FLUCTUATION NOISE IN PARTIALLY SATURATED DIODES*

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(Paper first received 26th May, and in revised form 18th November, 1938.)

SUMMARY

A previous paper† discussed the nature of fluctuation noise in thermionic valves which are either perfectly temperature-limited or perfectly space-charge-limited. It was suggested that the noise should be represented by a " shot noise " formula in the former, and a " thermal noise " formula in the latter. But since perfection is difficult to achieve, the case of more practical value is the valve which is mainly space-charge-limited, but not entirely free from temperature limitation. A method of quantitative estimation of the noise in diodes under such conditions (in terms of the current/voltage characteristic) is the subject of this further paper.

The " ideal " diode would have a characteristic such as that sketched in Fig. 1, curve (*a*), following a $\frac{3}{2}$-power law from the origin to a point $V_a = V_s$, and then changing abruptly to constant current. In an actual valve, however, the curve is more likely to be as shown by the dotted line (*b*), where the current is not zero at $V_a = 0$ (owing to electron emission velocity). The slope of the curve is always less than corresponds to the $\frac{3}{2}$-power law, and the change to temperature-limitation (constant-current condition) is comparatively gradual. From the previous paper we have hypotheses which would permit us to predict the noise from the i_a/V_a characteristic for both parts of curve (*a*); namely, as thermal noise in a certain equivalent resistance at a defined effective temperature for the $\frac{3}{2}$-power law (space-charge-limited) region, and as simple shot noise for the constant-current (temperature-limited) region. But we have to face the problem of curve (*b*), which does not belong wholly to either of these regions.

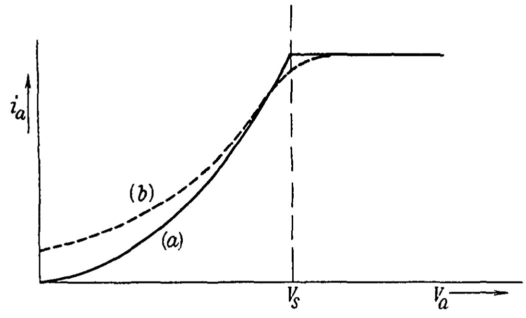

Fig. 1

Consider now a hypothetical system of two diodes of identical construction but different cathode temperatures, one of which, (*a*), is perfectly space-charge-limited, while the other, (*b*), is perfectly temperature-limited throughout the range of anode voltage chosen for examination. The temperature-limitation of (*b*) is suggested in Fig. 2 by the resistance in series with its heater, giving a lower cathode temperature and less emission. The currents in the two valves are accordingly given by the expressions

$$i_1 = aV_a^{3/2} \quad \dots \dots \quad (1)$$

$$i_2 = b \quad \dots \dots \quad (2)$$

and the total current i_0 will be

$$i_0 = i_1 + i_2 = aV_a^{3/2} + b \quad \dots \quad (3)$$

Now suppose it is impossible to measure separately the currents i_1 and i_2; it will still be possible to determine

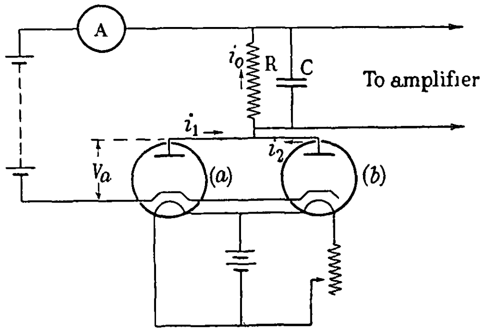

Fig. 2

them from a series of observations of i_0 at different values of V_a, by making use of the slope of the i_0/V_a characteristic. Thus

$$\frac{di_0}{dV_a} = (3/2)aV_a^{\frac{1}{2}} = \frac{3i_1}{2V_a}$$

$$\therefore \quad i_1 = \frac{2V_a}{3} \cdot \frac{di_0}{dV_a} = \frac{2V_a}{3R_a} \quad \dots \quad (4)$$

In fact the measurable slope conductance di_0/dV_a of the combination is equal to that of valve (*a*) alone, since valve (*b*) has zero slope conductance. By simple subtraction we then write

$$i_2 = i_0 - i_1 = i_0 - \frac{2V_a}{3R_a} \quad \dots \quad (5)$$

so that both constituent currents are known. The value of this is seen when we replace the two separate idealized valves by a single imperfect diode, having, for example, a non-uniform cathode temperature, in which some parts of the cathode reach temperature-limitation while other

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† " A Theory of Fluctuation Noise," *Journal I.E.E.*, 1938, vol. 82, p. 522.

parts are still space-charge-limited; by means of equations (4) and (5) we can determine what proportions of the anode current of such a diode are to be attributed to each regime.

Before investigating the fluctuation noise to be

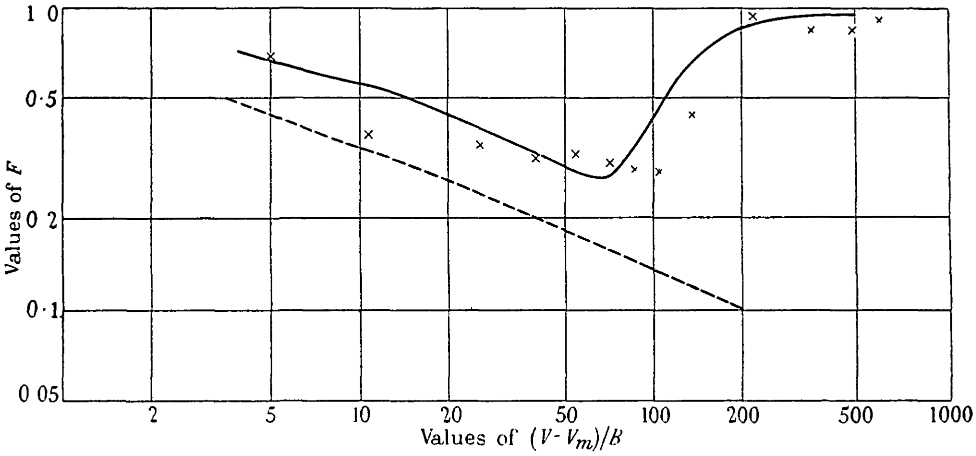

Fig. 3.—" Rauschrohre 6 "

Calculated curve ————
Jacoby and Kirchgessner measured points. × × ×
Schottky calculated curve of lower limit of noise - - - - - -

expected from the system, we must decide what units are to be employed in specifying it. The obvious course of using absolute units is open to the objection that it does not give a ready comparison between valves of different dimensions [different values of the constant a in equation (1)]. Following the example of previous

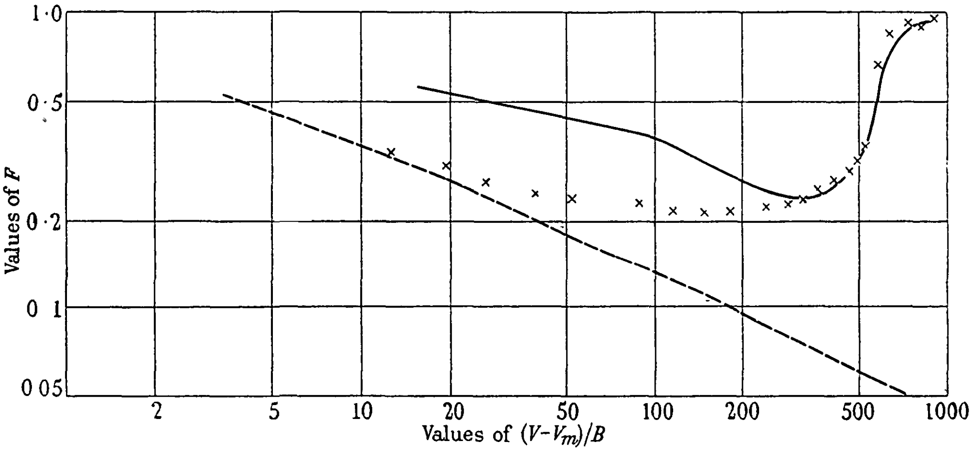

Fig. 4.—"Rauschröhre 7."
(Details as Fig. 3.)

authors, therefore, the results are preferably expressed as the ratio of the observed noise to the noise that would be generated by an equal current i_0 flowing through a temperature-limited valve connected in the same measuring circuit. This ratio has sometimes been called the " smoothing factor "; but that name is undesirable since it is suggestive of the " thermal noise plus shot noise " theory, which is not consistent with the hypotheses adopted by the author. The ratio in question will therefore be termed simply " the noise ratio." It is now necessary to distinguish between two possible noise ratios. First, one may take the ratio of the mean-square values, and this is the factor which Dr. F. C. Williams denotes by A;* alternatively, one may take the ratio of the root-mean-square values, and this is the factor denoted by F in the work of Prof. W. Schottky and his colleagues.†

The mean-square value is obviously the factor of greater primary significance, but the derived root-mean-square value will have practical advantages (as pointed out by Prof. Schottky) when the theory can be applied to amplifying valves instead of diodes. For the noise problem normally arises in voltage amplifiers, whose

* See, for example, *Journal I.E E.*, 1936, vol. 79, p. 349.
† See, for example, *Wissentschaftliche Veröffentlichungen aus den Siemens-Werken*, vol. 16, part 2.

signal output is specified linearly in volts, not as a squared value, so that signal/noise ratios will be more easily calculated if r.m.s. noise values are used. In practice F is obtained when required simply by taking the square root of A.

Returning to Fig. 2, let us calculate the total fluctuation current to be expected from the two currents i_1 and i_2, which are assumed both to flow through a noise-measuring system consisting of the load circuit R, C, and an amplifier. Considering a narrow frequency band df, valve (b) will contribute a "shot noise" fluctuation current of mean-square value

$$\overline{I_s^2} = 2i_2e \cdot df \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Now it was shown in the previous paper* that a valve of $\frac{3}{2}$-power law and slope resistance R_a appears as a thermal-noise source of mean-square e.m.f.

$$\overline{V^2} = 4(1 \cdot 2)R_a k(T/2) \cdot df \quad . \quad . \quad . \quad (7)$$

The mean square fluctuation *current* in such a valve must therefore be

$$\overline{I_t^2} = \frac{\overline{V^2}}{R_a^2} = \frac{4(1 \cdot 2)kT}{2R_a} \cdot df \quad . \quad . \quad . \quad (8)$$

Since we are concerned with mean-square values of random currents, the total fluctuation noise is the sum of the components from valve (a) and valve (b), namely

$$\overline{I_0^2} = \left\{2i_2e + \frac{4(1 \cdot 2)kT}{2R_a}\right\}df \quad . \quad . \quad . \quad (9)$$

But the noise from the whole current i_0 if it were entirely temperature-limited would be

$$\overline{I_x^2} = 2i_0e \cdot df \quad . \quad . \quad . \quad . \quad . \quad (10)$$

On dividing (9) by (10), the mean-square noise ratio is

$$\frac{\overline{I_0^2}}{\overline{I_x^2}} = A = \frac{i_2}{i_0} + \frac{1 \cdot 2kT}{i_0eR_a} \quad . \quad . \quad . \quad . \quad (11)$$

The factor F is then obtained as the square root of A.

This method of calculation has been applied to the two diodes described as "Rauschröhre 6" and "Rauschröhre 7" in a paper by H. Jacoby and L. Kirchgessner,† and Figs. 3 and 4 show a comparison of their published measurements of noise ratio with the values of F calculated by the method described above from their published i_a/V_a characteristics. In view of the fact that the published characteristics are printed to a size of graph of $2\frac{3}{8}'' \times 3''$ only, and as it was necessary to re-plot these and apply graphical differentiation to obtain values of R_a,‡ the agreement of theory with experiment is considered to be good. The abscissae are as in Jacoby and Kirchgessner's paper, in units of $(V - V_m)/B$, where $(V - V_m)$ is the potential difference between anode and potential minimum, and B is the equivalent in volts of the mean emission energy of the electrons ($B = kT/e$). The potential minimum has been calculated in terms of the ratio of the observed anode current at any voltage to the total cathode emission, the latter being assumed to be equal to the saturation value of anode current reached at high anode voltage.

As an indication of the values involved, a set of figures corresponding to the curve in Fig. 3 is given in the Table.

The best basis of comparison between diodes and triodes is in terms of the ratio of working anode current

Table

V	i	R_a	i_1	i_2	$\frac{6kT}{5eR_a}$ $\times 10^{-3}$	F
1·35	0·192	7 500	0·12	0·072	0·028	0·722
2·60	0·391	5 500	0·316	0·075	0·038	0·538
3·77	0·637	4 550	0·553	0·084	0·046	0·452
6·25	1·23	3 650	1·14	0·09	0·057	0·346
11·09	2·58	3 000	2·46	0·12	0·069	0·272
13·4	3·40	2 940	3·03	0·37	0·071	0·36
13·9	3·5	2 940	3·04	0·46	0·07	0·388
17·0	4·57	2 850	3·98	0·59	0·073	0·38
20·5	5·66	3 300	4·15	1·51	0·063	0·526
34·5	7·41	10 050	2·29	5·12	0·02	0·841
51·7	8·20	29 500	1·17	7·03	0·007	0·926
70·5	8·71	42 000	1·12	7·59	0·005	0·934
89·7	8·89	57 000	1·05	7·84	0·004	0·946

to total cathode emission. Let us assume that a triode amplifying valve has a working range of anode current from 10 % to 50 % of cathode emission. Then in Fig. 3 and the Table it will be seen that over this range for the diode the temperature-limited component of current i_2 contributes 60 % to 90 % of the noise, though it is only of the order of 10 % to 15 % of the anode current. This indicates at once the liability of any commercial valve to have a noise level far above that predicted by the thermal theory on the assumption of complete space-charge limitation. It also indicates the difficulty of checking the theory by published results of other workers, since very precise knowledge of the i_a/V_a characteristic is necessary to determine i_2 with the required accuracy. The d.c. characteristics of the valve have not hitherto been considered of fundamental importance to the theory of fluctuation noise, and the author has so far been unable to find published results for triodes which give suitable data.

* *Loc. cit.*

† *Wissentschaftliche Veröffentlichungen aus den Siemens-Werken*, vol. 16, part 2, p. 42.

‡ A curve of R_a against i_a is plotted in the published paper, but it was apparently obtained by measurement on a bridge; although satisfactory for the straighter parts of the characteristic, it has not the high precision required for the present purpose in the region of the upper bend. The graphical method therefore had to be used to obtain values of R_a.