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Variance Fluctuations in Flicker Noise and Current Noise*

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Variance fluctuations in band-limited vacuum-tube flicker noise, current noise in a germanium p–n junction, and current noise in a carbon resistor are measured by determining probability amplitude distributions of the noise signals with a multichannel pulse-height analyzer. The variances of 30 noise specimens, each 100 sec in duration, for the three noise sources are obtained from the width of the probability amplitude distributions, all of which obey a normal-distribution law. Fluctuations in variance are largest in the case of resistor current noise and least for current noise in a p–n junction. Even in the latter case, however, the variations are well in excess of artifacts resulting from sampling errors, as demonstrated by examining Nyquist noise signals. It is concluded that all three $1/f$ noise sources exhibit variance fluctuations to varying degrees.

I. INTRODUCTION

Experimental measurements[1] have demonstrated that variance fluctuations can be detected in the current noise of carbon resistors. That is, the total variance of a noise signal of a given duration differs from the total variance in other, similar intervals. This means that the noise is not stationary in the usual statistical sense such as is true, for example, for Nyquist noise.

A most striking feature of current noise in carbon resistors is the well-known $1/f$ noise-power spectrum.[2] It is pertinent to inquire if variance fluctuations are observed in other $1/f$ noise phenomena as well. Experimental investigation of the statistical properties of flicker noise in vacuum tubes[3] and current noise in semiconductor p–n junctions,[4] both known to exhibit a $1/f$ noise-power spectrum, are instituted to answer this question.

II. EXPERIMENTAL TECHNIQUE

Variances of the random noise signals are examined with an improved version of the same apparatus previously used.[1] Noise signals are amplified in a conventional amplifier tunable-filter system and sampled for a given duration at a 10 kHz rate. The signal level at each sample is converted to an electrical pulse and the distribution of pulse heights measured with a conventional pulse-height analyzer. The system is also capable of determining the noise-power spectra in the standard manner. Minor modifications include improved amplifier stability and the display of pulse amplitude distributions on an xy recorder rather than on an oscilloscope. These changes improve the performance of the system significantly, as discussed in the next section.

The statistical properties of current noise of several 50 000 Ω carbon resistors are investigated for comparison with previous results. To study variance fluctuations in the current noise of semiconductor p–n junctions, the resistor is replaced by a type 2N2000 germanium transistor with the collector and emitter connected together. The noise properties are examined at a reverse bias of 20 V. Flicker noise is examined by simply shorting the input of the first amplifier stage. The output noise signal is then that due to the vacuum tube in the first stage, a type 12AX7.

Amplifier gain is adjusted to present equal noise-signal levels to the analog-to-digital sampling circuit in spite of the rather wide differences in noise level between the three sources. The overall gain is calibrated by introducing known sinewave signals at the input terminals. This calibration is confirmed by measuring the Nyquist noise level of 25 000- and 100 000-Ω resistors connected to the input. Finally, as described below, the measured statistical properties of Nyquist noise from these resistors is found to agree with the Nyquist theorem.

The noise-power spectrum of each noise source is determined in the conventional manner to confirm that the noise spectrum has the $1/f$ behavior. To search for variance fluctuations, the variance of approximately 30 noise specimens, each of 100 sec duration, is obtained. The variance s is calculated from the width of each probability amplitude distribution displayed on the xy plotter. In agreement with previous results, all distributions obey a normal-distribution law.

For each noise type, the mean variance $\langle s \rangle$ of the 30 noise specimens is calculated. Then the variance of the variances $\langle s^2 \rangle$ is computed manually. It is found, as in the previous experimental results, that the mean variance in the case of Nyquist noise agrees with the total variance calculated from the experimental Nyquist noise spectrum.

III. RESULTS

Typical noise-power spectra for each of the three noise sources are shown in Fig. 1. All exhibit the $1/f$ characteristic, as anticipated, although the flicker-noise level of the vacuum tube becomes equal to shot

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[1] J. J. Brophy, Phys. Rev. **166**, 827 (1968).
[2] I. M. Templeton and D. K. C. MacDonald, Proc. Phys. Soc. (London) **B66**, 680 (1953).
[3] A. Van der Ziel, *Noise in Electron Devices*, L. D. Smullin and H. A. Haus, Eds. (John Wiley & Sons, Inc., New York, 1969), Chap. 2.
[4] F. J. Hyde, Proc. Phys. Soc. (London) **B69**, 231 (1956).

TABLE I. Measured statistical properties of noise sources.[a]

Noise signal	Signal bandwidth (Hz)	Sample length (sec)	Mean variance $\langle s\rangle$ V^2	Variance of variance $\langle s^2\rangle$ V^4	$\langle s\rangle^2/\langle s^2\rangle$
Nyquist noise 25 000-Ω resistor	8 to 10^4	100	4.7×10^{-12}	6.1×10^{-27}	3600
Current noise 50 000-Ω carbon resistor	8 to 10^4	100	5.4×10^{-9}	2.0×10^{-18}	15
Flicker noise 12AX7 tube	8 to 10^3	100	5.9×10^{-13}	3.9×10^{-27}	89
Current noise 2N200 germanium diode	8 to 10^4	100	2.7×10^{-10}	4.8×10^{-22}	150
Nyquist noise 100 000-Ω resistor	8 to 10^4	100	9.1×10^{-12}	2.9×10^{-26}	2900
Nyquist noise 100 000-Ω resistor	8 to 10^4	1	8.5×10^{-12}	1.2×10^{-25}	620

[a] Based on approximately 30 samples of each.

noise at about one kHz. For this reason, further examination of this noise source was restricted to the band from 8 to 10^3 Hz. The current noise level of the carbon resistor is approximately the same as that observed previously[1] in a different specimen. Conveniently, the germanium diode current-noise magnitude is not too different from the resistor-noise level.

The statistical properties of these noise sources are presented in the first four lines of Table I. The first line gives data for the carbon resistor in the absence of current, that is, for Nyquist noise. Because of the loading effect of the input circuit,[1] the noise level is equivalent to that of a 25 000-Ω resistor. Note that the ratio of the square of the mean variance to the variance

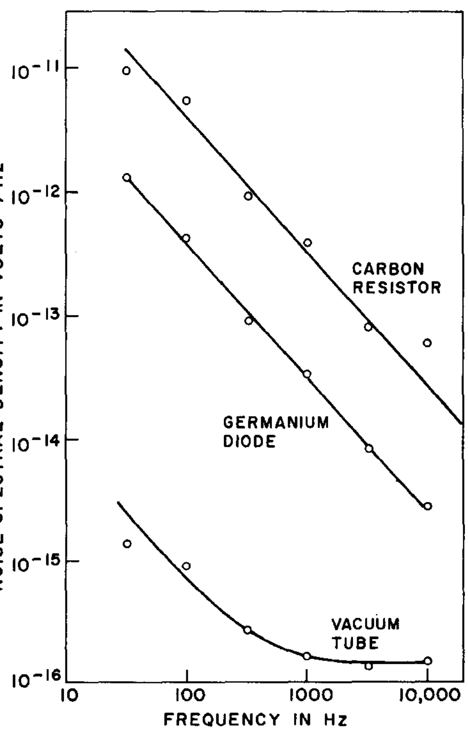

FIG. 1. Current noise spectra of a 50k-Ω carbon resistor, a 2N2000 germanium diode-connected transistor, and a 12AX7 vacuum tube.

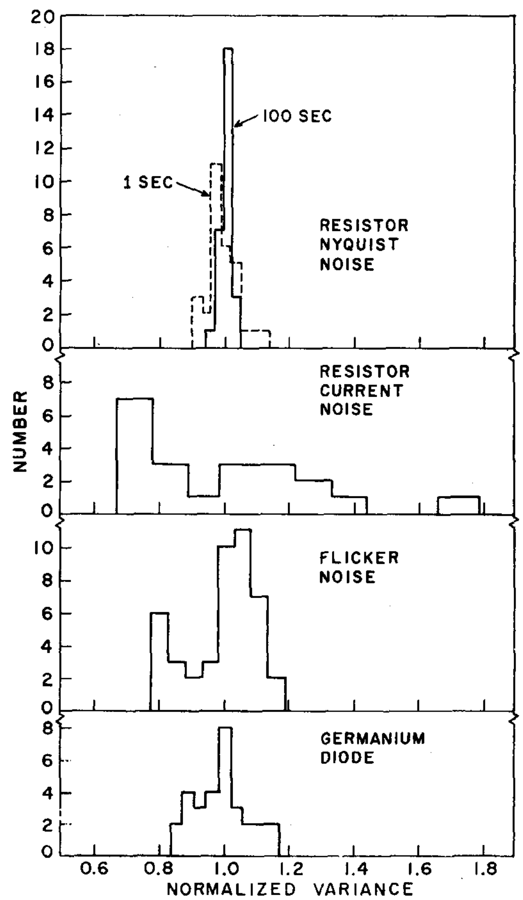

FIG. 2. Variance distributions for different noise sources.

of the variance, $\langle s\rangle^2/\langle s^2\rangle$, is 3600. In the case of Nyquist noise, the variance of the variance is determined by statistical uncertainty and experimental random errors so that a large value of this ratio indicates a more reliable experimental technique. The value of the ratio in Ref. 1 was 33 so that the performance of the present apparatus is improved by two orders-of-magnitude.

By comparison, the ratio $\langle s\rangle^2/\langle s^2\rangle$ is equal to 15 in the case of resistor current noise, compared to the value 2.6 previously reported. Since a small value of this ratio indicates large variance fluctuations, the previous specimen exhibits a "noisier noise" than the one presently reported. The value of the ratio is 89 in the case of flicker noise and 150 in the case of the germanium diode, as shown in lines 3 and 4. These results indicate that variance fluctuations are present in all three types of $1/f$ noise.

In the case of current noise in a germanium diode, the ratio is more than twenty times smaller than that observed for Nyquist noise, as shown in Table I. This would seem to be a sufficiently large factor to confirm the existence of variance fluctuations in this case. It has been pointed out,[5] however, that the experimental technique employed tends to yield greater variance fluctuations in $1/f$ noise signals than in Nyquist noise signals. This is so because both noise signals are sampled at the same 10-kHz rate and the large low-frequency components characteristic of $1/f$ noise introduce correlation between successive samples. Thus, there are, in effect, fewer independent samples in the case of $1/f$ noise compared to Nyquist noise, which has a white noise-power spectrum. Statistical fluctuations associated with fewer independent samples tend to cause variations in experimentally determined variances. As shown in Ref. 5, under the experimental conditions pertaining to Table I, there are 120 fewer independent samples in the case of $1/f$ noise compared to Nyquist noise.

The influence of this experimental effect is investigated by examining Nyquist noise of a 100 000-Ω resistor for two different durations, 100 sec and 1 sec. At the constant sampling rate of 10 kHz, the 100-sec duration results in 10^6 samples while the 1-sec duration yields only 10^4 samples. Therefore, there is a 100:1 ratio of independent samples since all samples are independent in the case of Nyquist noise. The results for 30 noise specimens of each duration are shown in lines 5 and 6 of Table I. The value of the ratio $\langle s\rangle^2/\langle s^2\rangle$ is 2900 for the 100-sec duration samples, which compares favorably with that listed in the first line of the table. The ratio is 620 for the 1-sec specimens, which is a factor of 5 smaller than that for the longer interval. These results do, in fact, illustrate the influence of statistical fluctuations arising from the number of samples, as expected.

[5] L. J. Greenstein and J. J. Brophy, J. Appl. Phys. **40**, 682 (1969).

Note that the ratio in line 6 is greater than that for the germanium diode current noise. Yet, on the basis of correlation effects the two sets of data are based on approximately the same number of independent samples. It is concluded that germanium diode noise exhibits variance fluctuations in excess of those expected on the basis of purely statistical effects.

Histograms of the data pertaining to Table I, shown in Fig. 2, are consistent with the above interpretation. In order to compare the various sets of data, individual variances are converted to a normalized variance by dividing by the mean variance for each noise source. The top figure shows data for the 100 000-Ω resistor. The spread of the data for the 100-sec case is quite small and is somewhat greater for the 1-sec case. Both histograms are symmetrical and suggest a normal distribution arising from statistical effects and experimental randomness.

The resistor current noise has a much greater spread. Furthermore, the distribution is skewed so that the most probable variance is less than the mean variance. This is consistent with previous results on other specimens.[1,5]

The spread in the data for both flicker noise and germanium diode current noise is less than in the case of resistor noise, but is greater than that corresponding to Nyquist noise. There is also a trend toward symmetry in these histograms. These limited data suggest that variance fluctuations in flicker noise and semiconductor diode current noise differ from variance fluctuations in current noise in a carbon resistor.

IV. CONCLUSIONS

Variance fluctuations are observed in $1/f$ noise signals generated in vacuum tubes and p–n junction diodes as previously reported in current noise in carbon resistors. The magnitude of the fluctuations in the two new noise sources examined are smaller than those seen in the latter case but are well in excess of statistical errors arising from correlation effects due to large low-frequency components characteristic of $1/f$ noise signals. In addition, the distributions of individual variances about the mean variance for flicker noise and semiconductor diode noise tend to be symmetrical rather than skewed toward smaller values as in the case of resistor current noise.

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