aligned for controlling the path length between the sample and the focusing elements (parabolic mirrors) so that the sample is kept in the focus of the THz beam.

### IV. RESULTS

Fig. 3 shows the THz time-domain waveform reflected from a Si wafer and a Teflon disk in front of a mirror. The peaks from the wafer surface and the interfaces Si-Teflon and Teflon-mirror are clearly reproduced. The mirror peak moves backwards on the time axis when the Teflon disk is replaced by an air slit. These waveforms were recorded with a photodiode and oscilloscope instead of the CCD camera.

The imaging properties of the setup are demonstrated in Fig. 4 where a point source is imaged by using a planar mirror as sample. The lateral resolution equals the diameter of the spot, 1.3 mm on the ZnTe crystal or 1.5 cm on the sample. This is close to the theoretical optimum at 1 THz with the given set of parabolic mirrors although the oval shape still indicates some potential for improvements.

Since we want to determine the amount of lateral movement of this spot we want to know how precisely we can measure its position in Fig. 4. Considering the noise level and the symmetry of the spot shape, we estimate this figure to 1/3 mm at the sample or 0.03 mm at the ZnTe crystal.

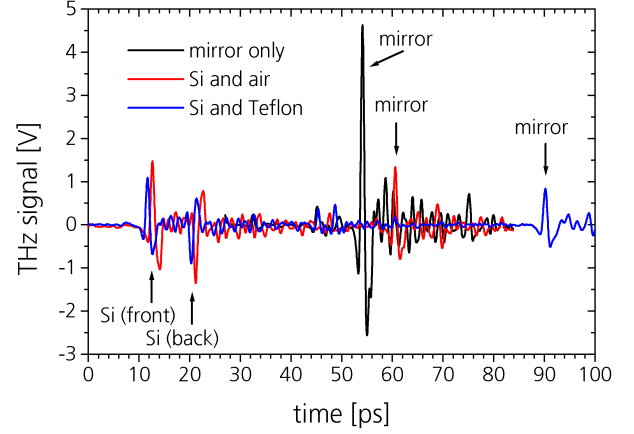


Fig. 3. THz time-domain waveform reflected from a Si wafer and a Teflon disk in front of a metallic mirror (see text).

### V. DISCUSSION

From the above considerations, the accuracy of the refractive index can be calculated. The error of  $n$ ,  $\Delta n$ , can be derived from the error of  $b$ ,  $\Delta b$ , using (4) to

$$\frac{\Delta n}{n} = \frac{1}{2} \frac{\Delta b}{b} \quad (6)$$

If the THz radiation were continuous, the images of the spots on both surfaces would interfere on the detector, and  $\Delta b$  would be given by the spot diameter. Since we use pulsed THz radiation, we can separate the spots due to their different arrival times, and  $\Delta b$  is given by the lateral precision calculated in the preceding section (and multiplied by  $\sqrt{2}$ ).

Thus, for example, for a 2 cm thick disk with  $n = 1.5$  at  $\alpha = 30^\circ$ , the spot positions differ by  $b = (12.2 \pm 0.5)$  mm so that  $n = 1.5 \pm 0.03$ .

### VI. CONCLUSION

The measurement of a refractive index of a sample with a THz triangulation and radar setup is possible. The accuracy can reach 2 %. Careful alignment of the setup is necessary for achieving this figure. The use of high-repetition rate lasers for THz imaging based on electro-optic sampling is possible but amplified laser systems are clearly superior in terms of the signal-to-noise ratio.

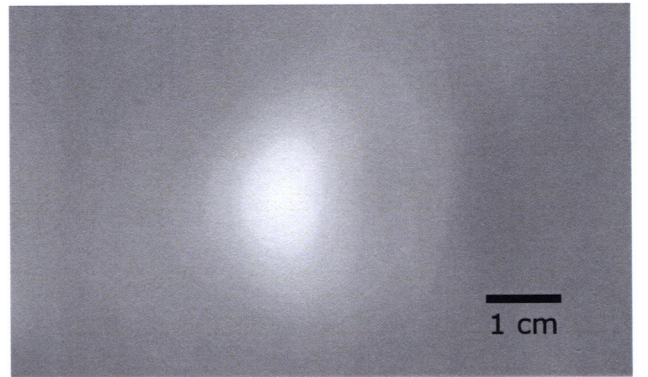


Fig. 4. THz image of a point source (measured for calibration purposes).