The Effect of Phase-Shifter Errors on the Performance of an Antenna-Array Beamformer

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(Invited Paper)

Abstract—**The paper considers the random phase errors in the phase shifters which are used in an antenna array to steer the beam in the look direction, and analyzes the effect of these errors on the performance of the optimal processor which maximizes the output SNR by deriving the expressions for the output signal power, residual interference power, output SNR, and the array gain as a function of the variance of these errors. The paper also considers the phase quantization error which arises in the digital phase shifters and shows how the performance of the optimal processor depends on the number of bits of the digital phase shifters.**

I. INTRODUCTION

PHASE SHIFTERS are commonly used in phased array radars [1], [2] and adaptive arrays [3]–[7] to control the phases of narrow-band signals received by or to be transmitted by antenna elements of an array. The phase shifters are chosen such that the narrow-band signals received by all the elements from a given direction, referred to as the "look direction," are phase aligned [8]. In this situation, the array beam is pointed or steered in the look direction. For broad-band signals, the beam steering is achieved by replacing the phase shifters by pure time delays [5], [9], [10].

Commonly used phase shifters are ferrite phase shifters and diode phase shifters [1], [11]. One of the important specification of these phase shifters which concerns an array designer is the root mean-square (RMS) phase error. One would expect that the reduction in the RMS phase error below certain limits would result in an increase in the cost of the phase shifters. Thus an array designer would like to choose the maximum permissible RMS phase error to reduce the total cost of the system. Hence, it is important to know how these errors degrade the array performance.

The aim of this paper is to analyze the effect of these errors on the performance of the optimal processor, which maximizes the SNR in the absence of errors [12], [13]. The performance criteria under consideration are the signal suppression, the reduction in output SNR, and the array gain.

The effects of random errors on the performance of beamformers have been studied by many authors [14]–[23]. These errors include the error due to signal fluctuation [15], element failure [17], imperfect knowledge of the signal direction [21], errors in the amplitudes and the phases of the weights of beamformers [16], [19], [20], and errors in the steering vectors [18], [22], [23].

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The study carried out in [16]–[18] is concerned with the effect of errors on the beam pattern and sidelobe levels of a beamformer. The analysis carried out in [19] is for a conventional beamformer, and it derives the expression for the main-beam gain as a function of the variance of the amplitude and the phase errors of the complex weights. The analysis of [20] assumes that the variance of the real and the imaginary parts of the complex weights are equal and studies the effect of these errors on the array gain.

The effect of errors on the optimal processor is considered in [21]–[23]. The study in [21] is concerned with the effect of mismatch which arises when there is imperfect knowledge of the signal direction, and is closely related to the work carried out in [22] and [23] but with a different approach. The signal suppression due to the presence of the signal component in the array correlation matrix, used in calculating the weights of the optimal processor, has been demonstrated in both the cases. The analysis in [21] is carried out using the generalized sine and cosine of the angle between the steering vectors corresponding to the actual direction and the known direction of signal source, whereas the analysis in [22] and [23] is carried out by modeling the errors in the steering vectors as random additive components to the steering vectors.

The phase-shifter errors, assumed to be random, considered in this paper are additive to the exponents and have a multiplicative effect on the steering vectors. Thus the effect of these errors on the performance of the processor is different from those considered previously.

In the next section, a review of the concepts of signal representation and optimal processor is presented and the notation and the terminology used throughout the paper is introduced. In Section III the random phase errors are introduced and their effect on the optimal processor is analyzed by deriving the expressions for the output signal power, residual interference power, output SNR, and the array gain as a function of the variance of these errors.

A comparison between the effects of random phase errors and the effect of random steering-vector errors on the performance of the optimal processor is made in Section IV. In Section V, a special case of random phase error, namely, the phase quantization error, which arises in digital phase shifters is considered. Section VI concludes the paper.

II. PRELIMINARY CONSIDERATION

In this section the concepts of the signal representation and the optimal processor are discussed and notation used throughout the paper is introduced.

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A. Signal Representation

Consider an array of L ominidirectional elements immersed in the far field of a sinusoidal point source of frequency f_0. Assuming the center of the coordinate system as time reference, the signal on the lth element of the array can be expressed in the complex notation [24] as

$$x_l(t) = m(t)e^{j2\pi f_0(t+\tau_l(\phi,\theta))} + n_l(t) + x_{Il}(t) \tag{1}$$

where n_l is the random-noise component on the lth element and it is assumed to have the following statistics:

$$E[n_l(t)] = 0, \qquad l = 1, 2, \cdots, L \tag{2}$$

$$E[n_l(t)n_k(t)] = \begin{cases} \sigma_w^2 & \text{if } l = k, \\ 0 & \text{otherwise,} \end{cases} \qquad l, k = 1, 2, \cdots, L \tag{3}$$

$$\tau_l(\phi, \theta) = \frac{r_l \cdot \hat{V}(\phi, \theta)}{v}. \tag{4}$$

r_l is the position vector of the lth element, $\hat{V}(\phi, \theta)$ is a unit vector in the direction of source (ϕ, θ) as shown in Fig. 1, v represents the speed of propagation of the plane wavefront, dot $(\cdot)$ represents the inner product, and $m(t)$ is a complex lowpass process with

$$E[m(t)] = 0 \tag{5}$$

$$E[m(t)m^*(t)] = p_s \tag{6}$$

where p_s is the source power and * denotes the complex conjugate. $x_{Il}(t)$ in (1) represents the component of directional interference induced on the lth element.

Let an L dimensional vector $X(t)$ represent the L signal waveforms derived from L elements of the array, that is

$$X(t) \triangleq [x_1(t), x_2(t), \cdots, x_L(t)]^T \tag{7}$$

and an L dimensional vector W represents the weights of the beamformer shown in Fig. 2, that is

$$W \triangleq [w_1, w_2, \cdots, w_L]^T \tag{8}$$

where the superscript T denotes the transpose. Let α_l, $l = 1, 2, \cdots, L$ be the phase delays to steer the array in the look direction. The phase delays are chosen such that the L output waveforms of the phase-shifted elements due to a source in the look direction are identical. For example, the array is presteered in the direction of the source (ϕ_0, θ_0) by choosing

$$\alpha_l = 2\pi f_0 \tau_l(\phi_0, \theta_0), \qquad l = 1, 2, \cdots, L. \tag{9}$$

Let an $L \times L$ dimensional diagonal matrix Φ_0 be defined as

$$\Phi_{0ll} \triangleq \exp(j\alpha_l). \tag{10}$$

From Fig. 2 an expression for the output $y(t)$ is given by

$$y(t) = \sum_{l=1}^{L} w_l^* \exp(-j\alpha_l) x_l(t)$$

$$= W^H \Phi_0^H X(t) \tag{11}$$

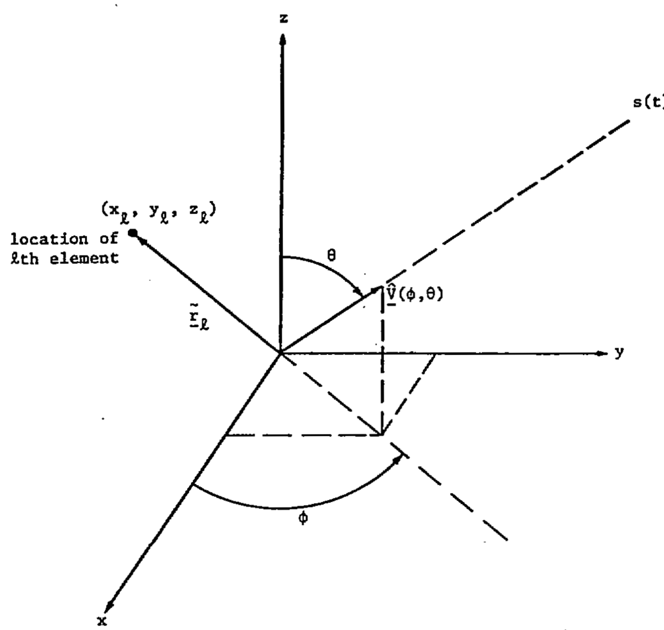

Fig. 1. Definition of coordinate system.

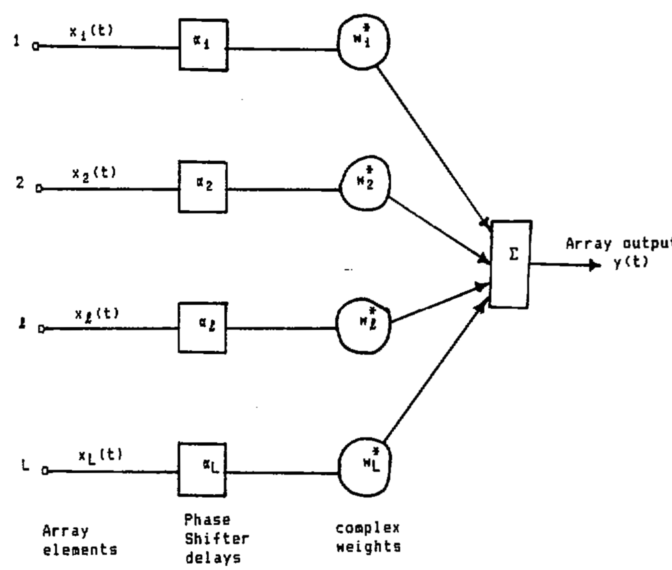

Fig. 2. Beamformer structure.

where the superscript H denotes the complex-conjugate transpose.

If the components of $X(t)$ can be modeled as zero-mean stationary processes, then for a given W the mean output power of the processor is given by

$$P(W) = E[y(t)y^*(t)]$$

$$= W^H \Phi_0^H R \Phi_0 W \tag{12}$$

where $E(\cdot)$ is the expectation operator and R is the array correlation matrix defined by

$$R \triangleq E[X(t)X^H(t)]. \tag{13}$$

From (1), (3), (6), (7), and (13) it follows that

$$R = p_s S_0 S_0^H + \sigma_w^2 I + R_I \tag{14}$$

where R_I is the array correlation matrix due to interferences only, I is an identity matrix, and S_0 is an L dimensional vector defined as

$$S_0 \triangleq [\exp\,(j2\pi f_0\tau_1(\phi_0,\theta_0)),\ \cdots,\ \exp\,(j2\pi f_0\tau_L(\phi_0,\ \theta_0))]^T \tag{15}$$

and is called steering vector [25], [26].

From (12) and (14) it follows that

$$P(W)=p_s W^H\Phi_0{}^H S_0 S_0{}^H \Phi_0 W + W^H W\sigma_w{}^2 + W^H\Phi_0 R_I \Phi_0 W. \tag{16}$$

B. Optimal Processor

Let $\hat{W}$ represent the weights of the processor shown in Fig. 2 when it maximizes the output SNR. These weights are referred to as the optimal weights, and the processor with these weights is referred to as the optimal processor. An expression for the optimal weights is given in [12] and [13]. For the processor with unit response in the look direction it becomes

$$\hat{W}=\frac{\Phi_0{}^H R_N{}^{-1} S_0}{S_0{}^H R_N{}^{-1} S_0} \tag{17}$$

where R_N is the array correlation matrix of interference and white noise only, that is

$$R_N \triangleq R_I + \sigma_w{}^2 I. \tag{18}$$

From (16) the total output power $\hat{P}$ of the optimal processor is

$$\hat{P}=p_s\hat{W}^H\Phi_0{}^H S_0 S_0{}^H\Phi_0\hat{W}+\hat{W}^H\hat{W}\sigma_w{}^2+\hat{W}^H\Phi_0{}^H R_I\Phi_0\hat{W}. \tag{19}$$

Substituting for $\hat{W}$, Φ_0, and S_0 in (19), one obtains after manipulation

$$\hat{P}=p_s+\sigma_w{}^2\beta+\frac{S_0{}^H R_N^{1} R_I R_N^{1} S_0}{(S_0{}^H R_N^{1} S_0)^2} \tag{20}$$

where

$$\beta \triangleq \frac{S_0{}^H R_N{}^{-1} R_N{}^{-1} S_0}{(S_0{}^H R_N{}^{-1} S_0)^2}. \tag{21}$$

One notes from (20) that the output signal power $\hat{P}_s$, output white noise power $\hat{P}_w$, and output interference power $\hat{P}_I$ of the optimal processor are given by

$$\hat{P}_s=p_s \tag{22}$$

$$\hat{P}_w=\sigma_w{}^2\beta \tag{23}$$

$$\hat{P}_I=\frac{S_0{}^H R_N{}^{-1} R_I R_N{}^{-1} S_0}{(S_0{}^H R_N{}^{-1} S_0)^2}. \tag{24}$$

Note from (23) that β is the ratio of the output white noise power of the optimal processor to the input white noise power, that is

$$\beta=\frac{\hat{P}_w}{\sigma_w{}^2}. \tag{25}$$

Let $\hat{P}_N$ denote the total output noise power of the optimal processor, that is

$$\hat{P}_N \triangleq \hat{P}_I+\hat{P}_w. \tag{26}$$

Substituting for $\hat{P}_I$ and $\hat{P}_w$ in (26), one obtains after manipulation

$$\hat{P}_N=\frac{1}{S_0{}^H R_N{}^{-1} S_0}. \tag{27}$$

III. RANDOM PHASE ERRORS

In this section the effect of random phase errors on the performance of the optimal processor is studied. The phase shifters with random phase errors are termed as "actual phase shifters" and the processor in this case is termed as "optimal processor with phase errors" (OPPE). It is assumed that random phase errors which exist in the phase shifters can be modeled as stationary processes of zero mean and equal variance and are uncorrelated from each other.

Let δ_l, $l = 1, 2, \cdots, L$ represent the phase error in the lth phase shifter. By assumption

$$E[\delta_l]=0, \qquad l=1,\ 2,\ \cdots,\ L \tag{28}$$

and

$$E[\delta_l\delta_k]=\begin{cases}\sigma^2 & \text{if } l=k,\\ 0 & \text{otherwise,}\end{cases} \qquad l,\ k=1,\ 2,\ \cdots,\ L. \tag{29}$$

Let $\tilde{\alpha}_l$, $l = 1, 2, \cdots, L$ represent the phase delays of the actual phase shifters. Then

$$\tilde{\alpha}_l=\alpha_l+\delta_l, \qquad l=1,\ 2,\ \cdots,\ L \tag{30}$$

where α_l, $l = 1, 2, \cdots, L$ are the phase delays of error-free phase shifters and are given by (9). Let a diagonal matrix Φ be defined as

$$\Phi_{ll} \triangleq \exp\,(j\tilde{\alpha}_l),\ l=1,\ 2,\ \cdots,\ L. \tag{31}$$

Substituting Φ for Φ_0 in (19) and taking expectation over random phase errors, one obtains an expression for the mean output power $\check{P}$ of the OPPE:

$$\check{P}=p_s\hat{W}^H E[\Phi^H S_0 S_0{}^H\Phi]\hat{W}+\hat{W}^H\hat{W}\sigma_w{}^2+\hat{W}^H E[\Phi^H R_I\Phi]\hat{W}. \tag{32}$$

It follows from (32) that the output signal power $\check{P}_s$, white noise power $\check{P}_w$, and the interference power $\check{P}_I$ of OPPE are given by

$$\check{P}_s=p_s\hat{W}^H E[\Phi^H S_0 S_0{}^H\Phi]\,\hat{W} \tag{33}$$

$$\check{P}_w=\sigma_w{}^2\|\hat{W}\| \equiv \sigma_w{}^2\beta \tag{34}$$

$$\check{P}_I=\hat{W}^H E[\Phi^H R_I\Phi]\,\hat{W} \tag{35}$$

where $\|\cdot\|$ denotes the Euclidian norm of a vector. Comparing

(23) and (34), one notes that the output white noise power is not affected by the random errors in the phase shifters. The effect of random phase errors on the output signal power and output interference power is now examined.

A. Signal Suppression

Rewrite (33) in the following form:

$$\check{P}_s = p_s \sum_{l,k}^{L} \hat{w}_l^* E[\Phi_{ll}^* S_{0l} S_{0k}^* \Phi_{kk}] \hat{w}_k. \tag{36}$$

Substituting for Φ and S_0 in (36), one obtains after rearrangement

$$\check{P}_s = p_s \sum_{l,k}^{L} \hat{w}_l^* \hat{w}_k E[\exp\ (-j(\delta_l - \delta_k)]$$

$$= p_s \sum_{\substack{l,k \\ l \neq k}}^{L} \hat{w}_l^* \hat{w}_k E[\exp\ -j(\delta_l - \delta_k)] + p_s \sum_{\substack{l,k \\ l = k}}^{L} \hat{w}_l^* \hat{w}_k \tag{37}$$

using

$$\exp\ (z) = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \cdots. \tag{38}$$

The first term in the right-hand side (RHS) of (37) becomes

$$p_s \sum_{l \neq k} \hat{w}_l^* \hat{w}_k E[\exp\ (j(\delta_k - \delta_l))]$$

$$= p_s \sum_{l \neq k} \hat{w}_l^* \hat{w}_k E\left[1 + j(\delta_k - \delta_l) - \frac{(\delta_k - \delta_l)^2}{2!} - \frac{j(\delta_k - \delta_l)^3}{3!} + \cdots\right]. \tag{39}$$

Assuming that the contribution of the higher-order terms is negligibly small, one obtains from (28), (29), and (39)

$$p_s \sum_{l \neq k} \hat{w}_l^* \hat{w}_k E[\exp\ (j(\delta_k - \delta_l))]$$

$$= p_s \sum_{l \neq k} \hat{w}_l^* \hat{w}_k (1 - \sigma^2)$$

$$= p_s (1 - \sigma^2) \sum_{l,k} \hat{w}_l^* \hat{w}_k - p_s (1 - \sigma^2) \sum_{l=k} \hat{w}_l^* \hat{w}_k$$

$$= p_s (1 - \sigma^2) \hat{W}^H C C^T \hat{W} - p_s (1 - \sigma^2) \| \hat{W} \| \tag{40}$$

where C is an L dimensional vector of 1's, that is,

$$C^T = [1,\ \cdots,\ 1]. \tag{41}$$

Noting that the second term in the RHS of (37) is $p_s \| \hat{W} \|$, one obtains from (17), (21), (37), and (40)

$$\check{P}_s = p_s - p_s \sigma^2 (1 - \beta). \tag{42}$$

Note that in the absence of directional interferences $\beta \equiv \| \hat{W} \| = 1/L$ and (42) becomes

$$\check{P}_s = p_s - p_s \frac{L-1}{L} \sigma^2. \tag{43}$$

Thus the output signal power of OPPE is suppressed. The suppression of the output signal power is proportional to the input signal power and the variance of the random errors. In the presence of directional interference, β increases and thus the reduction in the signal power is less than otherwise. In other words, the signal suppression is maximum in the absence of directional interference and is given by the second term in the RHS of (43).

B. Residual Interference Power

Rewrite (35) in the following form:

$$\check{P}_I = \sum_{l,k}^{L} \hat{w}_l^* E[\Phi_{ll}^* R_{Ilk} \Phi_{kk}] \hat{w}_k. \tag{44}$$

Using (10), (30), and (31) in (44), one obtains

$$\check{P}_I = \sum_{l,k}^{L} \hat{w}_l^* \Phi_{0ll}^* R_{Ilk} \Phi_{0kk} \hat{w}_k E[\exp\ (j(\delta_k - \delta_l))]$$

$$= \sum_{l=k}^{L} \hat{w}_l^* \Phi_{0ll}^* R_{Ilk} \Phi_{0kk} \hat{w}_k E[\exp\ (j(\delta_k - \delta_l))]$$

$$+ \sum_{l \neq k} \hat{w}_l^* \Phi_{0ll}^* R_{Ilk} \Phi_{0kk} \hat{w}_k E[\exp\ j(\delta_k - \delta_l)]. \tag{45}$$

Noting that the diagonal entries of R_I are the sum of total directional interferences power p_I, the first term in the RHS of (45) reduces to $p_I \| \hat{W} \|$. Following the steps (38)–(40), the second term in the RHS of (45) becomes $(1 - \sigma^2)[\hat{W}^H \Phi_0^H R_I \Phi_0 \hat{W} - p_I \| \hat{W} \|]$. Thus

$$\check{P}_I = (1 - \sigma^2) \hat{W}^H \Phi_0^H R_I \Phi_0 \hat{W} + \sigma^2 p_I \| \hat{W} \|. \tag{46}$$

Substituting for $\hat{W}$ in (46) it follows that

$$\check{P}_I = \hat{P}_I + \sigma^2 (\beta p_I - \hat{P}_I) \tag{47}$$

where p_I is the total power of the directional interferences at the input of the processor and $\hat{P}_I$ is the residual interference power of the optimal processor.

C. Array Gain

In this section the effect of random phase errors on the array gain of OPPE is examined. The array gain is defined to be the ratio of the output SNR to the input SNR, that is,

$$\text{Array Gain} \triangleq \frac{\text{output SNR}}{\text{input SNR}}. \tag{48}$$

Let $\check{\text{SNR}}_0$ be the output SNR of OPPE. Thus

$$\check{\text{SNR}}_0 = \frac{\check{P}_s}{\check{P}_N} \tag{49}$$

where

$$\check{P}_N = \check{P}_I + \check{P}_w \tag{50}$$

is the total output noise power of OPPE.

Substituting from (34) and (47) in (50) and using (21), (23), (24), and (26), one obtains after manipulation

$$\check{P}_N = \hat{P}_N[1+\sigma^2(\beta\hat{G}-1)] \tag{51}$$

where

$$\hat{G} \triangleq \frac{p_N}{\hat{P}_N} > 1 \tag{52}$$

is the array gain of the optimal processor.

From (42), (49), and (51) it follows that

$$\check{\text{SNR}}_0 = \frac{p_s}{\hat{P}_N}\left[\frac{1+\sigma^2(\beta-1)}{1+\sigma^2(\hat{G}\beta-1)}\right]. \tag{53}$$

If $\check{G}$ denote the array gain of OPPE, then it follows from (47), (52), and (53) that

$$\check{G} = \hat{G}\,\frac{1+\sigma^2(\beta-1)}{1+\sigma^2(\hat{G}\beta-1)}. \tag{54}$$

Let

$$\check{G}_1 \triangleq \check{G}|_{\sigma=\sigma_1} \tag{55}$$

and

$$\check{G}_2 \triangleq \check{G}|_{\sigma=\sigma_2}. \tag{56}$$

A simple algebraic manipulation using (54)–(56) shows that for

$$\sigma_2 > \sigma_1 \tag{57}$$

$$\check{G}_2 < \check{G}_1. \tag{58}$$

Thus the array gain of the optimal processor with random phase errors is a monotonically decreasing function of the variance of the error. In the next section the results presented in this section are compared with the result on the steering-vector error [23].

IV. COMPARISION WITH THE STEERING-VECTOR ERROR RESULTS

In this section a comparision between the effect of the random phase-shifter errors and the effect of the random steering-vector errors on the performance of the optimal processor is made. The result on the steering-vector errors is derived in [23]. The steering-vector errors are modeled [22], [23] as additive components to the steering vector in the look direction, that is

$$\tilde{S} = S_0 + \Gamma \tag{59}$$

where $\tilde{S}$ is the estimated steering vector which is used in computing the optimal weights and Γ is the vector of random errors. It is assumed that

$$E(\Gamma_i) = 0, \qquad i = 1, 2, \cdots, L \tag{60}$$

and

$$E[\Gamma_i^*\Gamma_j] = \begin{cases} \sigma_s^2 & \text{if } i=j \\ 0 & \text{otherwise} \end{cases}. \tag{61}$$

Table I compares both the results. For the purpose of the comparison, the results in both the cases are presented relative to the corresponding error-free values and thus are referred to as normalized. One observes the following from the comparison.

1) The output signal power decreases with the increase in the variance of the phase-shifter errors if $\beta < 1$, whereas in the case of the steering vector errors it is a monotonically increasing function of the variance of the errors. Note that for white noise only $\beta = 1/L$.

2) The output white noise power is not affected with the phase-shifter errors.

3) The total output noise is a monotonically increasing function of the variance of the steering-vector errors, whereas the total output noise power decreases with the increase in the variance of phase-shifter error if $\beta\hat{G} < 1$.

4) The array gains in both the cases are monotonically decreasing functions of the variance of random errors.

V. PHASE QUANTIZATION ERROR

In this section a special case of random error, namely, the phase quantization error, which arises in digital phase shifters is considered. In a p-bit phase shifter the minimum value of the phase which can be changed is $2\pi/2^p$. Thus it is assumed that the error which exists in a p-bit digital phase shifter is uniformly distributed between

$$-\frac{\pi}{2^p}$$

and

$$\frac{\pi}{2^p}.$$

For an uniformly distributed random variable x in the interval $(-C, C)$, it can easily be verified that

$$E[x^2] = \frac{C^2}{3}. \tag{62}$$

Substituting for

$$C = \frac{\pi}{2^p}$$

one obtains an expression for the variance σ_p^2 of the error in a p-bit phase shifter, which is given by

$$\sigma_p^2 = \frac{\pi^2}{3.2^{2p}}. \tag{63}$$

Substituting σ_p for σ in (42), (51), (53), and (54), one obtains the following expressions for the output signal power, output noise power, output SNR, and the array gain as a function of the variance of the phase quantization error:

$$\check{P}_s = p_s[1-\sigma_p^2(1-\beta)] \tag{64}$$

$$\check{P}_n = \hat{P}_n[1+\sigma_p^2(\beta\hat{G}-1)] \tag{65}$$

TABLE I

Type of error	Phase-shifter error	Steering-vector error
Normalized output signal power	$(1+\sigma^2\beta-\sigma^2)$	$(1+\sigma_s{}^2\beta)$
Normalized output white noise power	1	$1+\sigma_s{}^2\hat{P}_N{}^2 \operatorname{tr}(R_N{}^{-1}R_N{}^{-1})1/\beta$
Normalized total output noise power	$(1+\sigma^2\beta\hat{G}-\sigma^2)$	$[1+\sigma_s{}^2\hat{P}_N \operatorname{tr}(R_N{}^{-1})]$
Normalized array gain	$\left[\dfrac{1+\sigma^2\beta-\sigma^2}{1+\sigma^2\beta\hat{G}-\sigma^2}\right]$	$\left[\dfrac{1+\sigma_s{}^2\beta}{1+\sigma_s{}^2\hat{P}_N \operatorname{tr}(R_N{}^{-1})}\right]$

$$\check{\text{SNR}}_0=\left[\frac{1-\sigma_p{}^2(1-\beta)}{1+\sigma_p{}^2(\hat{G}\beta-1)}\right]\frac{p_s}{\hat{P}_N} \tag{66}$$

and

$$\check{G}=\hat{G}\,\frac{1+\sigma_p{}^2(\beta-1)}{1+\sigma_p{}^2(\hat{G}\beta-1)}\,. \tag{67}$$

VI. CONCLUSION

The paper has considered the random phase errors in the phase shifters which are used in an antenna array to steer the beam in the look direction, and has analyzed the effect of these errors on the performance of the optimal processor which maximizes the output SNR. The paper has derived the expressions for the output signal power, residual interference power, output SNR, and the array gain as a function of the variance of the random phase errors. The paper has shown that the output signal power of the processor gets suppressed in the presence of random phase errors and the suppression is proportional to the product of the input signal power and the variance of the random error. It is shown that the suppression is maximum in the absence of the directional interferences. The paper has shown that output white noise power is not affected by random phase errors and the array gain of the processor is a monotonically decreasing function of the variance of the random phase errors. The paper has also considered the phase quantization error which arises in the digital phase shifters and shown how the performance of the optimal processor depends on the number of the bits of the digital phase shifter.

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