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FLUCTUATION NOISE IN RADIO RECEIVERS*

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Summary—*This paper discusses fluctuation noise in radio receivers due to shot and thermal effects (Schottky) in the radio-frequency circuits. The r-f noise components beat with the carrier when a signal is being received and are transformed to audio components which are heard as a hissing noise.*

The mathematical theory for a receiver employing a square-law detector is given, and it is shown that the deflection of a meter measuring the average square of voltage or current due to the noise is proportional to the area under the curve representing the square of the over-all transmission against frequency. The over-all transmission is somewhat analogous to the over-all fidelity of the receiver. This result is similar to the well-known laws for simple linear networks without frequency transformation.

The method of calculating the noise due to the shot and thermal effects is discussed.

Finally a convenient method of measuring the specific noise (noise per frequency interval) in a radio receiver is described, and results for several typical commercial receivers are given. The method consists in comparing the noise, as referred to the antenna circuit, with the amplitudes of the side bands, mE_0, in a standard modulated signal.

ON ACCOUNT of the atomistic structure and nature of the universe many of the quantities with which we are concerned in macroscopic physical measurements do not in reality possess that steadiness with which we often associate them, but are in reality fluctuating quantities of which our ordinary measurements furnish merely average or statistical values. The fundamentally fluctuating nature of such quantities may often be brought into evidence by measurements of sufficient delicacy. The term "fluctuation" has been proposed† as a generic characterization of these phenomena, of which the following examples may be cited by way of illustration: fluctuation of pressure due to rain falling upon an object of finite size, such as a bounded flat surface; fluctuation of pressure upon the walls of a vessel containing a gas which arises from the random impacts of the molecules in consequence of their thermal agitation; Brownian movement of small particles in a gas or liquid brought about by random impacts of the molecules of the gas or liquid; fluctuation of current in an electrical conductor in consequence of the thermal agitation of its ions

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† R. Fürth, Schwankungserscheinungen in der Physik. T. C. Fry, "Probability and Its Engineering Uses," Ch. 11, p. 389, (New York, 1928).

and electrons (Schottky temperature effect); fluctuations of the photoelectric emission of electrons from metals in consequence of the random impact of light quanta; fluctuation in the thermionic emission; fluctuation in the saturation current carried by the electrons in a thermionic tube as a result of this random emission (Schottky "shot effect"); fluctuation in the e.m.f. of an electrolytic cell due to random motion of ions; fluctuations in the rate of magnetization of magnetic materials (Barkhausen effect); fluctuation of space-charge limited currents in a thermionic tube due to ionization of residual gas, evaporation of positive ions, etc.,; fluctuation of thermionic emission due to statistical disturbances in surface conditions, or work function, (Johnson-Schottky "flicker effect"). These examples of the statistical character of natural phenomena may be multiplied endlessly.

Attention was drawn to certain electrical fluctuation phenomena by W. Schottky[1] in 1918. Among the more important effects to which he drew attention were: (1) the fluctuation of current in ordinary electrical conductors as a consequence of the ordinary thermal agitation of the conducting particles; (2) the fluctuation of current in a tube carrying voltage-saturated thermionic current as a consequence of the random emission (and flow) of electrons. The first effect he called the "temperature effect," and the second the "schrot" (small-shot or shot) effect. Both effects are of some technical significance since the noise, or other disturbances produced by them, sets a definite limit upon the electrical amplification which may be usefully employed.

In a normal radio receiver the chief sources of fluctuation noise reside in the thermal agitation of electricity in the conductors (coils, antenna, etc.) and in the shot effect in the first tube. Naturally the contribution of noise from any source depends upon the amplification following it, so that if the amplification per r-f stage is reasonably high only the antenna-coupling circuits, first tube and its output circuit contribute significantly to the noise production.

Schottky's original theory of these disturbances was a statistical one based upon the general statistical relation $\bar{n}^2 = N\tau$. In this theory the time-form of the individual disturbances is not considered, and the results are therefore correct only when the duration of the disturbance is vanishingly short; however, in both thermal and shot effects the duration is actually so short that for the frequency regions of practical interest, Schottky's theory is quite adequate.

[1] W. Schottky, "On spontaneous current fluctuations in various electrical conductors," *Annalen der Physik*, **18**, 541, 1918.

The author has recently[2] published an alternative theory based upon Rayleigh's theorem which possesses some mathematical interest and is of slightly greater generality. In this theory the elementary disturbance is analyzed by Fourier's theorem to determine its spectral distribution characteristics. The total effect of one disturbance is obtained by integrating, over the frequency range, the product of the energy-per-frequency interval and the square of the transmission. Then, by Rayleigh's theorem, the average effect of a number of disturbances occuring at random at the rate of n per second is precisely equal to n times that of a single disturbance, and has the same frequency distribution.

In the pure shot effect the elementary disturbance, or event, is the pulse of current due to the passage of a single electron from cathode to anode; in the thermal effect it is the pulse of current occasioned by the flight of an electrified particle between collisions.

If the duration of the elementary disturbance be sufficiently short its spectrum will contain components *of all frequencies of equal amplitudes.* In this case, as in the case of heat radiation, we may not speak of the amplitude of a single component of a definite frequency, but must speak of an *amplitude-per-frequency interval.* In spectral distribution the fluctuation effects due to elementary disturbances of durations which are short compared with the reciprocal of the frequencies in the range under consideration are analogous to perfectly white light.

In a conventional radio receiver comprising a radio-frequency amplifier, detector, and audio-frequency amplifier, the disturbance components originating in the first r-f stage and its associated circuits are of radio frequency and are transmitted by the r-f amplifier to the detector, where by beating with each other, they are transferred to audio-frequency disturbances which are heard as a hissing noise. When a carrier is present at the detector the r-f noise components also beat with the carrier, forming audible noise components. In a radio receiver of average sensitivity (10μv) with a square-law detector the audio noise due to beating of the r-f noise components among themselves in the absence of a carrier is usually too weak to be detected. When a carrier is present, however, the audio noise produced by beating of the r-f noise components with the carrier is proportional to the carrier and may become quite noticeable. The hissing noise ordinarily heard during reception is predominantly of this latter type and we shall consider it exclusively in this discussion.

[2] Stuart Ballantine, "Shot effect in high frequency circuits," *Cont. from the Radio Frequency Laboratories*, No. 7, 1928; *Jour. Frank. Inst.*, **206**, 159, 1928.

II

Theory of Audio-Noise Production in a Receiver Employing a Square-Law Detector

The theory of fluctuation effects has been worked out for the case in which a linear transmission network connects the branch of the circuit in which the disturbance originates and the instrument employed for measuring $\bar{e}^2$ or $\bar{\imath}^2$ (time average of voltage- or current-squared).

In a radio receiver the situation is somewhat more complicated. The disturbances originate in the r-f circuits. We are not interested in the noise at the output terminals of the r-f amplifier, as given by the ordinary theory, but in the audio-frequency noise which appears after detection and audio amplification. The extension of the theory to this more complicated case is not difficult.

We may note that when in the case of a square-law detector the carrier is large compared with the noise components each r-f component of frequency f is transmitted through the system with a change of frequency $f-f_0$ (f_0 = carrier frequency) and of amplitude; also the principle of superposition holds since if $E_0 >> E$ the beating of the noise components among themselves is negligible compared with their beating with the carrier. The system then possesses a sort of quasi linearity and the ordinary theory is applicable.

Let us assume that the audio noise is measured at the output terminals of the audio amplifier by an instrument capable of indicating the time average of $\bar{e}^2$. Let us assume that the disturbance is a current pulse originating in a r-f branch of the receiver circuit.

In the form in which we shall use it the Rayleigh-Schuster[3] theorem may be stated as follows:

Rayleigh-Schuster Theorem:

Let $E_j(t)$ represent the output voltage produced in mesh, j by the current $I_k(t)$ in mesh k acting through a linear network; let a quantity $Z_{jk}(f)$, called the transimpedance, be defined as

$$Z_{jk} = E_j/I_k$$

when

$$e_j(t) = E_j e^{i2\pi f t} \text{ and } i_k = I_k e^{i2\pi f t}. \quad \text{Then} \tag{a}$$

$$\int_0^\infty e_j^2(t)dt = 2\int_0^\infty |Z_{jk}(f)|^2 I_k^2(f)\cdot df \tag{b}$$

[3] Rayleigh, *Phil. Mag.*, **27**, 466, 1889. A. Schuster, *Phil. Mag.*, **37**, 509, 1894, "Theory of Optics," 2nd Ed., p. 399, (London, 1909). J. R. Carson, "Electrical Circuit Theory and Operational Calculus," p. 185, (New York, 1926).

where

$$I_k^2(f) = \left[\int_{-\infty}^{\infty} i_k(t) \cos 2\pi ft \cdot dt\right]^2 + \left[\int_{-\infty}^{\infty} i_k(t) \sin 2\pi ft \cdot dt\right]^2, \tag{c}$$

and if $i_k(t)$ represents a disturbance recurring at random intervals at the average rate of n times per second, then

$$\bar{e}_j^2(t) = n\int_0^{\infty} e_j^2(t)\cdot dt = 2n\int_0^{\infty} E_k^2(f)\cdot df \tag{d}$$

$$= 2n\int_0^{\infty} |Z_{jk}(f)|^2 I_k^2(f)\cdot df \tag{e}$$

or in other words the frequency distribution of the square of the total disturbance is equal to n times that of a single disturbance and is precisely of the same form as a function of frequency.

In the above equations $|Z|$ denotes, as usual, the absolute value, or amplitude, of the complex impedance Z.

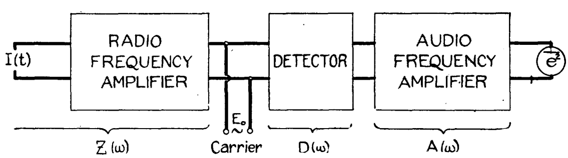

Fig. 1—Scheme of conventional radio receiver under discussion.

In the case of a radio receiver (Fig. 1) let $Z(f)$ represent the transimpedance between the mesh of the circuit in which the disturbance $i(t)$ originates and the output of the r-f amplifier, E_0 the carrier voltage at the detector, D, the detection factor, defined in terms of output voltage, and $A(f)$ the transmission of the audio amplifier (ratio of output voltage to input voltage). Then, remembering that an r-f component of frequency f is transformed to one of audio frequency $f-f_0$

$$Z_{jk}(f) = Z(f)E_0D(f - f_0, f)A(f - f_0). \tag{1}$$

Z_{jk} is of the nature of an over-all transimpedance, being proportional to the product of the transmissions of the detector and radio amplifier and the transimpedance of the r-f amplifier. The component factors are roughly suggested in Fig. 2. Z_{jk} is readily measured by impressing a carrier voltage E_0 on the detector, a current of variable radio frequency in the branch of the r-f amplifier in which the fluctuation effect has its origin and measuring the output voltage as a function of the radio frequency, f.

In view of (1) the Rayleigh-Schuster theorem gives for the total noise as measured by $\bar{e}^2$.

$$\bar{e}^2 = 2n \int_0^\infty I^2(f) \mid Z(f) E_0 D(f, f - f_0) A(f - f_0) \mid^2 df \tag{2}$$

where $I^2(f)$ is given by (c). In the case of most disturbances the duration of $i(t)$ is so small compared with $1/f_0$ that $I^2(\mathrm{f})$ (equation c) is given very approximately by

$$I^2(f) = \left[\int_{-\infty}^{\infty} i(t) \cdot dt \right]^2 \tag{3}$$

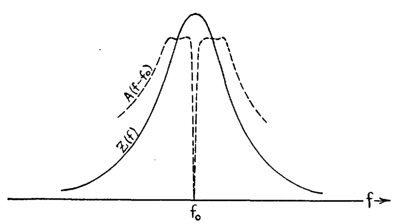

Fig. 2—Radio- and audio-transmission curves as functions of frequncy; f_0 = carrier frequency.

which is independent of frequency and equal to the square of the total electric change transferred by the current pulse. In this case

$$\bar{e}^2 = 2n \left[\int_{-\infty}^{\infty} i(t) \cdot dt \right]^2 \int_0^\infty \mid Z(f) E_0 D(f, f - f_0) A(f - f_0) \mid^2 \cdot df. \tag{4}$$

This result may be expressed as follows:

1. *In a radio receiver employing a square-law detector, and with a carrier voltage impressed upon the detector, the audio-frequency noise, as measured by an instrument indicating the average value of the square of voltage (or current), is proportional to the area under the curve representing the square of the over-all transimpedance (or of the transmission) from the r-f branch in which the disturbance originates to the measuring instrument as a function of frequency, and proportional to the square of the carrier voltage.*

2. *When the disturbance is a current pulse $i(t)$ of short duration τ, such that $\tau f_0 << 1$, recurring at random intervals at the average rate of n per second, the noise is proportional to the product of n and the square of the time integral of $i(t)$ or to the square of the total electric charge conveyed by the pulse.*

Shot Noise

The author has considered the calculation of $I^2(f)$ for the shot effect elsewhere.† For a pure electronic shot effect the time of passage of an electron from cathode to anode is of the order of 10^{-9} sec. and at ordinary frequencies (3) is sufficiently accurate. The equivalent shot circuit is shown in Fig. 3.

In calculating transimpedance from the plate circuit it is often convenient to replace the impressed current $i(t)$, Fig. 3b, by its equivalent series e.m.f., Fig. 3c. (Thevenin's theorem). This puts the shot-

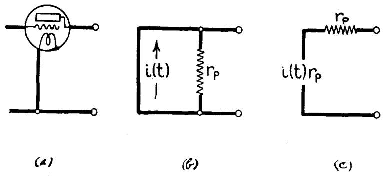

Fig. 3—Equivalent circuits for calculation of shot effect; $i(t)$ represents elementary current pulse due to passage of one electron.

noise source on the same footing as the signal e.m.f. and is convenient in making comparisons of them. If ϵ is the electronic charge and if I_s denotes the saturation component of current through the tube then

$$nI^2(f) = n\epsilon^2 = I_s\epsilon \tag{5}$$

which is the well-known result.

The shot noise is then given by (4) and (5) as

$$\bar{e}^2 = 2I_s\epsilon \int_0^\infty | Z(f)E_0D(f_1f - f_0)A(f - f_0) |^2 \, df. \tag{6}$$

A further detailed discussion of the shot noise generation in vacuum tubes is reserved for a separate paper now in preparation.[4]

Thermal Noise

In electrical conductors in which the mean-free-path of the conducting particles is short the energy acquired by these particles from the electric field is small compared with that due to ordinary thermal agitation and the shot-effect merges into a thermal agitation effect. An ordinary inductance coil wound with copper wire or a high-resistance unit (e.g., one megohm) at room temperature are examples of such

† See footnote 2.

[4] Since the present paper was written a discussion of the noise production, in tubes and their associated circuits has been published by F. B. Llewellyn, Proc. I.R.E., **18**, 243; February, 1930. Since the present paper is confined to general theory, and matters not considered by Dr. Llewellyn, it has not been considered necessary to make any changes in its original form.

conductors. In these the conducting particles fly in all directions between collisions and constitute a fluctuating electric current. The "elementary event," corresponding to the flight of one electron from the cathode to anode in a vacuum tube in the case of the shot effect, is now the flight of an electron between collisions. If the mechanism of conduction were sufficiently well known the $i(t)$ corresponding to the elementary event and the spectral distribution could be calculated. In the absence of such detailed information, we are obliged to calculate the thermal effect in a more general fashion.

Such a general calculation of the frequency distribution of energy in the thermal effect has been performed by Nyquist[5] for a system in equilibrium. Upon the assumption of equipartition of energy and em-

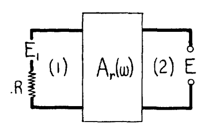

Fig. 4—Thermal noise circuit.

ploying the principle of detailed balancing Nyquist showed that the energy per frequency interval is

$$e^2 df = 4kTR(f)df \tag{7}$$

where k = Boltzmann's constant $= 1.372 \times 10^{-23}$ watts/dyne; τ = absolute temperature; $R(f)$ = effective resistance of the source of the noise.

In Fig. 4 the noise is generated in the resistance R in branch 1 of a network. If we denote by A_r(f) the voltage produced across the terminals of the detector by a sinusoidal e.m.f. of unit amplitude in series with R, then the audio- noise output may be expressed as follows:

$$\begin{aligned} \bar{e}^2 &= 4kT\int_0^\infty R(f) \mid A_r(f)E_0D(f,f-f_0)A_a(f-f_0)\mid^2 df \\ &= 5.48 \times 10^{-23}\, T\int_0^\infty \text{etc.} \qquad \text{(degrees K, ohms).} \end{aligned} \tag{8}$$

If there are several sources

$$\bar{e}^2 = 4k\int_0^\infty \left\{\sum_k T_k R_k(f) \mid A_r(f)\mid^2\right\} \mid E_0 D(f,f-f_0)A_a(f-f_0)\mid^2 \cdot df. \tag{9}$$

In the thermal effect the spectral distribution function $I^2(f)$ of the elementary current pulse $i(t)$ thus equals $2kT/R$.

[5] H. Nyquist: *Phys. Rev.*, **32**, 110, 1928.

III

Measurement of Receiver Noise

When the external noise level is low the fluctuation noise generated in the receiver will limit the sensitivity that may usefully be employed; this factor is therefore an important one in the performance rating of the receiver. Other factors, such as selectivity, sensitivity, fidelity, etc., of receiver performance have become the subject of standardized measurement and it seems desirable to standardize a suitable technique for noise measurement. The following procedure, which the author has found convenient, may be of interest in this connection.

If the "noise" is measured by the deflection of a meter indicating E^2 or I^2 it will, as we have seen, depend upon the electrical "fidelity" of the receiver. A receiver of good fidelity will be noisier than one of poor fidelity. Hence we cannot use the over-all noise alone as a basis of comparison. Just what audio-transmission range should be provided

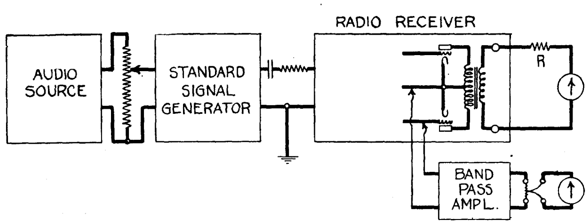

Fig. 5—Apparatus for measurement of specific noise in a radio receiver in terms of side-band amplitude in standard modulated signal.

in a receiver, in view of the presence of noise, is a matter of judgment and the decision depends upon many factors. In view of this it has seemed to me preferable to measure, instead of the total noise as given by (4), the *specific noise*, which is defined as the *noise per frequency interval*. This is the quantity which, multiplied by the transmission-squared and integrated over the frequency range, gives the total noise. The use of this quantity effectively divorces the question of noise generation from that of fidelity and permits a direct comparison to be made of the inherent noise generation in two receivers.

The experimental arrangement employed for this measurement is shown in Fig. 5. The scheme here is to single out a band of frequencies, wide enough to give a measurable deflection due to noise, but narrow enough to avoid troubles due to hum in a-c receivers on the low-frequency side and to avoid variations in the fidelity of the receiver on the higher-frequency side. The band-pass amplifier is designed to transmit frequencies from 300 to 1500 cycles. Its actual transmission curve is ascertained and a correction factor calculated

by mechanical integration. For better accuracy the fidelity curve of the receiver is combined with the transmission of the filter before integration.

The noise is compared with the amplitude of the side bands in a standard symetrically modulated signal of the type $e=E_0(1+m \sin at) \sin \omega t$, and expressed in terms of mE_0 (side-band amplitude). A measurement is made as follows:

The resistance R is that giving maximum output at 400 cycles and is that ordinarily used in measurements of the sensitivity by the standard method. The receiver sensitivity is adjusted to its maximum value and the standard signal generator (S.S.G.) input is adjusted for standard output in R at 400 cycles with 30 per cent modulation. After setting the signal and receiver in this fashion the 400-cycle modulation is cut off the S.S.G. (carrier remains) and the band-pass amplifier gain is adjusted until a readable deflection *due to noise* is obtained. The 400-cycle modulation voltage is then reduced *to a very low value* and reconnected to the S.S.G. being then adjusted until the reading of the noise meter is exactly doubled. The noise is then expressed as the value of mE_0 required for this equality of side-band signal and noise.

This may be repeated for other settings of the receiver sensitivity, and other operating conditions.

It should be noted that the "noise," measured in terms of mE_0 is not $\bar{e}^2$ but $(\bar{e}^2)^{1/2}$, a quantity which may be called the *noise amplitude*.

The deflection due to the modulation is

$$\text{Def.} = k(E_0^2 A_r^2 m D A_a)^2 \tag{10}$$

where E_0 is the input carrier voltage, A_r and A_a are respectively the transmissions of radio and audio amplifiers, D is the detection factor, m is the degree of modulation, and k is a factor expressing the transmission of the band-pass amplifier and meter at 400 cycles.

Throughout the range covered by the band-pass amplifier the transmissions from the various sources of noise to the detector are approximately uniform with respect to frequency so that very approximately the deflection due to the noise will be

$$\text{Def.} = kA_r^2E_0^2N^2 \int_0^\infty \left| A_r'(f)D(f_1 f \frac{f_0}{-})A_a(f) \right|^2 \left| K(f) \right|^2 \cdot df. \tag{11}$$

$K(f)$ represents the transmission of the BP amplifier (400 cycles $=100$ per cent. If the deflections are equal:

$$E_0 m = NA_r'/A_r \times \sqrt{M} \tag{12}$$

where M is a correction factor which expresses the effect of nonuniformity in the transmission over the frequency range. We see that E_0m does not actually measure the specific noise N at its source since A (the radio transmission from antenna to detector) does not in general equal A_r' (radio transmission from source of noise to detector) but it expresses the noise-signal ratio, or the noise due to all sources as referred to the antenna circuit. This is really the quantity of fundamental significance from the point of view of over-all performance. For design work other methods are available for the analysis of the several sources of noise.

One or two precautions may be mentioned. Since the amplitude of the noise is usually low difficulty may often be encountered in a-c receivers due to hum. The contribution of the hum to the noise deflection should be carefully ascertained in such cases. The suppression of all frequencies up to 300 cycles by means of the band-pass amplifier is usually sufficiently effective in avoiding this embarrassment. As to the measuring instrument, the author has found the use of ordinary vacuum-tube voltmeters unsatisfactory for the measurement of fluctuation effects. This is due to the fact that the amplitudes fluctuate considerably so that the instrument is actually overloaded, as far as the peaks are concerned, long before a sufficient deflection has been obtained. The same difficulty has been observed by A. W. Hull.[6] The use of a thermocouple completely avoids this difficulty and is much to be preferred. A detailed report of measurements of receiver noise will be given elsewhere, but a few sample results may be given here in conclusion.

TABLE I

RELATIVE SPECIFIC FLUCTUATION NOISE (REFERRED TO THE ANTENNA) IN A GROUP OF RFL BROADCAST RECEIVERS

Receiver	Relative Noise
A1	2.3
A2	2.75
B	1.4
C	1.0
D	5.7
E1	3.2
E2	3.0

The above measurements relate to a group of broadcast receivers designed for licensees of the Radio Frequency Laboratories and illustrate typical variations among various receivers. The results refer to a common sensitivity to which all receivers were adjusted.

[6] A. W. Hull, *Phys. Rev.*, **25**, 172. 1925.