1) Assume no image signal (V_{pim}=0) and V_{ps}=1mV_{peak}. Calculate ***rms*** output signal power (S_o) and output noise power ($N_o/\Delta f$) density and noise factor F (15 pt).
(Note: K=1.38x10^{-23} J/K, T=300 K)

$v_{op} = \frac{2}{\pi} g_m R_L v_p \rightarrow v_{op}^2 = \left(\frac{2}{\pi} g_m R_L v_{ps}\right)^2$

$\rightarrow \overline{v_{op}^2} = \left(\frac{2}{\pi} g_m R_L v_p\right)^2 \frac{1}{2}$

$\rightarrow S_o = \overline{v_{op}^2} \cdot \frac{1}{R_L} = \left(\frac{2}{\pi} g_m R_L v_p\right)^2 \frac{1}{2R_L}$

$= \left(\frac{2}{\pi} \cdot 3k\Omega \cdot 500 \cdot 1m\right)^2 \cdot \frac{1}{2 \cdot 500} = 117.13\,mW$

$= -39.31\,dBm$

$\overline{v_{no}^2} = 4kT\left(R_s \cdot g_m^2 + \frac{\gamma}{\alpha} \cdot g_m + \frac{2}{R_L}\right) R_L^2$

$= 4 \cdot 1.38 \times 10^{-23} \cdot 300 \left(50 \cdot (3k\Omega)^2 + \frac{1.5}{0.9} \cdot 3k\Omega + \frac{2}{500}\right) 500^2$

$= 490.45 \times 10^{-18}\, V_{rms}^2/Hz$

2) Repeat 1) when W/L increased twice while maintaining same bias current (10 pt).

$F = \frac{1}{\left(\frac{2}{\pi} \cdot g_m \cdot R_L\right)^2} \cdot \frac{\overline{v_{no}^2}}{\overline{v_{ni}^2}} = \frac{1}{\left(\frac{2}{\pi} \cdot g_m \cdot R_L\right)^2} \frac{4kT\left(R_s \cdot g_m^2 + \frac{\gamma}{\alpha} g_m + \frac{2}{R_L}\right) R_L^2}{4kTR_s}$

$= \left(\frac{\pi}{2}\right)^2 \left(1 + \frac{\gamma}{\alpha} \cdot \frac{1}{g_m R_s} + \frac{2}{g_m^2 R_L R_s}\right) = 5.057 \Rightarrow 7.04\,dB$

W/L ↑ ×2 → g_m ↑ × $\sqrt{2}$

⇒ S_o ↑ × 2

$\Rightarrow F_{new} = \left(\frac{\pi}{2}\right)^2 \left(1 + \frac{\gamma}{\alpha} \frac{1}{g_m R_s} \cdot \frac{1}{\sqrt{2}} + \frac{2}{g_m^2 \cdot R_L \cdot R_s} \times \frac{1}{2}\right) = 4.26$

$\Rightarrow 6.3\,dB$

3) What's output signal power if the image tone has same magnitude as signal, i.e., V_{pim}=V_{ps}=1mV_{peak} (5 pt) ?

$v_o = \frac{4}{\pi}\left(\sin\omega_{LO}t \cdot V_{ps}\cos\omega_s t + \sin\omega_{LO}t \cdot V_{pim}\cos\omega_{im}t\right)$

$= \frac{2}{\pi} V_{ps}\left(\sin(\underbrace{\omega_{LO} - \omega_{RF}}_{=+\omega_{IF}})t + \sin(\underbrace{\omega_{LO} - \omega_{im}}_{=-\omega_{IF}})t\right)$

$= \frac{2}{\pi} V_{ps}\left(\sin\omega_{IF}t - \sin\omega_{IF}t\right)$

$= 0.$