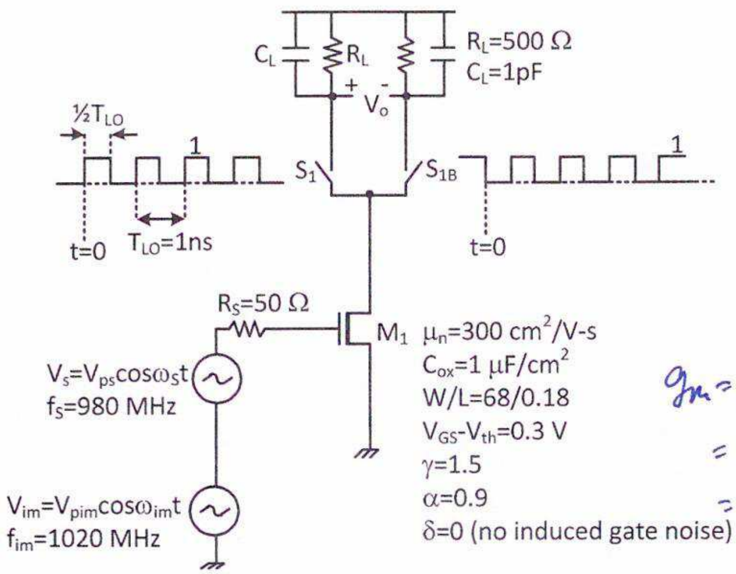
ECE5220 RFIC Final Exam, Due: 4PM, 05/10/2012 (Thursday)   Name: solution **Grade:**

1. In the mixer shown below, DC characteristic of the NMOS, M1, is set by square-law characteristic, i.e.

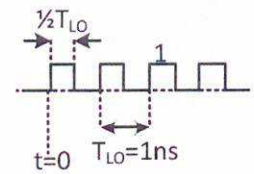
$$I_{DS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2.$$

Assume that Rs << $1/\omega C_{gs}$ and NMOS has only drain thermal noise current, i.e., no gate induced noise and parasitic gate resistance. The drain thermal noise coefficient is γ. S_1 and S_{1B} are ideal differential switches driven by ideal rectangular pulse trains whose duty cycle is 50%. **R_L is noisy load resistor**.



$$g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})$$
$$= 300 \times 1\mu \times \frac{68}{0.18} \times 0.3$$
$$= 34\ m\mho$$

*You may need this series expression for question #1.



$$\text{rect}(t) = \frac{1}{2} + \sum_{n=1}^{n=\infty}\left(\frac{2}{(2n-1)\pi}sin(2n-1)\omega_{LO}t\right)$$

1) Assume no image signal (V_{pim}=0) and V_{ps}=1mV_{peak}. Calculate ***rms*** output signal power (S_o) and output noise power ($N_o/\Delta f$) density and noise factor F (15 pt).
(Note: K=1.38x10^{-23} J/K, T=300 K)

$$V_{op} = \frac{2}{\pi} g_m R_L V_p \rightarrow V_{op}^2 = \left(\frac{2}{\pi} g_m R_L V_{ps}\right)^2$$
$$\rightarrow \overline{V_{op}^2} = \left(\frac{2}{\pi} g_m R_L V_p\right)^2 \frac{1}{2}$$
$$\rightarrow S_o = \overline{V_{op}^2} \cdot \frac{1}{R_L} = \left(\frac{2}{\pi} g_m R_L V_p\right)^2 \frac{1}{2R_L}$$
$$= \left(\frac{2}{\pi} \cdot 34m \cdot 500 \cdot 1m\right)^2 \cdot \frac{1}{2 \cdot 500} = 117.13\,nW$$
$$= -39.31\,dBm$$

$$\overline{V_{no}^2} = 4KT\left(R_S \cdot g_m^2 + \frac{\gamma}{\alpha} \cdot g_m + \frac{2}{R_L}\right) R_L^2$$
$$= 4 \cdot 1.38\times10^{-23} \cdot 300 \left(50 \cdot (34m)^2 + \frac{1.5}{0.9} \cdot 34m + \frac{2}{500}\right) 500^2$$
$$= 490.45\times10^{-18}\, V_{rms}^2/Hz$$

2) Repeat 1) when W/L increased twice while maintaining same bias current (10 pt).

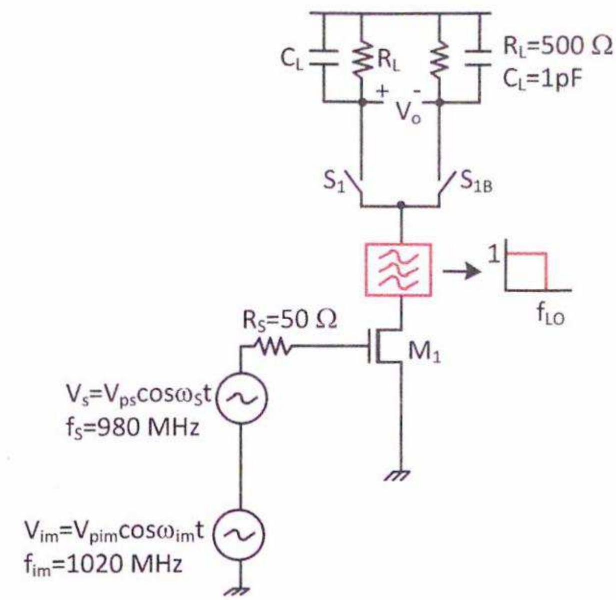
$$F = \frac{1}{\left(\frac{2}{\pi} \cdot g_m \cdot R_L\right)^2} \cdot \frac{\overline{V_{no}^2}}{\overline{V_{ni}^2}} = \frac{1}{\left(\frac{2}{\pi} \cdot g_m \cdot R_L\right)^2} \frac{4KT\left(R_S \cdot g_m^2 + \frac{\gamma}{\alpha} g_m + \frac{2}{R_L}\right) R_L^2}{4KTR_S}$$

$$= \left(\frac{\pi}{2}\right)^2 \left(1 + \frac{\gamma}{\alpha} \cdot \frac{1}{g_m R_S} + \frac{2}{g_m^2 R_L R_S}\right) = 5.057 \Rightarrow 7.04\,dB$$

$$W/L \uparrow \times 2 \rightarrow g_m \uparrow \times \sqrt{2}$$
$$\Rightarrow S_o \uparrow \times 2$$
$$\Rightarrow F_{new} = \left(\frac{\pi}{2}\right)^2 \left(1 + \frac{\gamma}{\alpha} \frac{1}{g_m R_S} \cdot \frac{1}{\sqrt{2}} + \frac{2}{g_m^2 \cdot R_L \cdot R_S} \times \frac{1}{2}\right) = 4.26$$
$$\Rightarrow 6.3\,dB$$

3) What's output signal power if the image tone has same magnitude as signal, i.e., V_{pim}=V_{ps}=1mV_{peak} (5 pt) ?

$$V_o = \frac{4}{\pi}\left(\sin W_{LO}t \cdot V_{ps}\cos W_s t + \sin W_{LO} t \cdot V_{pim} \cos W_{im} t\right)$$
$$= \frac{2}{\pi} V_{ps}\left(\sin(\underbrace{W_{LO} - W_s}_{= +W_{IF}})t + \sin(\underbrace{W_{LO} - W_{im}}_{= -W_{IF}})t\right)$$
$$= \frac{2}{\pi} V_{ps}\left(\sin W_{IF}t - \sin W_{IF}t\right)$$
$$= 0.$$



4) Now, in order to remove the image tone, let's apply ideal brick-wall type low pass filter (LPF) as shown above. Calculate output noise power and noise factor. Assume that the LPF is noiseless (5 pt).
(Note: the image rejection filer will also reject noise and help to improve SNR. In general, the M1 can be regarded as LNA and the switches (S1 and S1B) as mixer, respectively. You could understand the role of a filter between LNA and mixer through this example.)

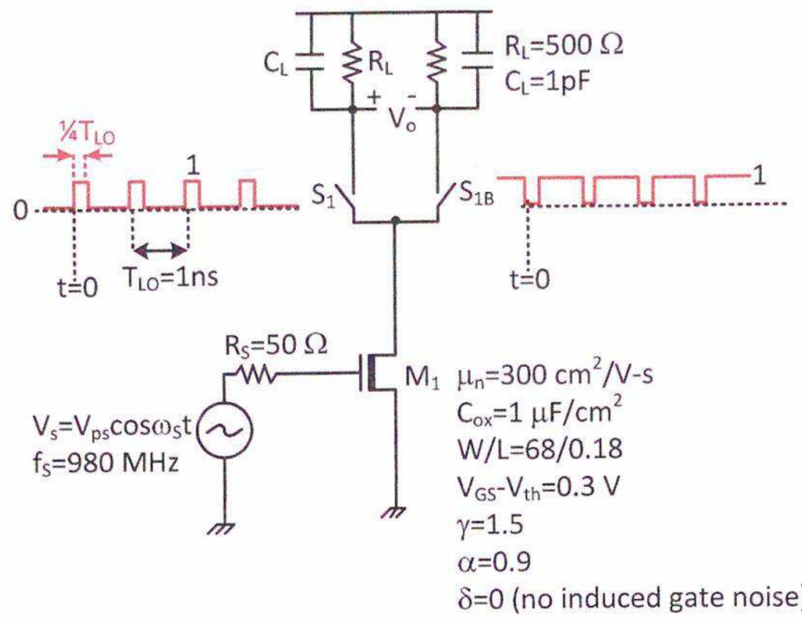
$$\overline{V_{no}^2} = \underbrace{\left(\frac{2}{\pi}\right)^2}_{\text{due to filtering}} \cdot 4KT\left(R_s \cdot g_m^2 + \frac{\gamma}{\alpha} g_m\right) R_L^2 + \underbrace{4KTR_L \times 2}_{\text{No-filtering of } R_L\text{-noise}}$$

$$= 208.62 \times 10^{-18}\ V_{rms}^2/Hz$$

$$F = \frac{1}{\left(\frac{2}{\pi} g_m R_L\right)^2} \cdot \frac{\overline{V_{no}^2}}{\overline{V_{ni}^2}}$$

$$= \frac{1}{\left(\frac{2}{\pi} \cdot 34m \times 500\right)^2} \cdot \frac{208.62 \times 10^{-18}}{828 \times 10^{-21}}$$

$$= 2.15 \Rightarrow 3.327\ dB \Rightarrow \text{about 3dB improvement.}$$



5) Now let's change the LO clock duty cycle from 50% to 25% as shown above, and all the other conditions are same as 1) and no image signal. Calculate ***rms*** output signal power (S_o) and output noise power (N_o) and noise factor F (15 pt).
(Note: non-50% duty cycle also generates RF leakage to IF output node due to DC component of LO waveforms.)

Fundamental tone

$50\% \rightarrow \frac{2}{\pi}$

$25\% \rightarrow \frac{2}{\pi} \sin\left(\pi \cdot \frac{1}{4}\right) = \frac{2}{\pi} \cdot \frac{1}{\sqrt{2}}$

$\therefore S_{o,new} = S_{o,old} \times \left(\sin\frac{\pi}{4}\right)^2 = 117.13\ nW \times \frac{1}{2}$
$= 58.565\ nW$

Noise power will be the same as in 50% duty-cycle

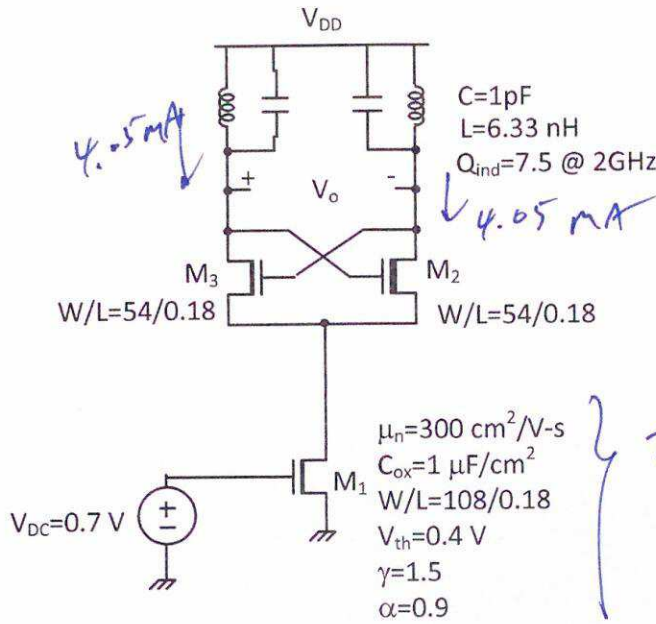
$\therefore F_{,new} = F_{old} \times \frac{1}{\left(\sin\frac{\pi}{4}\right)^2} = \frac{5.057}{1/2} = 10.114$

$\Rightarrow 10.049\ dB$

2. In the VCO shown below, DC characteristic of the NMOS, M1-3, is set by square-law characteristic,

$$I_{DS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2.$$

Assume that Cgs of M2-3 is negligible compared with C in the LC-tank. For simplicity, let's further assume that dominant noise sources are M1 and parasitic resistance from LC-tank, and noises from M2-3 are negligible. M1 has only drain thermal noise current, i.e., no gate induced noise and parasitic gate resistance.



$Q_{ind} = 7.5$

$\Rightarrow R_{ind} = Q_{ind} \cdot \omega_{osc} \cdot L$

$= 7.5 \times 2\pi \cdot 2G \cdot 6.33\,nH$

$= 596.59\,\Omega$

$I_{DC} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{th})^2$

$= \frac{1}{2} \cdot 300 \times 1\mu \times \frac{108}{0.18} \times 0.3^2$

$= 8.1\,mA$

$\Rightarrow g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{th})$

$= 54\,m\mho$

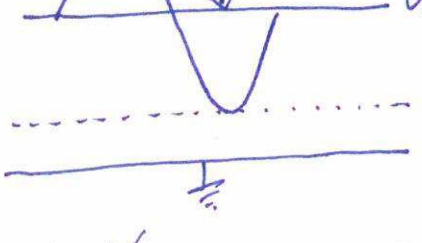
1) Calculate the possible maximum output signal swing in peak-to-peak value and set minimum supply voltage VDD that allows the maximum output swing (5 pt).
(Note: Each NMOS needs at least minimum $V_{ds,sat}$ during full output swing excursion to sustain oscillation.)

$(V_{ds,sat})_{M1} = V_{GS} - V_{th} = 0.7 - 0.4 = 0.3\,V$

$(V_{ds,sat})_{M2,M3}$ @ $I_{DS} = 8.1\,mA = (V_{ds,sat})_{M1} \times \sqrt{2}$

$= 0.424\,V$

because size of M2, M3 is half of M1.



$\frac{1}{2} I_{DC} \times R_{ind} \times \frac{2}{\pi} = \frac{1}{2} \times 8.1m \times 596.59\,\Omega \times \frac{2}{\pi}$

$V_{DD,min}$ $= 1.538\,V$

$(V_{ds,sat})_{M1} + (V_{ds,sat})_{M2,M3} = 0.724\,V$

$\therefore V_{DD,min} = 1.538 + 0.724 = 2.262\,V$

2) Under the bias condition determined in 1), calculate phase noise contributions from parasitic resistance from inductor at 10 MHz offset frequency (10 pt).

differential & no-correlation

$$(\overline{\Phi_n^2})_{\text{due to } R_{ind}} = \left(\frac{1}{2Q\frac{\Delta\omega}{\omega_0}}\right)^2 \times \frac{4KTR_{ind}\times 2}{\frac{1}{2}V_p^2\times 4}$$

differential & correlation

$$= \left(\frac{1}{2\times 7.5}\frac{2G}{10M}\right)^2 \times \frac{4\times 1.38\times 10^{-23}\times 300\times 596.6\times 2}{\frac{1}{2}\cdot(1.538)^2\times 4}$$

$$= 742.52\times 10^{-18}\ \text{rad}^2/\text{Hz}$$

$$= -151.29\ \text{dBc/Hz @ 10MHz offset}$$

3) Under the bias condition determined in 1), calculate phase noise contributions from M1 at 10 MHz offset frequency (10 pt)

(Note: You don't need to calculate phase noise contribution due to flicker noise of M1, since at 10MHz offset dominant noise source will be thermal noise of M1.)

$$(\overline{\Phi_n^2})_{\text{due to } M1} = \left(\frac{1}{2Q\frac{\Delta\omega}{\omega_0}}\right)^2 \times \frac{4KT\frac{\gamma}{\alpha}g_{m1}\cdot R_{ind}^2}{\frac{1}{2}V_p^2\times 4}$$

$$= \left(\frac{1}{2\times 7.5}\cdot\frac{2G}{10M}\right)^2 \times \frac{4\times 1.38\times 10^{-23}\times 300\times\frac{1.5}{0.9}\times 54m\times 596.59^2}{\frac{1}{2}(1.538)^2\times 4}$$

$$= 19.934\times 10^{-15}\ \text{rad}^2/\text{Hz}$$

$$= -137.\ \text{dBc/Hz @ 10MHz offset}$$

4) Under the bias condition determined in 1), calculate total phase noise at 10 MHz offset frequency (5 pt).

$$(\overline{\Phi_n^2})_{\text{total}} = 742.52\times 10^{-18} + 19.934\times 10^{-15}$$

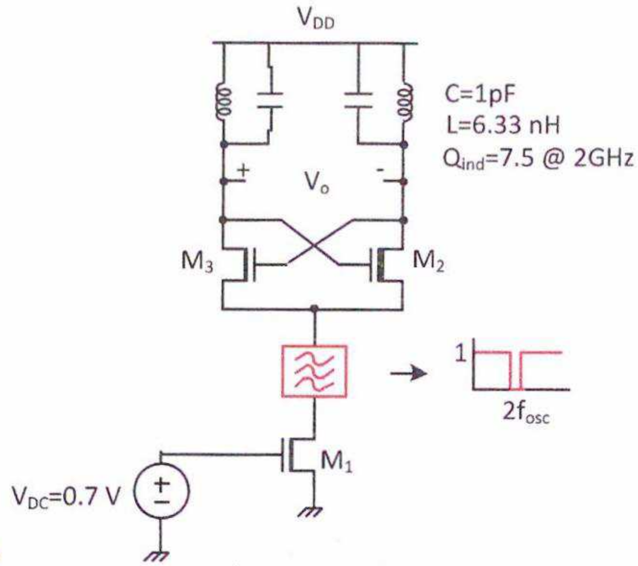
$$\approx 19.934\times 10^{-15}\ \text{rad}^2/\text{Hz}$$

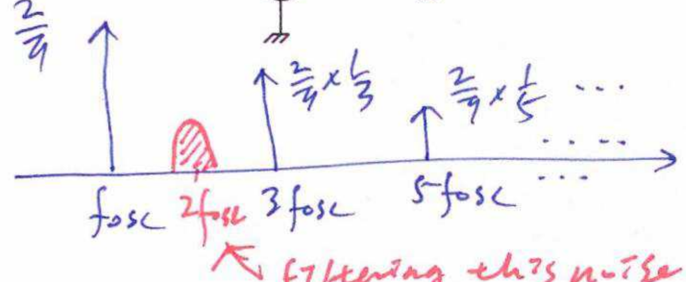
$$= -137\ \text{dBc/Hz @ 10MHz offset}$$

$$= -117\ \text{dBc/Hz @ 1MHz offset}$$

⇒ This is a typical number for NMOS VCO at GHz range.

5) Now, let's use ideal filter which will reject noise at $2xf_{osc}$ from M1 completely (f_{osc} is output oscillation frequency). Assume the filter is noiseless. Calculate phase noise contribution from M1 at 10MHz offset. How much improvement will it achieve compared with 3) through filtering the second harmonic noise? (20 pt).





$$\overline{i_{no}^2} = 4KT\frac{\gamma}{\alpha}g_{m1}\left(1-\left(\frac{2}{\pi}\right)^2\left(1^2+\frac{1}{3}^2\right)\right)$$

double side band noise per tone

details $\Rightarrow$ $\overline{i_{no}^2} = \underbrace{2}_{\text{2-sided spectrum}}\times \underbrace{2KT\frac{\gamma}{\alpha}g_m}_{\text{1-side noise current power}}\left(\frac{2}{\pi}\right)^2\left\{\underbrace{\sum_{n=1}^{\infty}\left(1+\frac{1}{3}^2+\frac{1}{5}^2+\frac{1}{7}^2+\cdots\right)\times 2}_{\text{power of fundamental \& harmonic tones}} - \underbrace{\left(1+\frac{1}{3}^2\right)}_{\text{rejection by filtering}}\right\}$

$$= 4KT\frac{\gamma}{\alpha}g_m - 4KT\frac{\gamma}{\alpha}g_m\left(\frac{2}{\pi}\right)^2\left(1+\frac{1}{3}^2\right)$$

$$\therefore \left(\overline{\Phi_n^2}\right)_{\text{due to } M1} = \left(\frac{1}{2Q\frac{\Delta\omega}{\omega_0}}\right)^2\frac{4KT\frac{\gamma}{\alpha}g_m R_L^2\left(1-\left(\frac{2}{\pi}\right)^2\left(1+\frac{1}{9}\right)\right)}{\frac{1}{2}V_p^2\times 4}$$

$$= 10.958\times 10^{-15}\ rad^2/Hz = -139.6\ dBc/Hz$$

$\Rightarrow$ 2.6 dB improvement.