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Noise Analysis of Bipolar Harmonic Mixer

Hiroshi Tanimoto and Ryuta Ito
Department of Electrical and Electronic Engineering
Kitami Institute of Technology
165, Koen-cho, Kitami 090-8507, JAPAN

***Abstract*—A noise analysis of bipolar harmonic mixer (BHMIX) for use with direct conversion receiver mixers is presented. The optimal conditions (LO amplitude, total collector current) of using BHMIX are given. This analysis considers thermal noise from base resistance and shot noise from collector current. These noises are treated as amplitude modulated white noises, which are characterized as cyclostationary noise. Assuming that input RF signal is very small, the BHMIX circuit is modeled as LO driven linear time varying circuit to calculate dependence of output noise on the LO amplitude. The analysis agreed well with measured data.**

I. Introduction

Even harmonic type of mixers (EHMIX) have been drawing much attention, because it is free from self mixing, in principle, when it is used in a direct conversion receiver. A bipolar harmonic mixer (BHMIX) is a kind of EHMIX, which uses a bipolar differential pair as a harmonic mixer core[1].

We have presented BHMIX's nonlinear analysis for conversion gain, third and second order input intercept points (IIP3 and IIP2) due to offset of bipolar differential pair[2]. However, there is very few report on its noise analysis. We have found only a reference reporting a noise analysis of the BHMIX under some simplifying assumptions[3]. This paper succeeded qualitatively to explain signal-to-noise ratio (SNR) dependence on local oscillator (LO) signal drive; however, the analysis does not take folding effect into account for collector shot noises.

In this paper, we present a complete noise analysis of BHMIX considering thermal noise of base resistances and collector shot noise with an assumption of static hypertangent input/output transfer characteristic for the bipolar differential pair.

II. Noise Analysis of BHMIX

Figure 1 shows a circuit diagram of the BHMIX in its simplest form. Usually, either LO signal or RF signal is fed in a balanced signal by using two identical differential pairs, in order to avoid even-harmonic intermodulation distortions due to finite common-mode signal rejection[1]. However, we analyze the circuit of Fig. 1 for simplicity, assuming perfect balance of the differential pair.

In the noise analysis of a mixer, we have to consider time varying nature of the output noises, unlike linear circuits as LNA etc., because the mixer is a nonlinear time-varying circuit. This means that the output noises are nonstationary, and we cannot directly make use of familiar linear noise analysis techniques. However, the noise sources are very small compared to the periodic LO drive signal, we can model the mixer as a linear periodically time varying (LPTV) circuit[4]. In addition, both thermal noise and shot noise outputs are modeled by periodically modulated stationary noises, i. e, cyclostationary noises[5].

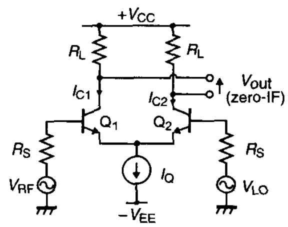

Fig. 1. Bipolar harmonic mixer circuit (principle).

The output signal V_{out} in Fig. 1 contains several major noise contributions; i. e, thermal noise from base resistances, r_{b}, and collector shot noises of Q_1 and Q_2. Thermal noises of the load resistors R_{L}, base shot noises of Q_1 and Q_2, and the noise from the tail current I_{Q} are ignored for simplicity.

The equivalent input thermal noise voltage of r_{b} at the base terminal in Fig. 1 is constant; however, corresponding output noise can be modeled as a modulated version of white noise, because the gain of the differential amplifier periodically changes with the LO drive signal.

The collector shot noise produced by a constant bias current is a stationary white noise. For the BHMIX case, in contrast, the collector bias current is not constant but varies with the LO drive signal. Assuming that the noise generating mechanisms are very much faster than the LO drive signal frequency, we can model the output noise by an amplitude modulated white noise[5]. Hence, the collector shot noise may also be modeled by an amplitude modulated stationary white noise with time varying envelope in our case.

As the output noises are small, we can calculate the noise outputs from small-signal equivalent circuit shown in Fig. 2 as usual; however, the small signal quantities in Fig. 2 are not constant, because Q_1 and Q_2 are nonlinear devices and their operating points are periodically changing with the differential input voltage:

$$\begin{cases} I_{\text{C1}} = \dfrac{\alpha_{\text{F}} I_{\text{Q}}}{2}\left(1 + \tanh\dfrac{V_{\text{RF}} - V_{\text{LO}}}{2V_{\text{T}}}\right) \\ I_{\text{C2}} = \dfrac{\alpha_{\text{F}} I_{\text{Q}}}{2}\left(1 - \tanh\dfrac{V_{\text{RF}} - V_{\text{LO}}}{2V_{\text{T}}}\right) \end{cases} \tag{1}$$

where, $\alpha_{\text{F}} \approx 1$ is a forward current gain of a common base transistor, $V_{\text{T}} = kT/q$ is the thermal voltage with R_{S} neglected. Using those collector current values with $V_{\text{RF}} = 0$, transconductances and input impedances are obtained by

$$g_{mi} = I_{\text{C}i}/V_{\text{T}}, \quad r_{\pi i} = \beta_{\text{F}}/g_{\text{m}i}, \quad (i = 1, 2), \tag{2}$$

where, $\beta_{\text{F}} \gg 1$ is a forward current gain of a common emitter transistor.

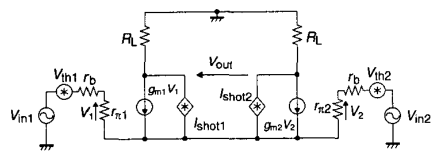

Fig. 2. Small-signal equivalent circuit for BHMIX.

A. Noise Spectrum of Amplitude Modulated White Noise

An amplitude modulated white noise can be modeled by the following expressions for time and frequency domain:

$$i_{\text{noise}}(t) = a(t) \times w(t) \overset{\mathcal{F}}{\Leftrightarrow} I_{\text{noise}}(\omega) = A(\omega) * W(\omega), \qquad (3)$$

where $a(t)$ is a modulating function (envelope), $w(t)$ denotes a formal white noise in time domain, and the upper case functions are Fourier transforms of corresponding lower case time functions, and $*$ denotes convolution. Since the envelope $a(t)$ is a positive T-periodic function, where $T = 2\pi/\omega_{\text{LO}}$, it can be represented as a Fourier series:

$$a(t) = \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_{\text{LO}}t} \overset{\mathcal{F}}{\Leftrightarrow} A(\omega) = \sum_{n=-\infty}^{+\infty} c_n \delta(\omega - n\omega_{\text{LO}}), \qquad (4)$$

where, ω_{LO} is an angular frequency of the LO signal, and

$$c_n = \frac{1}{T}\int_{-T/2}^{+T/2} a(t)\, e^{-jn\omega_{\text{LO}}t} dt. \qquad (5)$$

Thus, we have

$$I_{\text{noise}}(\omega) = \sum_{n=-\infty}^{+\infty} c_n \delta(\omega - n\omega_{\text{LO}}) * W(\omega) = \sum_{n=-\infty}^{+\infty} c_n W(n\omega_{\text{LO}}). \quad (6)$$

This indicates that the resulting noise spectrum is white again, as $W(\omega) = 1$ is constant over an entire frequency range. Using the Parseval's theorem, the noise power is given by

$$\int_{-\infty}^{+\infty} |i_{\text{noise}}(t)|^2\, dt = \int_{-\infty}^{+\infty} |I_{\text{noise}}(\omega)|^2\, d\omega = \sum_{n=-\infty}^{+\infty} |c_n|^2. \qquad (7)$$

Assuming that the shot noise and thermal noise are very small compared to the LO signal, and have no correlation with each other, we can calculate the total output noise simply adding them by rms way. Before doing this, we must calculate their envelopes next.

B. Output Shot Noise Envelope

As I_{shot} changes with instantaneous LO signal drive, $x(t)$, we calculate I_{shot} as a function of x and I_{Q}.

In Fig. 2, $I_{\text{shot}i} = \sqrt{2qI_{Ci}}$, $(i = 1, 2)$ are collector shot noise sources. The output noise component of shot noise is obtained by solving the nodal equation for Fig. 2 with $V_{\text{in1}} = x$ and $V_{\text{in2}} = 0$:

$$\overline{I^2_{\text{shot}}} = \frac{qI_{\text{Q}}\,\text{sech}^2\frac{x}{2V_{\text{T}}}\left(1 + \cosh\frac{x}{V_{\text{T}}} + 4\beta_{\text{F}} + 2\beta_{\text{F}}^2\right)}{(1+\beta_{\text{F}})^2} \qquad (8)$$

$$\approx 2qI_{\text{Q}}\,\text{sech}^2\frac{x}{2V_{\text{T}}} \quad (\beta_{\text{F}} \gg 1) \qquad (9)$$

where, q is a charge of an electron, $V_{\text{T}} = kT/q$ is the thermal voltage, k is Boltzmann constant, T is an absolute temperature.

Equation (8) indicates that the output shot noise amplitude is an even function of x.

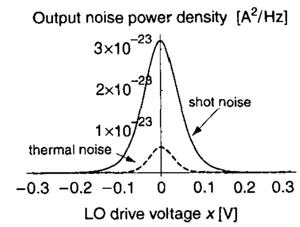

Fig. 3. Output noise power density vs. LO drive voltage. Calculated for I_{Q} = 0.1 [mA], and r_{b} = 50 [Ω], V_{LO} = 50 [mV].

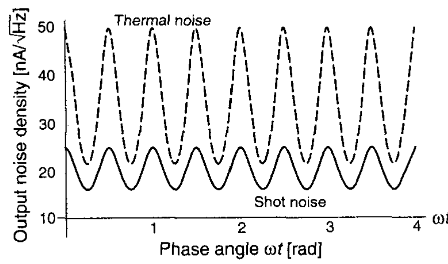

Fig. 4. Envelopes of modulated shot noise and thermal noise. Calculated for I_{Q} = 2 [mA], and r_{b} = 50 [Ω], V_{LO} = 50 [mV].

C. Output Thermal Noise Envelope

We consider thermal noise of only base resistance r_{b} for simplicity. Putting equivalent thermal noise generators V_{th1} and V_{th2} in series with base terminals, then neglect r_{b}. This is because $r_{\text{b}} \ll r_\pi$ holds in most cases. V_{th1} and V_{th2} have rms voltage of $\sqrt{4kTr_{\text{b}}}$ each at the input. In order to compare contribution of thermal noise with shot noise, thermal noise needs to be referred to output current I_{thermal}.

Small signal transconductance g_{m} at operating point V_{in1} is given by

$$g_m \approx \frac{d}{dx} I_{\text{Q}} \tanh\left(\frac{x}{2V_{\text{T}}}\right) = \frac{I_{\text{Q}}}{2V_{\text{T}}}\,\text{sech}^2\left(\frac{x}{2V_{\text{T}}}\right). \qquad (10)$$

Here, α_{F} is assumed to be unity and is dropped. Now, the output rms thermal noise is given by:

$$\overline{I^2_{\text{thermal}}} = 8kTr_{\text{b}} \times {g_m}^2 = \frac{2qr_{\text{b}}}{V_{\text{T}}} I_{\text{Q}}^2 \sec^4\left(\frac{x}{2V_{\text{T}}}\right). \qquad (11)$$

This also is an even function of x.

Comparing (11) with (8), we notice that while shot noise $\overline{I^2_{\text{shot}}}$ is proportional to I_{Q}, thermal noise $\overline{I^2_{\text{thermal}}}$ is proportional to I_{Q}^2. This indicates that the thermal noise dominates over the shot noise as I_{Q} becomes larger.

Figure 3 shows an example of the relation between output noise power density and static LO drive voltage x.

D. Output Noise as a Function of LO Amplitude

Figure 4 shows envelope examples for the shot noise and the thermal noise at the output terminal, where sinusoidal LO signal,

$$x(t) = V_{\text{LO}} \sin \omega_{\text{LO}} t, \qquad (12)$$

is applied to the BHMIX.

Fourier coefficients c_n are calculated for (8) and (11), by numerically integrating (5). Then, $|c_n|^2$ is accumulated until relative error of the sum becomes less than 10^{-5}. For example,

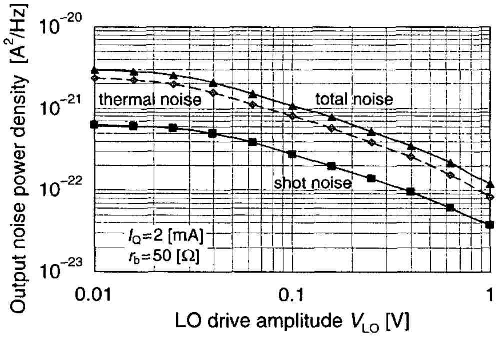

Fig. 5. Output noise power densities versus LO drive amplitude. Calculated for $I_Q = 2$ [mA], and $r_b = 50$ [Ω].

the sum was taken up to $n = 50$ for $V_{LO} = 1$ [V]. Smaller V_{LO} values need much less terms. Here, $\beta_F = 100$ was assumed for shot noise calculation. Note that $\sum |c_n|^2$ values scale with I_Q, r_b and β_F, so that we need not to recalculate them for various bias conditions.

Reference [3] calculated only $|c_0|$ for shot noise component, while $\sum |c_n|^2$ was calculated for thermal noise of r_b. Thus it may under estimate the total noise power.

Figure 5 shows an example of the result for a realistic condition; i. e, I_Q=2 [mA] and $r_b = 50$ [Ω]. Thermal noise component is dominant over shot noise component in the total noise power in this condition.

III. Equivalent Input Noise and Noise Figure

Equivalent input noise V_{neq} can be calculated by converting the total output noise back into the input terminal voltage. This requires a knowledge of conversion gain dependence on LO signal amplitude for the BHMIX. We can calculate the small signal conversion gain by the procedure given in Appendix. See [2] for detail. Resulting normalized conversion gain, η, is plotted in Fig. 8 to the normalized LO signal amplitude.

Because the normalized conversion gain, η, is defined by

$$\eta \equiv \frac{I_{\mathrm{diff}}}{\alpha_F I_Q} \Big/ \frac{V_{RF}}{2V_T}, \tag{13}$$

where V_{RF} is a RF signal amplitude, V_{neq} can be calculated by

$$V_{neq} = \frac{1}{\eta} \frac{2V_T}{\alpha_F I_Q} I_{ntotal}, \tag{14}$$

where, I_{ntotal} stands for the total output noise current value (rms).

Once the equivalent input noise V_{neq} is calculated, this noise source resides in series with base terminal at the input of the BHMIX; thus, V_{neq} can be directly compared with a thermal noise voltage of the signal source impedance R_S. Thus, noise figure, NF, can readily be calculated by the following equation:

$$\mathrm{NF} = 10 \log\left(1 + \frac{V_{neq}^2}{4kTR_S}\right). \tag{15}$$

Figure 6 shows some calculated NF curves for various I_Q values. This clearly indicates that the minimum NF occurs around $V_{LO} = 0.15$ [V]. These were calculated for $r_b = 50$ [Ω]; however, shot noise component dominates for $I_Q = 100$ [μA], and thermal noise component dominates for $I_Q > 1$ [mA] cases. Therefore, LO amplitude of $V_{LO} = 0.15$ [V] is concluded to be the optimal operating point from NF point of view.

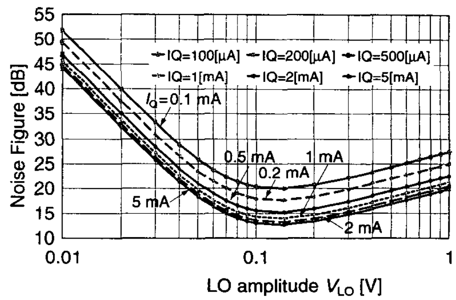

Fig. 6. Calculated relation between noise figure and LO amplitude for various I_Q ($r_b = 50$ [Ω]).

From Fig. 6, we see that NF improvement saturates with I_Q. So, practical limit of I_Q may be around several milliamperes, considering power dissipation.

IV. Comparison with Measured Data

Calculated noise by using the proposed method is compared with a measured data. The measured data are taken from [6]. The measured circuit consists of two identical BHMIX's to form a single balanced mixer. The RF input is divided in two and down converted by two identical BHMIX's and the outputs are combined into one at the output. Thus, the NF value must be the same for a single BHMIX.

The BHMIX chip was fabricated by using a bipolar process with $f_T = 20$ [GHz], and its input impedance for LO port is 25 [Ω] (single ended), and RF port impedance is 50 [Ω] (balanced). As the mixer was designed for use with PHS system, the RF frequency is about 2 [GHz] so that the LO frequency is 1 [GHz]. Its supply current was 5 [mA] including bias circuitry[6]. Estimated parameter values from the description, $I_Q = 2$ [mA] and $r_b = 50$ [Ω], were used for theoretical calculations in this section.

The calculated equivalent input noise voltage by our method is overlaid on the measured data in Fig. 7. Also, calculated V_{neq} by the method of [3] is plotted on it. Measured and calculated results fit fairly well, but our method fits better. As it is predicted, method in [3] gives 2 to 4 [dB] lower estimate due to excluding the folding effect in shot noise calculation for large LO amplitude. Both methods match the measured result very well for low LO signal level region. This is because the thermal noise dominates in this region for $I_Q = 2$ [mA].

Our proposed analysis method agreed very well with the measured data at a very high frequency, 2 [GHz], even though it assumes only static nonlinearity.

V. Conclusion

A noise analysis method is proposed for bipolar harmonic mixer for use with direct conversion receivers. Thermal noise and shot noise are modeled as amplitude modulated white noises, which are treated as cyclostationary noises. The analysis can include all the folding noises from high frequencies. The total output noise can be calculated by weighting and summing two universal output noise curves, once tail current I_Q, base resistance r_b and β_F are given. The measured noise and

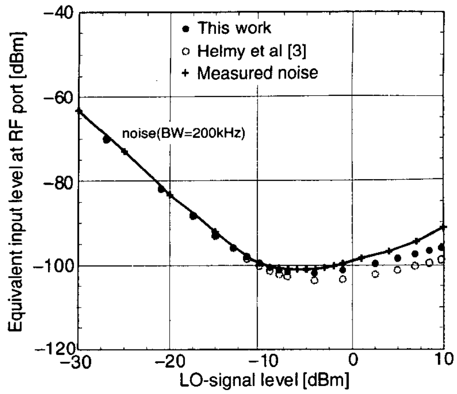

Fig. 7. Comparison of measured noise data[1] and analysis results. Calculated for $I_Q = 2$ [mA], and $r_b = 50$ [Ω].

calculated noise agrees very well even though the method assumes only static nonlinearity. The optimal LO signal drive amplitude is determined to be about $V_{LO} = 0.15$ [V]. It has been clarified that the thermal noise of base resistance dominates over shot noise for typical operation conditions.

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Appendix

Analysis of the conversion gain for BHMIX is explained here. The input/output relation of the differential pair can be described by the following equation:

$$I_{\rm diff} \equiv I_{\rm C1} - I_{\rm C2} = \alpha_{\rm F} I_{\rm Q} \tanh\left(\frac{V_{\rm diff}}{2V_{\rm T}}\right), \quad (16)$$

where,

$$V_{\rm diff} = V_{\rm in1} - V_{\rm in2}. \quad (17)$$

To simplify (16), introduce normalized variables:

$$y \equiv I_{\rm diff}/(\alpha_{\rm F} I_{\rm Q}), \quad x \equiv V_{\rm diff}/(2V_{\rm T}) \quad (18)$$

Now (16) becomes

$$y = \tanh x. \quad (19)$$

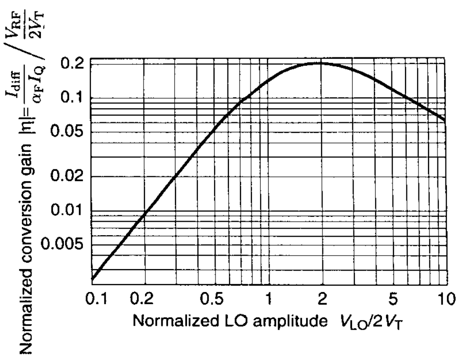

Fig. 8. Normalized conversion gain for BHMIX.

When there is no RF input signal, only an LO signal

$$x(t) = V_{\rm LO} \cos \omega_{\rm LO} t \quad (20)$$

is input, an output y_0 is

$$y_0 = \tanh(V_{\rm LO} \cos \omega_{\rm LO} t). \quad (21)$$

When a small RF signal Δx is present beside the LO signal,

$$\begin{aligned} y &= y_0 + \Delta y & (22)\\ &= \tanh(V_{\rm LO} \cos \omega_{\rm LO} t + \Delta x) & (23)\\ &\approx y_0 + \frac{\partial y_0}{\partial x}\Delta x. & (24) \end{aligned}$$

$$\therefore \frac{\Delta y}{\Delta x} \approx \frac{\partial y_0}{\partial x} = \frac{1}{\cosh^2(V_{\rm LO} \cos \omega_{\rm LO} t)} \quad (25)$$

This is an even function of LO signal, so that it is a periodic function with a fundamental frequency of $2\omega_{\rm LO}$.

As we are considering the BHMIX as a direct conversion down converter, the Fourier coefficient $|c_2|$ for (25) becomes the conversion gain, η. Figure 8 shows a conversion gain versus LO amplitude. From Fig. 8, the largest conversion gain occurs around $V_{\rm LO}/(2V_{\rm T}) \approx 2$, i. e, $V_{\rm LO} \approx 100$ [mV].