

From ②,

$$I_{D1} + I_{D2} - 2\sqrt{I_{D1} \cdot I_{D2}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS1} - V_{GS2})^2$$

$$\therefore 2\sqrt{I_{D1} \cdot I_{D2}} = 2I_{D5} - \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS1} - V_{GS2})^2 \quad - (3)$$

apply

From ①,

$$\begin{aligned} (I_{D1} - I_{D2})^2 &= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS1} - V_{GS2})^2 (2I_{D5} + 2\sqrt{I_{D1} \cdot I_{D2}}) \\ &= 2I_{D5} \mu_n C_{ox} \frac{W}{L} (V_{GS1} - V_{GS2})^2 \\ &= \frac{1}{4} \left(\mu_n C_{ox} \frac{W}{L} \right)^2 (V_{GS1} - V_{GS2})^4 \end{aligned}$$

$$\therefore I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS1} - V_{GS2}) \sqrt{\frac{8I_{D5}}{\mu_n C_{ox} \frac{W}{L}} - (V_{GS1} - V_{GS2})^2} \quad - (4)$$

From ④, $I_{D1} - I_{D2} = i_o$
 $V_{GS1} - V_{GS2} = v_{in}$

$$\therefore i_o = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} v_{in} \sqrt{\frac{8I_{D5}}{\mu_n C_{ox} \frac{W}{L}} - v_{in}^2} \quad - (5)$$

$$\begin{aligned} \therefore \text{overall } G_m &= \left(\frac{\partial i_o}{\partial v_{in}} \right)_{v_{in}=0} \\ &= \left(\frac{1}{2} \mu_n C_{ox} \frac{W}{L} \frac{\frac{8I_{D5}}{\mu_n C_{ox} \frac{W}{L}} - 2v_{in}^2}{\sqrt{\frac{8I_{D5}}{\mu_n C_{ox} \frac{W}{L}} - v_{in}^2}} \right)_{v_{in}=0} \\ &= \sqrt{\mu_n C_{ox} \frac{W}{L} \cdot 2I_{D5}} \end{aligned}$$