Single-view Metrology and Camera Calibration

Computer Vision

Jia-Bin Huang, Virginia Tech

Many slides from S. Seitz and D. Hoiem
Administrative stuffs

• HW 2 due 11:59 PM on Oct 9th

• Ask/discuss questions on Piazza

• Office hour
Perspective and 3D Geometry

• **Projective geometry and camera models**
  • What’s the mapping between image and world coordinates?

• **Single view metrology and camera calibration**
  • How can we estimate the camera parameters?
  • How can we measure the size of 3D objects in an image?

• **Photo stitching**
  • What’s the mapping from two images taken without camera translation?

• **Epipolar Geometry and Stereo Vision**
  • What’s the mapping from two images taken with camera translation?

• **Structure from motion**
  • How can we recover 3D points from multiple images?
Last Class: Pinhole Camera

\[
P = \begin{bmatrix} u \\ v \end{bmatrix}
\]

Principal Point \((u_0, v_0)\)

Camera Center \((t_x, t_y, t_z)\)

\[
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}
\]

\[
f
\]

\[
Z
\]

\[
X
\]

\[
Y
\]

\[
Z
\]
Last Class: Projection Matrix

\[ x = K[R \ t]X \rightarrow \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & s & u_0 \\ 0 & \alpha f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \]
Last class: Vanishing Points

Vanishing point

Vertical vanishing point (at infinity)

Vanishing line

Vanishing point

Vanishing point
This class

• How can we calibrate the camera?

• How can we measure the size of objects in the world from an image?

• What about other camera properties: focal length, field of view, depth of field, aperture, f-number?
How to calibrate the camera?

\[ x = K \begin{bmatrix} R & t \end{bmatrix} X \]

\[
\begin{bmatrix}
w_u \\
w_v \\
w
\end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ 1 \\
\end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \\
\end{bmatrix}
\]
Calibrating the Camera

Method 1: Use an object (calibration grid) with known geometry
- Correspond image points to 3d points
- Get least squares solution (or non-linear solution)

\[
\begin{bmatrix}
w_u \\
w_v \\
w
\end{bmatrix}
= \begin{bmatrix}
m_{11} & m_{12} & m_{13} & m_{14} \\
m_{21} & m_{22} & m_{23} & m_{24} \\
m_{31} & m_{32} & m_{33} & m_{34}
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix}
\]

Known 2d image coordinates

Known 3d locations

Unknown Camera Parameters
Known 2d image coords

\[
\begin{bmatrix}
    su \\
    sv \\
    s
\end{bmatrix} =
\begin{bmatrix}
    m_{11} & m_{12} & m_{13} & m_{14} \\
    m_{21} & m_{22} & m_{23} & m_{24} \\
    m_{31} & m_{32} & m_{33} & m_{34}
\end{bmatrix}
\begin{bmatrix}
    X \\
    Y \\
    Z \\
    1
\end{bmatrix}
\]

Known 3d locations

\[m_{31}uX + m_{32}uY + m_{33}uZ + m_{34}u = m_{11}X + m_{12}Y + m_{13}Z + m_{14}\]
\[m_{31}vX + m_{32}vY + m_{33}vZ + m_{34}v = m_{21}X + m_{22}Y + m_{23}Z + m_{24}\]
\[0 = m_{11}X + m_{12}Y + m_{13}Z + m_{14} - m_{31}uX - m_{32}uY - m_{33}uZ - m_{34}u\]
\[0 = m_{21}X + m_{22}Y + m_{23}Z + m_{24} - m_{31}vX - m_{32}vY - m_{33}vZ - m_{34}v\]
Unknown Camera Parameters

Known 2d image coords

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

0 = $m_{11}X + m_{12}Y + m_{13}Z + m_{14} - m_{31}uX - m_{32}uY - m_{33}uZ - m_{34}u$

0 = $m_{21}X + m_{22}Y + m_{23}Z + m_{24} - m_{31}vX - m_{32}vY - m_{33}vZ - m_{34}v$

• Method 1 – homogeneous linear system. Solve for m’s entries using linear least squares

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 & -u_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 & -v_1 \\ \vdots \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n & -u_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n & -v_n \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$[U, S, V] = \text{svd}(A)$

$M = V(:, \text{end})$;

$M = \text{reshape}(M, [], 3)'$;
• Method 2 – nonhomogeneous linear system. Solve for m’s entries using linear least squares

\[ \begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \]

Known 3d locations

Ax=b form

\[
\begin{bmatrix}
X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 \\
0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 \\
\vdots \\
X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n \\
0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n
\end{bmatrix}
= \begin{bmatrix}
u_1 \\
v_1 \\
\vdots \\
u_n \\
v_n
\end{bmatrix}
\]

\[ M = \text{A} \backslash Y; \]

\[ M = [M; 1]; \]

\[ M = \text{reshape}(M, [\text{ }] , 3)'; \]
Calibration with linear method

• Advantages
  • Easy to formulate and solve
  • Provides initialization for non-linear methods

• Disadvantages
  • Doesn’t directly give you camera parameters
  • Doesn’t model radial distortion
  • Can’t impose constraints, such as known focal length
  • Doesn’t minimize projection error

• Non-linear methods are preferred
  • Define error as difference between projected points and measured points
  • Minimize error using Newton’s method or other non-linear optimization
Can we factorize $M$ back to $K \begin{bmatrix} R & | & T \end{bmatrix}$?

- Yes!
- You can use $RQ$ factorization
  - (note – not the more familiar $QR$ factorization).

$R$ (right diagonal) is $K$, and $Q$ (orthogonal basis) is $R$.

$T$, the last column of $\begin{bmatrix} R & | & T \end{bmatrix}$, is $\text{inv}(K) \times$ last column of $M$.
  - Need post-processing to make sure that the matrices are valid.
  - See http://ksimek.github.io/2012/08/14/decompose/

Slide credit: J. Hays
Calibrating the Camera

Method 2: Use vanishing points

- Find vanishing points corresponding to orthogonal directions
Calibration by orthogonal vanishing points

• Intrinsic camera matrix
  • Use orthogonality as a constraint
  • Model K with only $f$, $u_0$, $v_0$

For vanishing points

$$p_i = KRX_i$$

$$X_i^T X_j = 0$$

$$X_i = R^{-1}K^{-1}p_i$$

$$p_i^T(K^{-1})^T(R)(R^{-1})(K^{-1})p_i = 0$$

• What if you don’t have three finite vanishing points?
  • Two finite VP: solve $f$, get valid $u_0$, $v_0$ closest to image center
  • One finite VP: $u_0$, $v_0$ is at vanishing point; can’t solve for $f$
Calibration by vanishing points

• Intrinsic camera matrix

$$p_i = KRX_i$$

• Rotation matrix
  • Set directions of vanishing points
    • e.g., $X_1 = [1, 0, 0]$
  • Each VP provides one column of $R$
  • Special properties of $R$
    • $\text{inv}(R) = R^T$
    • Each row and column of $R$ has unit length

$$p_i = Kr_i$$
How can we measure the size of 3D objects from an image?
Perspective cues
Perspective cues
Perspective cues
Ames Room
Comparing heights

Vanishing Point
Measuring height
Which is higher – the camera or the man in the parachute?
The cross ratio

• A Projective Invariant
  • Does not change under projective transformations

The cross-ratio of 4 collinear points

\[ \frac{\mathbf{P}_3 - \mathbf{P}_1}{\mathbf{P}_3 - \mathbf{P}_2} \frac{\mathbf{P}_4 - \mathbf{P}_2}{\mathbf{P}_4 - \mathbf{P}_1} \]

Can permute the point ordering

• \( 4! = 24 \) different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry
Measuring height

scene points represented as

\[
P = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}
\]

image points as

\[
p = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}
\]

\[
\frac{\|B - T\| \|\infty - R\|}{\|B - R\| \|\infty - T\|} = \frac{H}{R}
\]

scene cross ratio

\[
\frac{\|b - t\| \|v_z - r\|}{\|b - r\| \|v_z - t\|} = \frac{H}{R}
\]

image cross ratio

Slide by Steve Seitz
Measuring height

vanishing line (horizon)

\[ v \approx (b \times b_0) \times (v_x \times v_y) \]

\[ t \approx (v \times t_0) \times (r \times b) \]

image cross ratio

\[ \frac{\|t - b\| \|v_z - r\|}{\|r - b\| \|v_z - t\|} \]
Measuring height

vanishing line (horizon)

What if the point on the ground plane $b_0$ is not known?

- Here the guy is standing on the box, height of box is known
- Use one side of the box to help find $b_0$ as shown above
What about focus, aperture, DOF, FOV, etc?
Adding a lens

- A lens focuses light onto the film
  - There is a specific distance at which objects are “in focus”
    - other points project to a “circle of confusion” in the image
  - Changing the shape of the lens changes this distance
A lens focuses parallel rays onto a single focal point
- focal point at a distance $f$ beyond the plane of the lens
- Aperture of diameter $D$ restricts the range of rays
The eye

• The human eye is a camera
  • Iris - colored annulus with radial muscles
  • Pupil (aperture) - the hole whose size is controlled by the iris
  • Retina (film): photoreceptor cells (rods and cones)

Slide source: Seitz
Depth of field

Changing the aperture size or focal length affects depth of field

Varying the aperture

Large aperture = small DOF

Small aperture = large DOF
Shrinking the aperture

- Why not make the aperture as small as possible?
  - Less light gets through
  - Diffraction effects
Shrinking the aperture
Relation between field of view and focal length

Field of view (angle width)  

\[ fov = \tan^{-1} \left( \frac{d}{2f} \right) \]  

Film/Sensor Width  
Focal length
Dolly Zoom or “Vertigo Effect”

http://www.youtube.com/watch?v=NB4bikrNzMk

How is this done?

Zoom in while moving away

http://en.wikipedia.org/wiki/Focal_length
Poltergeist (1982)
Variables that affect exposure

• [http://graphics.stanford.edu/courses/cs178-10/applets/exposure.html](http://graphics.stanford.edu/courses/cs178-10/applets/exposure.html)
Review

How tall is this woman?

How high is the camera?

What is the camera rotation?

What is the focal length of the camera?
Things to remember

• Calibrate the camera?
  • Use an object with known geometry
  • Use vanishing points

• Measure the size of objects in the world from an image?
  • Use perspective cues

• Camera properties
  • focal length,
  • field of view,
  • depth of field,
  • aperture,
  • f-number?
Next class

• Image stitching