Wye – Delta Transformations

Also known as T-Π transformations
Objective of Lecture

- Describe the equations that relate the resistances in a Wye (Y) and Delta (Δ) resistor network.
  - Wye networks are sometimes called T networks and Delta networks are occasionally called Π networks.
  - Chapter 8.8 Principles of Electric Circuits
  - Chapter 2.7 Fundamentals of Electric Circuits
- Describe a bridge circuit in terms of wye and delta subcircuits.
  - Chapter 6.5 Electric Circuit Fundamentals
  - Chapter 8.9 Principles of Electric Circuits
Wye and Delta Networks

- 3 terminal arrangements – commonly used in power systems

Wye (Y)

Delta (Δ)
T and Π

- Drawn as a 4 terminal arrangement of components.
T and \( \Pi \)

- 2 of the terminals are connects at one node. The node is a distributed node in the case of the \( \Pi \) network.
<table>
<thead>
<tr>
<th>To transform a Delta into a Wye</th>
<th>To transform a Wye into a Delta</th>
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</thead>
<tbody>
<tr>
<td>$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$</td>
<td>$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$</td>
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<tr>
<td>$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$</td>
<td>$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$</td>
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<tr>
<td>$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$</td>
<td>$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$</td>
</tr>
</tbody>
</table>
Simplification

- If $R_1 = R_2 = R_3 = R$, then $Ra = Rb = Rc = 3R$
- If $Ra = Rb = Rc = R'$, then $R_1 = R_2 = R_3 = R'/3$
Uses

- Distribution of 3 phase power
- Distribution of power in stators and windings in motors/generators.
  - Wye windings provide better torque at low rpm and delta windings generates better torque at high rpm.
Bridge Circuits

- Measurement of the voltage $V_{CD}$ is used in sensing and full-wave rectifier circuits.

  - If $R_A = R_B = R_C = R_D$, $V_{CD} = 0V$

  - In sensing circuits, the resistance of one resistor (usually $R_D$) is proportional to some parameter – temperature, pressure, light, etc., then $V_{CD}$ becomes a function of that same parameter.
Bridge Circuits (con’t)

- Back-to-back Wye networks
Bridge Circuit (con’t)

- Or two Delta networks where $R_{c1} = R_{c2} = \infty \Omega$. 
Bridge Circuits (con’t)

- Alternatively, the bridge circuit can be constructed from one Delta and one Wye network where $R_c = \infty \Omega$. 
Bridge Circuits (con’t)

- Original circuit redrawn.
  - $V_{CD} = V_C - V_D$
  - If $R_A = R_B = R_C = R$ and $R_D = R - \delta R$
    
    $$V_C = \left[\frac{R}{R_D} + R\right]V_T$$
    
    $$V_D = \left[\frac{R_D}{R_D} + R\right]V_T$$

    $$V_{CD} = V_C - V_D$$

    $$V_{CD} = \left[\frac{(R - R_D)}{(R_D + R)}\right]V_T$$

    $$V_{CD} = \left[\frac{1}{1 - \delta R/R}\right]V_T$$
Summary

- There is a conversion between the resistances used in wye and delta resistor networks.
- Bridge circuits can be considered to be a combination of wye-wye, delta-delta, or delta-wye circuits.
  - Voltage across a bridge can be related to the change in the resistance of one resistor if the resistance of the other three resistors is constant.