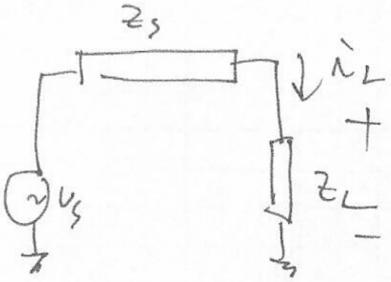
HW #1. Solution problem-1



1) Under mismatched conditions,

$$Z_S = R_S + jX_S$$

$$Z_L = (R_S + \Delta R_S) - j(X_S + \Delta X_S)$$

$$\Rightarrow V_L = \frac{Z_L}{Z_S + Z_L} \cdot V_S$$

$$= \frac{R_S + \Delta R_S - j(X_S + \Delta X_S)}{2R_S + \Delta R_S - j\Delta X_S} V_S$$

$$\Rightarrow i_L = \frac{1}{2R_S + \Delta R_S - j\Delta X_S} V_S$$

- Voltage across $R_L \Rightarrow V_{RL} = \dfrac{R_S + \Delta R_S}{2R_S + \Delta R_S - j\Delta X_S} V_S$

- Current through $R_L \Rightarrow i_{RL} = \dfrac{1}{2R_S + \Delta R_S - j\Delta X_S} V_S$

$$\therefore P_{RL} = Re\left(V_{RL} \cdot i_{RL}^*\right)$$

$$= \frac{(R_S + \Delta R_S) V_S^2}{(2R_S + \Delta R_S)^2 + (\Delta X_S)^2}$$

$$= \boxed{\frac{V_S^2}{4R_S} \frac{\left(1 + \frac{\Delta R_S}{R_S}\right)}{\left(1 + \frac{\Delta R_S}{2R_S}\right)^2 + \left(\frac{\Delta X_S}{2R_S}\right)^2}} \quad \text{(Ans)}$$

If V_S is peak value, then

$$P_{RL} = \boxed{\frac{1}{2} \cdot \frac{V_S^2}{4R_S} \frac{\left(1 + \frac{\Delta R_S}{R_S}\right)}{\left(1 + \frac{\Delta R_S}{2R_S}\right)^2 + \left(\frac{\Delta X_S}{2R_S}\right)^2}} \quad \text{(Ans)}$$

For both cases

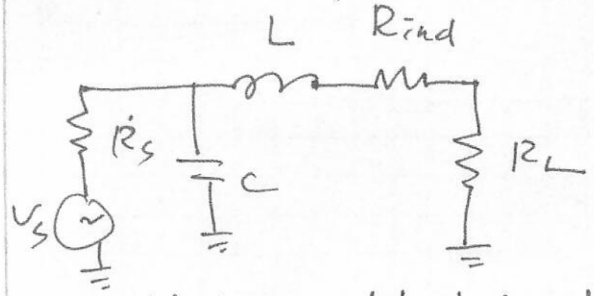
$$P_L = P_{max} \frac{1 + \frac{\Delta R_s}{R_s}}{\left(1 + \frac{\Delta R_s}{2R_s}\right)^2 + \left(\frac{\Delta X_s}{2R_s}\right)^2}$$

2) $$\frac{P_L}{P_{max}} = \boxed{\frac{1 + \frac{\Delta R_s}{R_s}}{\left(1 + \frac{\Delta R_s}{2R_s}\right)^2 + \left(\frac{\Delta X_s}{2R_s}\right)^2}} \quad (\text{Ans})$$

3) if $\Delta X_s = 0$

$\frac{\Delta R_s}{R_s}$	P_L / P_{max}
5% ~~(0.05)~~	
$\rightarrow \frac{\Delta R_s}{R_s} = 0.05$	$\rightarrow 0.9994 \rightarrow 99.94\%$
$\rightarrow \frac{\Delta R_s}{R_s} = -0.05$	$\rightarrow 0.9993 \rightarrow 99.93\%$
10%	
$\rightarrow \frac{\Delta R_s}{R_s} = 0.1$	$\rightarrow 0.9977 \rightarrow 99.77\%$
$\rightarrow \frac{\Delta R_s}{R_s} = -0.1$	$\rightarrow 0.9972 \rightarrow 99.72\%$
20%	
$\rightarrow \frac{\Delta R_s}{R_s} = 0.2$	$\rightarrow 0.9917 \rightarrow 99.17\%$
$\rightarrow \frac{\Delta R_s}{R_s} = -0.2$	$\rightarrow 0.9877 \rightarrow 98.77\%$
50%	
$\rightarrow \frac{\Delta R_s}{R_s} = 0.5$	$\rightarrow 0.96 \rightarrow 96\%$
$\rightarrow \frac{\Delta R_s}{R_s} = -0.5$	$\rightarrow 0.8888 \rightarrow 88.88\%$

HW #1. solution problem-2



Under matched conditions

1) $$V_L = \frac{1}{2} V_s \frac{R_L}{sL + R_{ind} + R_L}$$

$$i_L = \frac{1}{2} V_s \frac{1}{sL + R_{ind} + R_L}$$

$$\therefore P_L = Re\left(V_L \cdot I_L^*\right) = \frac{V_s^2}{4R_L} \frac{1}{\left(1+\frac{R_{ind}}{R_L}\right)^2 + \left(\frac{\omega_0 L}{R_L}\right)^2}$$

$$= \frac{V_s^2}{4R_s} \cdot \frac{R_s}{R_L} \frac{1}{\left(1+\frac{R_{ind}}{R_L}\right)^2 + \left(\frac{\omega_0 L}{R_L}\right)^2}$$

$$\frac{R_s}{R_L} = Q_T^2 + 1$$

$$\frac{R_{ind}}{R_L} = \frac{Q_T}{Q_{ind}}$$

$$\frac{\omega_0 L}{R_L} = Q_T$$

apply

$$= \frac{V_s^2}{4R_s}\left(Q_T^2+1\right) \frac{1}{\left(1+\frac{Q_T}{Q_{ind}}\right)^2 + Q_T^2}$$

2) $$V_{ind} = \frac{1}{2} V_s \frac{R_{ind}}{sL + R_{ind} + R_L}, \quad i_{ind} = i_L = \frac{1}{2} V_s \frac{1}{sL + R_{ind} + R_L}$$

$$\therefore P_{ind} = Re\left(V_{ind} \cdot I_{ind}^*\right) = \frac{V_s^2}{4R_L} \frac{R_{ind}/R_L}{\left(1+\frac{R_{ind}}{R_L}\right)^2 + \left(\frac{\omega_0 L}{R_L}\right)^2}$$

$$= \frac{V_s^2}{4R_s} \cdot \frac{R_s}{R_L} \cdot \frac{R_{ind}}{R_L} \cdot \frac{1}{\left(1+\frac{R_{ind}}{R_L}\right)^2 + \left(\frac{\omega_0 L}{R_L}\right)^2}$$

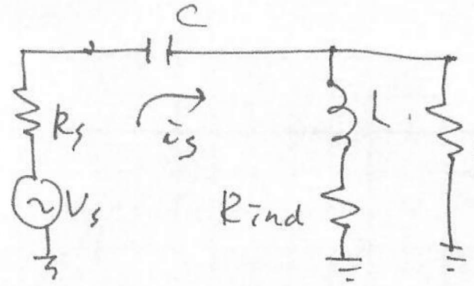
$$= \frac{V_s^2}{4R_s}\left(Q_T^2+1\right) \frac{Q_T}{Q_{ind}} \frac{1}{\left(1+\frac{Q_T}{Q_{ind}}\right)^2 + Q_T^2}$$

3)
$$\eta = \frac{P_L}{P_L + P_{ind}} = \frac{1}{1 + \frac{Q_T}{Q_{ind}}} = \frac{Q_{ind}}{Q_T + Q_{ind}}$$

~~4) same as 3)~~ or

$$\eta = \frac{1}{1 + \frac{R_{ind}}{R_L}} = \frac{R_L}{R_{ind} + R_L}$$

4) $\cancel{Q_{th}}$ $\eta = \dfrac{Q_{ind}}{Q_T + Q_{ind}}$.



Under matched conditions,

$$i_S = \frac{V_s}{2R_s}$$

1) $$i_{RL} = \frac{V_s}{2R_s} \cdot \frac{SL + R_{ind}}{SL + R_{ind} + R_L}$$

$$V_{RL} = i_{RL} \cdot R_L$$

$$\therefore P_{RL} = Re\left(V_{RL} \cdot i_{RL}^*\right) = R_L \cdot i_{RL} \cdot i_{RL}^*$$

$$= \frac{V_s^2}{4R_s^2} \cdot R_L \frac{R_{ind}^2 + \omega_0^2 L^2}{(R_L + R_{ind})^2 + \omega_0^2 L^2}$$

$$= \frac{V_s^2}{4R_s} \cdot \frac{R_L}{R_s} \frac{\left(R_{ind}/R_L\right)^2 + \left(\omega_0 L/\cancel{R_{ind}}\right)^2}{\left(1 + \frac{R_{ind}}{R_L}\right)^2 + \left(\frac{\omega_0 L}{R_L}\right)^2}$$

$$\frac{R_L}{R_s} = Q_T^2 + 1$$

$$\frac{\omega_0 L}{R_L} = \frac{1}{Q_T}$$

$$\frac{R_{ind}}{R_L} = \frac{1}{Q_T \cdot Q_{ind}}$$

apply

$$= \frac{V_s^2}{4R_s}\left(Q_T^2 + 1\right) \frac{\left(\frac{1}{Q_T \cdot Q_{ind}}\right)^2 + \left(\frac{1}{Q_T}\right)^2}{\left(1 + \frac{1}{Q_T \cdot Q_{ind}}\right)^2 + \left(\frac{1}{Q_T}\right)^2}$$

$$= \frac{V_s^2}{4R_s}\left(Q_T^2 + 1\right) \frac{1 + Q_{ind}^2}{\left(1 + Q_T \cdot Q_{ind}\right)^2 + Q_{ind}^2}$$

2) $\dot{i}_{Rind} = \dfrac{V_s}{2R_s} \dfrac{R_L}{sL + R_{ind} + R_L}$

$V_{Rind} = \dot{i}_{Rind} \cdot R_{ind}$

$$\therefore P_{ind} = Re\left(V_{Rind} \cdot \dot{i}_{Rind}^{*}\right)$$

$$= \frac{V_s^2}{4R_s^2} \cdot R_{ind} \frac{R_L}{(R_{ind}+R_L)^2 + \omega_o^2 L^2}$$

$$= \frac{V_s^2}{4R_s} \cdot \frac{R_L}{R_s} \cdot \frac{R_{ind}/R_L}{\left(1+\frac{R_{ind}}{R_L}\right)^2 + \left(\frac{\omega_o L}{R_L}\right)^2}$$

$$= \frac{V_s^2}{4R_s}(Q_T^2+1) \frac{\frac{1}{Q_T \cdot Q_{ind}}}{\left(1+\frac{1}{Q_T \cdot Q_{ind}}\right)^2 + \left(\frac{1}{Q_T}\right)^2}$$

$$= \frac{V_s^2}{4R_s}(Q_T^2+1) \frac{Q_T \cdot Q_{ind}}{(1+Q_T \cdot Q_{ind})^2 + Q_{ind}^2}$$

3) $$\eta = \frac{P_L}{P_L + P_{ind}} = \frac{(R_{ind}/R_L)^2 + (\omega_o L/R_L)^2}{(R_{ind}/R_L)^2 + (\omega_o L/R_L)^2 + R_{ind}/R_L}$$

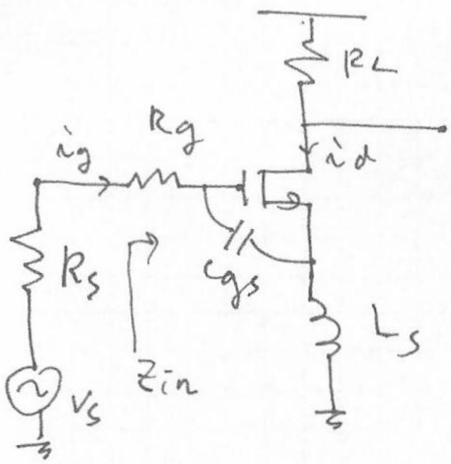
$$= \frac{1+Q_{ind}^2}{1+Q_{ind}^2 + Q_T \cdot Q_{ind}}$$

if $Q_{ind} \gg 1$

$$\approx \frac{Q_{ind}}{Q_{ind}+Q_T}$$ ← same as problem-2).

4) $$\eta = \frac{Q_{ind}}{Q_{ind}+Q_T}$$

HW#1 Solution problem-4



current gain, β

$$\boxed{\beta = \frac{i_d}{i_g} = \frac{g_m}{sC_{gs}}}$$

1) $$Z_{in} = R_g + \frac{1}{sC_{gs}} + (1+\beta) sL_s$$

$$= R_g + \frac{1}{sC_{gs}} + \left(1 + \frac{g_m}{sC_{gs}}\right) sL_s$$

$$= R_g + \frac{1}{sC_{gs}} + sL_s + \frac{g_m L_s}{C_{gs}}$$

2) $$V_{gs} = i_g \cdot \frac{1}{sC_{gs}}$$

$$= \frac{V_s}{R_s + Z_{in}} \cdot \frac{1}{sC_{gs}} = \frac{\frac{1}{sC_{gs}}}{R_s + \left(R_g + \frac{g_m L_s}{C_{gs}}\right) + sL_s + \frac{1}{sC_{gs}}} \cdot V_s$$

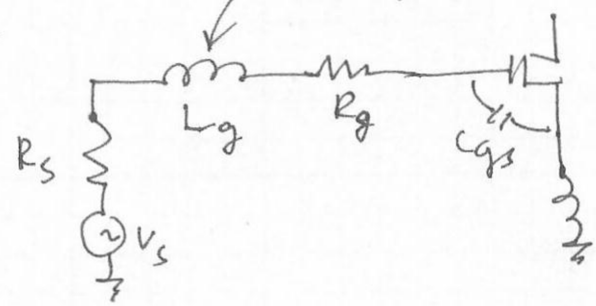
$$i_{out} = g_m V_{gs} = i_g \cdot \beta$$

$$= \frac{g_m / sC_{gs}}{R_s + \left(R_g + \frac{g_m L_s}{C_{gs}}\right) + sL_s + \frac{1}{sC_{gs}}} \cdot V_s$$

3) $$A_V = \frac{V_{out}}{V_s} = \frac{-i_{out} \cdot R_L}{V_s} = - \frac{g_m / sC_{gs} \cdot R_L}{\left(R_s + R_g + \frac{g_m L_s}{C_{gs}}\right) + sL_s + \frac{1}{sC_{gs}}}$$

4) $Z_{in} = \underbrace{R_g + \frac{g_m L_s}{C_{gs}}}_{= R_s} + \underbrace{sL_s + \frac{1}{sC_{gs}}}$

Since $\omega_o L_s < \frac{1}{\omega_o C_{gs}}$,
we need inductance
apply L_g ← to resonate the capacitance.



$$\omega_o (L_g + L_s) = \frac{1}{\omega_o C_{gs}}$$

$$\therefore L_g = \frac{1}{\omega_o^2 C_{gs}} - L_s$$

5) Under matched condition

$$i_g = \frac{V_s}{2R_s} \rightarrow V_{gs} = i_g \frac{1}{sC_{gs}} = \frac{1}{2R_s sC_{gs}} \cdot V_s$$

$$\rightarrow i_{out} = g_m V_{gs} = i_g \cdot \beta$$

$$= \frac{V_s}{2R_s} \cdot \frac{g_m}{sC_{gs}}$$

6) $A_V = \frac{V_{out}}{V_s} = \frac{-i_{out} \cdot R_L}{V_s} = -\frac{1}{2R_s} \cdot \frac{g_m}{sC_{gs}} \cdot R_L$

⇒ Under matched condition, gain will be increased by a factor of $\left(1 + \frac{sL_s + \frac{1}{sC_{gs}}}{2R_s}\right)$.

$$\Rightarrow 1 + \frac{sL_s + \frac{1}{sC_{gs}}}{2R_s} = 1 - \frac{sL_g}{2R_s} = 1 - j\frac{1}{2}\frac{\omega_o L_g}{R_s} \approx 1 - j Q$$

∴ if Q is large enough, the gain factor is Q, where Q is Quality factor of matching circuit.

7) L_s provides real impedance for matching.